Occurrence frequency of subcritical Richardson number assessed by global high-resolution radiosonde and ERA5 reanalysis

Jia Shao¹; Jian Zhang²*; Wuke Wang¹; Shaodong Zhang⁴; Tao Yu²; Wenjun Dong⁵,⁶

¹ College of Informatics, Huazhong Agricultural University, Wuhan 430070, China
² Hubei Subsurface Multi-scale Imaging Key Laboratory, School of Geophysics and Geomatics, China University of Geosciences, Wuhan 430074, China
³ School of environmental studies, China University of Geosciences, Wuhan 430074, China
⁴ School of Electronic Information, Wuhan University, Wuhan 430072, China
⁵ Center for Space and Atmospheric Research (CSAR), Embry-Riddle Aeronautical University, Daytona Beach, FL, USA
⁶ Global Atmospheric Technologies and Sciences (GATS), Boulder, CO, USA

Correspondence to:
Dr. Jian Zhang (Email: zhangjian@cug.edu.cn)
Abstract. Kelvin Helmholtz instability (KHI) is most likely to be the primary source for clear-air turbulence that is of importance in pollution transfer and diffusion and aircraft safety. It is indicated by the critical value of the dimensionless Richardson ($Ri$) number, which is predicted to be $1/4$ from linear stability analysis. However, $Ri$ is fairly sensitive to the vertical resolution of the dataset; a higher resolution systematically leads to a finer structure. The study aims to evaluate the performance of ERA5 reanalysis in determining the spatial-temporal variabilities of subcritical $Ri$ by comparing it against a near-global high-resolution radiosonde dataset during years 2017 to 2022 and further highlight global climatology and dynamical environment of subcritical $Ri$. Overall, the occurrence frequency of $Ri<1/4$ is inevitably underestimated by the ERA5 reanalysis over all climate zones at all heights from near-ground up to 30 km, compared to radiosonde, due largely to the severe underestimation in wind shears. Otherwise, the occurrence frequency of $Ri<1$ in ERA5 is climatologically consistent with that from $Ri<1/4$ in radiosondes in the free troposphere, especially over the midlatitude and subtropics in the Northern/Southern Hemisphere. Therefore, we argue that threshold value of $Ri$ could be approximated as 1 rather than $1/4$ when using ERA5-based $Ri$ as proxy for KHI. The occurrence frequency of subcritical $Ri$ revealed by both datasets exhibits significant seasonal cycles over all climate zones. In addition, it is positively correlated with the standard derivation of orography at low-levels and is exceptionally strong over the Niño 3 region at heights of 6–13 km. Furthermore, high occurrence of subcritical $Ri$ would likely be accompanied by strong wind speeds and intensive orographic or non-orographic gravity waves.

Key words: High-resolution radiosonde; ERA5 reanalysis; Wind shears; Richardson number; Gravity waves
Introduction

Kelvin Helmholtz instability (KHI) is a common phenomenon in the atmospheric boundary layer and the free atmosphere (Muschinski and Wode, 1998), and its wavelengths and depths span a wide range of scales throughout the atmosphere, varying from few meters or less to 10s of km (Fritts et al., 2011). It contributes to vertical mixing of heat, momentum, and constituents, and it acts to limit the maximum shears, just to name a few (Fritts et al., 2011). KHI along with gravity wave (GW) breaking are the most recognized instabilities in stably stratified flows (Fritts and Rastogi, 1985). KHI arises preferentially from micro- and mesoscale wind shear intensification, with maximal occurrence frequency near synoptic scale upper-level frontal zones near jet streams, with mountain waves, and above the tops of severe thunderstorms (North et al., 2014). Large wind shear is commonly associated with regions where stability changes rapidly (e.g., near the top of the boundary layer) and the large wind gradient in jet stream (Grasmick and Geerts, 2020). In a changing climate, wind shear in the North Atlantic upper-level jet stream could be increased (Lee et al., 2019), which may increase clear-air turbulence at cruise altitudes. In turn, KHI can reduce wind shears and alter tracer gradients where turbulence and mixing are most intense (Fritts et al., 2022).

KHI influences depend on the spatial scales at which they lead to turbulence (Fritts et al., 2022). Turbulence is by far the most common cause of serious injuries to aircraft (Williams and Joshi, 2013). Convective instability, shear instability, KHI, and GW breaking are known to be the major sources for turbulence (Sharman et al., 2012; Ko et al., 2019; 2022; Lazarus et al., 2021). KHI requires a sufficiently large Reynolds number and a Richardson (Ri) number sufficiently below 1/4 to enable KHI formation and subsequent secondary instability leading to turbulence (Fritts et al., 2022). Ri is not a good guide to instability character in general, and Ri>1/4 does not assure flow stability for superpositions of mean and GW motions. Despite these caveats, Ri<1/4 does provide a reasonable guide to expected local KHI structure in cases where clear KH
billows arise, according to the simulation in the mesosphere and lower thermosphere region (Fritts et al., 2014). Values of $Ri$ close to zero favor strong instability, deep billows, and relatively intense turbulence, whereas values of $Ri$ closer to 1/4 favor weak instability, shallow billows (Fritts et al., 2011). The Richardson number criterion can be applied as a turbulence diagnostic in numerical model output (e.g. Sharman and Pearson, 2017), and it has been used as such in climatological studies on the occurrence of clear air turbulence (Jaeger and Sprenger, 2007). Kunkel et al. (2019) includes a brief discussion on the capability of ECMWF models based on case studies to resolve subcritical Richardson numbers, and argues that the threshold value of $Ri$ ($Rit$) taken as 1 might be a good proxy for observed KHI. A very recent study by Lee at al. (2023) also sets $Rit$ from 0–1 in their climatology on the upper troposphere and lower stratosphere turbulence diagnostics. Moreover, Zhang et al. (2022) shows that over half of turbulence exists below $Ri<1$ when the environment is beneficial for the development of turbulence.

Turbulent mixing is of crucial importance to mass, energy, momentum transfer, the dispersion of pollutants, and stratosphere-troposphere exchange. In numerical models, turbulent dissipation rate, turbulent diffusivity and other parameters representing turbulent mixing efficiency are the most basic parameters, which need to be accurately parameterized to evaluate the impact of turbulence effect on matter and energy distribution (Gavrilov et al., 2005). However, due to the intermittent nature of turbulence it is generally not resolved in (global) numerical weather prediction models, even at nowadays common/states of the art horizontal resolutions of the order of tens of kilometers (Sandu et al., 2019), and it presents a challenge both in observation and numerical modeling (Sharman et al., 2012; Homeyer et al., 2014; Plougonven and Zhang, 2014). For this reason, the indices of turbulence, such as large wind shear, small $Ri$ and the negative squared Brunt-väisälä frequency, could be a great tool to characterize turbulence (Jaeger et al., 2007).

The Richardson number is estimated by the finite differences across thin layers and is quite sensitive to the vertical resolution of measurements (Haack et al., 2014). Thus, a proper estimation of $Ri$ requires a high-resolution measurement of temperature and
wind speed. The near-global distributed radiosonde site offers a unique opportunity to investigate the climatology of subcritical Ri occurrence frequency. The overview of subcritical Ri occurrence by using a near-global high-resolution (10-m) radiosonde data was presented in Zhang et al. (2022), and a close association between subcritical Ri occurrence frequency and turbulence fraction has been found. However, the global climatology characteristic of subcritical Ri remains most unclear, especially over oceans where the radiosonde network has a poor coverage.

By comparison, ERA5 global reanalysis can provide a seamless coverage of temperature and wind, and it is the latest generation latest version of the European Centre for Medium-Range Weather Forecasts (ECMWF) atmospheric reanalysis and is based on the state-of-the-art Integrated Forecasting System (IFS) Cy41r2 (Hersbach et al., 2020; Gu et al., 2023) has 137 model levels (Hersbach et al., 2020). Its predecessor, ERA–Intrin, was found in particular wind shear a factor of 2–3 lower simulated based on high-resolution radiosondes (Houchi et al., 2010). Moreover, results show that whatever the location and the geophysical conditions considered, biases between ERA-Interim and balloon wind measurements increase as a function of altitude (Duruisseau et al., 2017). Recent studies have suggested that the structure and variability of the trade winds in the lower troposphere are reasonably reproduced in the ERA5 reanalysis based on the EUREC4A field campaign (Savazzi et al., 2022). However, the similar comparison between ERA5 and high-resolution radiosonde across a near-global area has largely been undetermined. The proper estimation of wind shear and Brunt-Väisälä frequency is essential for the determination of Ri.

Thus, our objectives are to: (1) Evaluate the performance of ERA5 at different heights and climate zones in estimating wind shear and small Richardson number occurrence frequencies, in comparison with a large high-resolution radiosonde dataset spanning the years from 2017 to 2022. (2) Based on the validation and comparison results, we pose a question: how to use ERA5 for subcritical Ri estimation? (3) The global climatology of subcritical Ri occurrence based on versatile measurements and model products. (4) The dynamic environment (GWs and mean flow) of subcritical Ri. These works would be valuable for the understanding of the global distribution of
subcritical $R_i$, and furthermore, turbulence fraction. To this end, this analysis is organized as follows. Section 2 shows the data and methods used. Section 3 represents the climatological variation of subcritical $R_i$ and its comparison with radiosonde. Section 4 ends with a summary.

2 Data and methods

2.1 High-resolution radiosonde dataset

As described in Guo et al. (2021) and Zhang et al. (2022), a high vertical resolution radiosonde (HVRRS) dataset gained from several organizations was adopted, spanning January 2017 to October 2022, in a total of 5.8 years. The organizations include the China Meteorological Administration (CMA), the U.S National Oceanic and Atmospheric Administration (NOAA), the Global Climate Observing System (GCOS) Reference Upper-Air Network (GRUAN), the Centre for Environmental Data Analysis of the United Kingdom (CEDA), University of Wyoming, Deutscher Wetterdienst, and ECMWF. In total, around 0.95 million radiosonde profiles from 434 radiosonde stations released at regular synoptic times of 0000 UTC and 1200 UTC were collected to determine the value of $R_i$. These profiles were sampled at 0.5 Hz or 1 Hz, corresponding to a vertical resolution of approximately 10 m or 5 m. Thus, all the profiles were evenly interpreted to 10 m resolution in vertical by applying a cubic spline interpolation. In addition, the sounding with the burst height lower than 10 km above ground level (a.g.l.) was directly discarded for further study. Meteorological variables, including temperature and wind speed, were prepared for the $R_i$ estimation.

One of the shortages of radiosonde measurements is its inadequate concentration over the polar and ocean regions (Xia et al., 2021). The geographical distribution of total profile number of each radiosonde station is demonstrated in Figure S1 in the supporting information. The released radiosoundings over Europe, the United States, and Australia have good geographical coverage and time duration. Over some islands
of oceans (e.g., the Pacific Ocean) there are dozens of stations that can provide high-resolution measurement. Over the polar regions, there are around thirty stations.

2.2 ERA5 reanalysis and the collocation procedure

ERA5 is the latest version of ECMWF meteorological reanalysis, benefiting from a decade of developments in model physics, core dynamics, and data assimilation (Hersbach et al., 2020). The wind and temperature fields are modelled by the ERA5 reanalysis on a spatial resolution of 0.25° latitude/longitude and a temporal resolution of 1 hour. The reanalysis has 137 model levels, giving a vertical resolution of approximately 300 m in the middle and upper troposphere. The vertical resolution of ERA5 is illustrated in Figure S2. Compared to ERA5, the HVRRS does not provide global seamless observations. Thus, the collocation procedure between reanalysis and HVRRS goes as follows: (1) the matched grid of ERA5 reanalysis is the nearest neighbor of radiosonde station; (2) the regular synoptic start time of radiosonde and reanalysis needs to keep exact the same; (3) the pressure coordinate of model level of reanalysis that follows a hybrid sigma-pressure coordinate, is converted into geometric geopotential height/altitude to match with HVRRS.

In addition, the standard deviations of orography (SDOR) and the gravity wave dissipation due to the effects of stress associated with unresolved valleys, hills and mountains in ERA5 reanalysis are extracted.

The relative error between HVRRS-based and ERA5-based quantities is estimated by the ratio of deviations between HVRRS and ERA5 derived quantities to the HVRRS one.

2.3 The occurrence frequency of subcritical $Ri$ and its uncertainty

Based on a linear theory, the threshold $Ri$ ($Ri_t$) defines the threshold where the air flow changes from stability to turbulence, and it is usually suggested to be $1/4$ (Haack et al., 2014). $Ri$ is formulated as:

$$ Ri = \frac{N^2}{S^2} \quad (1) $$
where $N$ is the Brunt-Väisälä frequency ($\sqrt{\frac{g d \theta}{\theta dz}}$), $S$ is the vertical wind shear ($\sqrt{\frac{d u}{dz}^2 + \frac{d v}{dz}^2}$), and the overbar denotes a moving average in a 200-m bin step to eliminate the influence of measurement noises and small-scale fluctuations, such as turbulence and small-scale waves. For a vertical resolution of 10-m, the averaged parameter at altitude $i$ can be represented as $\overline{A(i)} = \frac{1}{N} \sum_{i=10}^{i+10} A(i)$, where $A$ denotes wind shear or Brunt-Väisälä frequency and $N$ is the number of vertical bin. In addition, horizontal winds measured under radiosonde at the scale of a few tens of meters are affected by the chaotic movements of the gondola due to the pendulum and to the balloon's own movements (Ingleby et al., 2022). However, it is hard to quantify the movement in present study. For 10-m radiosondes, the moving average in a step of 200 m could offset the effect of chaotic movements, at least to some extent. In this case, the matching quantities that include $Ri$, wind shear, and the Brunt-Väisälä frequency between radiosonde and ERA5 profiles are actually handled in averaged 200 m intervals. The Richardson number calculated from Eq.(1) depends on the vertical resolution of the underlying data, as well as on the averaging interval. Ultimately, this influences the estimated occurrence frequency for subcritical Richardson numbers as a proxy for KHI. We resample the HVRRS data to 50 m and 100 m, and range the length scale of overbar from 100 m to 500 m, to diagnose the uncertainties raised by the length scale of segments and the vertical resolution of dataset. As indicated in Figure 1, under the same length scale of overbar, a sparser vertical grid inevitably leads to a lower occurrence frequency of subcritical $Ri$. For instance, as the length scale set to 100 m, the occurrence frequency of $Ri<1/4$ at 0–2 km above sea level (a.s.l.) decreases from 22% when vertical resolution is equal to 10 m to 16% for a vertical resolution of 50 m. Moreover, a longer length-scale of segment generally yields a smaller occurrence frequency. For example, as the vertical resolution of radiosonde is equal to 10 m, the occurrence frequency at 10–15 km decreases from 9% when the length scale of segment equals 100 m to 1% when it equals 500 m. It is interesting to note that the occurrence frequency under a vertical resolution of 50 m and a segment interval of 100 m is a bit
larger than that under a vertical resolution of 10 m and a segment of 200 m, possibly
implying the fact that a shorter segment interval could be expected for a sparser vertical
resolution.

2.4 Gravity wave energy

The GW energy is extracted based on the broad spectral method, according to Wang
and Geller (2003). In this method, the magnitude of measured zonal wind (u),
meridional wind (v), and temperature (T) consisting of background states (u₀, v₀ and
T₀) that are determined by applying a second-order polynomial fit (Chen et al., 2018;
Zhang et al., 2022) and perturbations. Therefore, total perturbations are derived as:

\[(u', v', T') = (u, v, T) - (u_0, v_0, T_0)\]  \(2\)

The perturbations could include measurement noises, KH waves, GWs, and
planetary waves. Only the perturbations with vertical wavelengths of 0.3–6.9 km are
considered as GWs (Wang and Geller, 2003). The mean vertical wavelength of GWs is
about 2 km (Wang et al., 2005), and therefore, the lowermost threshold of 0.3 km could
have little influence on the GW energy. However, the retrieval of the largest wavelength
is not well determined, which is acknowledged as the radiosonde’s “observational filter”
(Alexander, 1998). By applying this band-pass filter, the average gravity-wave kinetic
energy per unit mass (energy density) and the average potential energy density can be
expressed as:

\[E_k = \frac{1}{2} \left[ \overline{u'^2} + \overline{v'^2} \right]\]  \(3\)
\[E_p = \frac{1}{2} \frac{g \overline{T'}^2}{N^2}\]  \(4\)

where \(g\) is the gravitational constant, \(\overline{T'} = T'/\overline{T}\) the normalized perturbation
temperature, and the overbar indicates an averaging over the tropospheric segment,
which is chosen as 2–8.9 km for all regions, expect the polar region, and it is selected
as 2–7.4 km for the polar region (Wang and Geller, 2003). Eventually, the total GW
energy \(E_t\) is the sum of \(E_k\) and \(E_p\).
3 Results and Discussions

3.1 Comparisons of wind shear between HVRRS and ERA5 reanalysis

The variations in vertical shear of horizontal wind speed and the squared Brunt-väisälä frequency entirely determine the Ri magnitude. Figure 2 provides an overview of the spatial distribution of wind shear at heights of 0–2 km a.g.s.l. and 10–15 km a.g.s.l. obtained from the HVRRS and ERA5 reanalysis. HVRRS-based wind shear is taken from Eq.(1), with a vertical resolution of 10 m. The shear at heights of 0–2 km a.g.s.l. estimated by ERA5 reanalysis demonstrates a strong spatial variation, and it is largely dependent on underlying terrains and latitudes (Fig.2a). For example, large values can most likely be observed along the coastline, which could be attributed to the prevailing sea-breeze circulation. As compared to the HVRRS, these shears are slightly underestimated by 3.30±5.37 m/s/km, based on all sounding measurements (Fig.2b). Nevertheless, a close association between averaged ERA5-reterived shears and HVRRS-determined shears can be noticed in terms of geospatial distribution, with a correlation coefficient of 0.48±36 (Fig. 2b).

It is noteworthy that shear in the ERA5 reanalysis at heights of 10–15 km a.g.s.l. is significantly underestimated compared to the HVRRS, especially at middle latitudes, with a mean absolute error for all stations of about 87.40±8 m/s/km (Table 1). The underestimation could partly be due to the coarse vertical resolution (around 300-m) in the ERA5 reanalysis in this height interval. However, the spatial distribution of the ERA5 shear still exhibits a significant positive correlation with the HVRRS shear, with a correlation coefficient of 0.44±35 (Fig.2d).

Following Houchi et al. (2010), the monthly shears over seven typical climate zones are separately investigated (Fig. 3), which are defined as follows: polar (70°–90°), mid latitudes (40°–70°), subtropics (20°–40°), and tropics (20°S–20°N). Over the polar region in the Northern/Southern Hemisphere, HVRRS-based shears are exceptionally strong in the lower stratosphere compared to those in the troposphere (Fig.3a, g), which could be attributed to the stratospheric polar jet. However, the similar
altitude variation can hardly be found in ERA5-based shears that are dramatically underestimated by around 12.44 m/s/km in the lower stratosphere (Fig. 3h, n, also seen in Table 1). The results in midlatitudes reach a similar conclusion (Fig. 3b, f, i, m). Over subtropical regions, HVRSS-based shears are consistent strong at heights of 16–21 km a.g.l., just above the subtropical jet stream (Fig. 3c, e). However, in the ERA5 reanalysis, the region with consistently strong shears can be noticed at approximately 16 km a.g.l. (Fig. 3j, l), which is about 3 km lower than that in the HVRSS. One possible reason might be that the model fails to resolve the further increasing shear in the lower stratosphere. In the tropics, the signature of quasi-biennial oscillation (QBO) can be identified in the lower stratosphere (Fig. 3d, k).

The comparison between HVRSS-based and ERA5-based shears at three typical regimes is tabulated in Table 1. These metrics highlight that ERA5-based shears are underestimated by approximately 3.92, 4.24, 7.65, 8.47, 11.99, and 12.45 m/s/km at heights of 0–2 km, 10–15 km, and 20–25 km a.g.l., respectively, which are roughly consistent with Houchi et al. (2010).

By comparison, the ERA5-acquired $N^2$ averaged over four height intervals (e.g., 0–5, 5–10, 10–15, 15–20 km a.g.l.) from the surface to 30 km a.s.l. is reliably estimated at all heights, with a relative error of around 11.4%, as illustrated in Figure S3. This finding indicates that the ERA5 reanalysis can properly present the static stability of the background atmosphere, but it is not properly coincident with radiosonde in terms of the small-scale variability of dynamical structures. Due to a lack of global measurement of the fine-structure of the upper-air wind, however, the accuracy of ERA5-resolved shears is hard to be globally validated.

### 3.2 Occurrence frequency of $R_i<1/4$ in HVRSS and ERA5 reanalysis

As a prominent example, the monthly occurrence frequency of $R_i<1/4$ over the Corpus Christi station (27.77° N, −97.5° W) during years from January 2017 to October 2022 is illustrated in Figure 4. As a result, the monthly occurrence rate of $R_i<1/4$ in the planetary boundary layer (PBL) regime low troposphere determined from
HVRRS is lower than the ERA5-based one, with mean values of around 10.6% and 16.9%, respectively. In the lowermost 2 km, the vertical resolution of ERA5 reanalysis is less than 200 m, and it is less than the moving segment interval in Eq.(1). The high occurrence frequency in the low troposphere (PBL) regime could be likely related to the negative or small $N^2$. Especially during the daytime, the planetary boundary layer (PBL) is well mixed due to strong turbulence induced by uprising thermals (Song et al., 2018). In addition, an obvious seasonal cycle of occurrence frequencies is revealed by HVRRS in the middle and upper troposphere and has a maximum in winter (December–January–February) and spring (March–April–May) seasons, which is consistent with the finding in Zhang et al. (2019). In the vicinity of jet streams, the occurrence frequency of $Ri<1/4$ is generally enhanced by large wind shears. However, the ERA5 reanalysis does not provide such a seasonal cycle pattern, and the occurrence frequency of $Ri<1/4$ is significantly underestimated by around 8% (Fig.4b), which could be attributed to the underestimation in wind shears. In the lower stratosphere, both the HVRRS and ERA5 reanalysis provide a low estimation of occurrence frequencies, with a value of around 1%. Furthermore, on a large spatial scale the occurrence frequency of $Ri<1/4$ retrieved by ERA5 reanalysis is remarkably underestimated in the free atmosphere, as compared to the HVRRS. The annual variation of the occurrence frequency of $Ri<1/4$ over seven climate zones at 10–15 km a.g.l. at 10 to 15 km a.s.l. indicated by HVRRS and ERA5 reanalysis is further demonstrated in Figure 5. It is clearly seen that the occurrence frequency of $Ri<1/4$ provided by ERA5 reanalysis is underestimated in all months, over all climate zones, possibly implying that, in the free atmosphere, the threshold value of 1/4 in Eq.(1) is too small for the ERA5 reanalysis to capture the occurrence of KHI. However, the ERA5 reanalysis data is non-uniformly sampled in altitude. Its vertical resolution drops from about 100-m in the boundary layer to about 500-m in the lower stratosphere. In contrast, radiosondes have a vertical resolution of 10-m at all heights. Therefore, we selected four typical heights and vertically interpolated the radiosonde to the same height resolution as ERA5 for comparison. The four height intervals are 0.8–1.3 km, 2.2–3.2 km, 6–15 km and 20–21 km a.g.s.1., as shown in Table
In these height intervals, the vertical resolution of ERA5 is about 100-m, 200-m, 300-m and 400-m respectively. Even at the same vertical resolution, ERA5 still seriously underestimates the value of \( OF(Ri<1/4) \) at all heights and all climate zones. These results indicate that the greatest difficulty in evaluating subcritical \( Ri \) with ERA5 is that its simulation of wind shears might be seriously underestimated compared with radiosonde. As illustrated in Table 3, even accounting for the fact that ERA5 has a comparable vertical resolution of radiosonde, wind shears in ERA5 reanalysis are still underestimated by around 50.31\%-48.74\%-43.64\%-62.25\% at 0.8–1.3 km, 2.2–3.2 km, 6–15 km and 20–21 km a.g.s., respectively. In order to obtain an occurrence frequency of subcritical \( Ri \) from ERA5 reanalysis that is comparable with radiosonde-based \( OF(Ri<1/4) \), the \( Rit \) for ERA5 should be set larger than 1/4. For instance, at 0.8–1.3 km and 2.2–3.2 km a.g.s., the \( Rit \) equals 1 could be a proper choice for ERA5 reanalysis, rather than 1/4 (Table 2). More generally, \( 0.5 < Rit < 1.5 \) could be more suitable for ERA5 reanalysis, compared to \( Rit = 1/4 \).

Due to the huge change in the vertical resolution of ERA5, it could be difficult to interpolate ERA5 into uniform data vertically with a relatively high resolution. Therefore, the question posed here is, what is the proper threshold value of \( Rit \) in predicting the occurrence of KHI when using the ERA5 reanalysis, compared to HVRRS? The occurrence frequency of \( Rit < 1/4 \) indicated by the HVRRS, the ERA5-determined occurrence frequencies produced by \( Rit < 0.25, Rit < 0.5, Rit < 1, Rit < 1.5, \) and \( Rit < 2 \) at all heights up to 30 km a.g.s. are demonstrated in Figure 6. It is notable that over all climate zones and in the free atmosphere, occurrence frequencies of \( Rit < 0.25 \) and \( Rit < 0.5 \) obtained from the ERA5 reanalysis are underestimated, but the frequencies of \( Rit < 1.5 \) and \( Rit < 2 \) are generally overestimated. The occurrence frequency of \( Rit < 1 \) gives a close estimation both in magnitude and spatial variation compared to HVRRS over all climate zones.

Furthermore, the correlation coefficients between HVRRS-determined occurrence frequency and the ERA5-determined frequencies indicated by different threshold values of \( Rit \) at height levels of 0 to 30 km are illustrated in Figure 7. It is worth noting that, in the troposphere, the ERA5-based frequencies indicated by \( Rit < 1, \)
$Ri < 1.5$, and $Ri < 2$ are highly positively correlated with those from the HVRRS, with a correlation coefficient of around 0.6 over all climate zones. In the lower stratosphere, however, these coefficients rapidly decline to 0.1, which can be explained by the low occurrence frequency in this height regime.

Combined the findings in Figures 6 and 7, in the free troposphere, we can conclude that the ERA5-determined occurrence frequency of $Ri < 1$ is closest to the frequency of $Ri < 1/4$ based on the HVRRS. In the free atmosphere, KHI is the dominant source for clear-air turbulence (CAT) that is a well-known hazard to aviation. Therefore, the global characterization of KHI occurrence frequency in the free atmosphere obtained from ERA5 reanalysis could be of importance for understanding the spatial-temporal variation of CAT. In the following sections, the occurrence frequency of subcritical $Ri$ (hereinafter $OF(Ri < Rit)$) is based on $Ri < 1$ in ERA5 reanalysis and $Ri < 1/4$ in HVRRS, unless otherwise noted.

Finally, it is noteworthy that $OF(Ri < Rit)$ includes the component of $Ri < 0$ that indicates potential for convective instability. However, both ERA5 and HVRRS are difficult to totally avoid $Ri < 0$ when calculating $Ri$. Therefore, we evaluated the proportion of $Ri < 0$ in all $Ri < Rit$ in the two datasets to evaluate the possible contribution from convection, as shown in Figure 8. For HVRRS, the proportion of $OF(Ri < 0)$ drops sharply from about 40% in the PBL region-low troposphere to about 18% at 5–15 km a.g.l. Similarly, for ERA5 its proportion drops from about 40% in the lowermost part of the atmosphere to about 2% at 5–16 km a.g.l. These findings indicate that, in the free atmosphere, $OF(Ri < Rit)$ is mainly composed of $OF(0 < Ri < Rit)$, which implies that local instabilities constitute most of the dynamic instability.

### 3.3 The $OF(Ri < Rit)$ climatology

For a first hint the global distributions of $OF(Ri < Rit)$ provided by the ERA5 reanalysis at 0–2 km, 5–10 km, 10–15 km, and 15–20 km a.g.l. and 10–15 km a.s.l. are displayed in Figure 9. $OF(Ri < Rit)$ in the PBL region-low troposphere is considerably spatially heterogeneous. Over subtropical oceans in the Northern/Southern Hemisphere,
the intense $OF(Ri<Rit)$ can be noticed and has a magnitude of around 50% (Fig. 9a). In addition, over the Sahara Desert the $OF(Ri<Rit)$ reaches as high as 65%. Interestingly, the spatial variation in $OF(Ri<Rit)$ ensembled by years 2017 to 2022 keeps high consistency with that of planetary boundary layer height (PBLH) over oceans, such as the Pacific Ocean near Japan and the Atlantic Ocean near U.S., as shown in Figure S4. However, at 0–2 km a.g.s.l., the spatial variation of $OF(0< Ri < Rit)$ exhibits a large difference with that of $OF(Ri<Rit)$ in terms of magnitude, as shown in Figure S5. It is around 40% (20%) lower than that of $OF(Ri<Rit)$ over subtropical oceans (Australia and North Africa). At heights of 5–10 km a.g.s.l., intensive $OF(0< Ri < Rit)$ can be viewed over the subtropic regions and has a value of around 10% (Fig. 9b), which is likely attributed to upper tropospheric jets. In the upper troposphere over the tropical oceans, $OF(Ri<Rit)$ is as high as 30% (Fig. 9c), possibly as a result of the maximal heating effect by mesoscale convective systems (e.g., Houze 1982). In the lower stratosphere, $OF(Ri<Rit)$ sharply decreases to around 0.1% (Fig. 9d).–

In comparison, the spatial-temporal variability of $OF(Ri<Rit)$ indicated by HVRRS keeps highly consistency with that of ERA5 reanalysis over all climate zones and in the free troposphere, except in the stratosphere of polar region (Figure 10). Seasonal cycles can be detected by both the HVRRS and ERA5 reanalysis over all climate zones, especially over subtropics and midlatitude regions. However, the ERA5-based $OF(Ri<Rit)$ can only reflect the large scale structure of the cycles, and it is hard to quantify the detailed variation like the HVRRS does.

Furthermore, the seasonal variation of $OF(Ri<Rit)$ with $Rit=1/4$ for HVRRS and $Rit=1$ for ERA5 for all climate zones is further analyzed in Figure 11. In the midlatitudes and subtropics, the $OF(Ri<Rit)$ exhibits maximum values in the low troposphere, as well as a local minimum in the middle troposphere and a local maximum at altitudes around 9 km. In the stratosphere the occurrence frequencies decrease to values of the order of 1% (Fig. 11b,c,e,f). Over tropic regions, a primary peak can be clearly noticed at around 13 km, with a maximum of 12% for the HVRRS and 20% for the ERA5 reanalysis (Fig. 11d, k). The seasonality over the tropical region could be related to some large scale flow features like the Summer Asian Monsoon and...
the tropical easterly jet (Rojas Raman et al., 2009; Sunilkumar et al., 2015; Kaluza et al., 2021). Over polar regions, the tropospheric $OF(Ri<\text{Rit})$ is significantly lower than that over other climate zones, with values ranging from around 4% at heights of 2–8 km to 1% in the lower stratosphere (Fig.11a,g).

In Table 4, the mean $OF(Ri<\text{Rit})$ magnitudes over seven climate zones and at three typical altitude regimes are listed. At 0–2 km a.g.l., the ERA5-based $OF(Ri<\text{Rit})$ is about 24% larger than that of the HVRSS-based one. At 10–15 km a.g.l., the ERA5-based $OF(Ri<\text{Rit})$ is reasonably well estimated, except that it is overestimated by around 5.928% over the tropics region. In addition, ERA5 underestimates $OF(Ri<\text{Rit})$ by around 0.5% in the lower stratosphere.

According to Fig.9a, it seems that low-level continental $OF(Ri<\text{Rit})$ is dependent on underlying terrain. W. However, the vertical resolution of ERA5 in the PBL decreases sharply, leading to the fact that the resolution of the PBL data over the region with high elevations can be significantly lower than that of regions with low elevations, which could bring great challenges to the analysis of the impact of topography on low-level $OF(Ri<\text{Rit})$. Therefore, we investigate the association of low-level HVRSS-determined $OF(Ri<\text{Rit})$ with the standard deviation of orography (SDOR). At heights of 1–2 km a.g.l., the underlying terrain with a large SDOR generally corresponds to a high $OF(Ri<\text{Rit})$, with a correlation coefficient between $OF(Ri<\text{Rit})$ and SDOR of 0.24. Then, the coefficient decreases to 0.15 at 3–4 km a.g.l. (Fig.12b), and eventually, it equals 0.14 at 5–6 km a.g.l. (Fig.12c). These findings indicate that, complex terrain may locally enhance $OF(Ri<\text{Rit})$.

Moreover, it is quite evident from Fig.9b and Fig.S5 that both $OF(Ri<\text{Rit})$ and $OF(0<\text{Ri}<\text{Rit})$ are largely enhanced over the tropical ocean associated with the El Niño Southern Oscillation (ENSO). The most of the enhanced $OF(Ri<\text{Rit})$ can be identified over the Niño 3 region ($5^\circ$ N–$5^\circ$ S, 150$^\circ$ W–90$^\circ$ W), and the time-height cross section of $OF(Ri<\text{Rit})$ during years of 2000 to 2022 is illustrated in Figure 13. The $OF(Ri<\text{Rit})$ at height region of 6–13 km are evidently large, with values of around 3540%, which is about 1520% larger than the climatological mean value (Fig.10a). More specifically, $OF(Ri<\text{Rit})$ during time periods of La Niña events is obviously stronger than that during
the El Niño periods. The identification of ENSO events is based on Ren et al. (2018), Li et al. (2022), and Lv et al. (2022). It is also worth recalling here that the wind shear does not exhibit such an anomaly over the Niño 3 region (Fig.2c), implying that the $OF(Ri<R_l)$ anomaly could likely be attributed to the ENSO-related tropical convective heating in the upper troposphere, leading to a low Brunt-Väisälä frequency.

3.4 The dynamical environment of $OF(Ri<R_l)$ in the free troposphere

In the free troposphere, the percentage of $OF(Ri<0)$ relative to $OF(Ri<R_l)$ is generally less than 20% (Fig. 8), where the convection activity is generally weak, KHI is preferentially generated from strong wind shear, which may be closely associated with mean flows and wave activities. The propagation of GW could raise strong wind shear, and therefore generate KHI. Thereby, we investigate the joint distribution of $OF(Ri<R_l)$ with tropospheric GW total energy and wind shear (Figure 14). The latitudinal variation of GW total energy exhibits a double-peak structure, with two peaks at around 30° in the Northern/Southern Hemisphere (Fig.14a). The joint distribution of $OF(Ri<R_l)$ with GW energy and wind shear indicates that large $OF(Ri<R_l)$ (for instance, larger than 10%) generally corresponds to GW energy larger than 10 J/kg or wind shear exceeds 14 m/s/km (Fig. 14b). Also, $OF(0<Ri<R_l)$ exhibits a similar distribution (Figure S6). Overall, $OF(Ri<R_l)$ obviously increases with GW total energy (Figure S9a), possibly implying that the propagation of GWs could enhance wind shear and therefore, the burst of KHI.

In addition, the interaction between low-level wind and mountain barrier could be a source of orographic GWs (Zhang et al., 2022). We take orographic GW dissipation in ERA5 reanalysis, which is the accumulated conversion of kinetic energy in the mean flow into heat over the whole atmospheric column, as an indicator of the strength of orographic GWs. It is interesting to note that monthly averaged orographic GW dissipation and monthly ERA5-determined $OF(Ri<R_l)$ at heights from ground up to 30 km demonstrates a close association (Figure S7). For instance, in the middle troposphere, they are positively associated over mountainous areas such as the Rocky
Mountains and the Alps Mountain, with correlation coefficients of around 0.5. These findings also suggest that during months with strong unresolved orographic gravity wave activity, which then modify the flow and stability parameters of the resolved flow, leading to a low Richardson number and result in an enhanced OF(Ri<\text{Rit})\text{.} Nevertheless, it is hard to quantify the effect of resolved orographic GWs on Ri here.

At jet heights (10–15 km a.g.l.), a large shear can be easily induced by strong wind speed. Figure 15 demonstrates the joint distribution of OF(Ri<\text{Rit}) with wind speed and wind shear. Generally, similarly, large OF(Ri<\text{Rit}) larger than 10\% (>10\%) can be easily found when the wind speed–shear exceeds 20–25 m/s/km. In addition, OF(0<\text{Ri}<\text{Rit}) can draw a similar conclusion (Figure S8). In the middle and upper troposphere, OF(Ri<\text{Rit}) almost linearly increases with wind speed (Figure S9b).

In a short conclusion, in the free troposphere, the occurrence of KHI would favor the dynamical environment with strong-intensive orographic or non-orographic GW activities and relatively large mean flow\text{(}around 25 m/s\text{)}.

4 Conclusion and remarks

The occurrence of KHI is potential crucial for many implications, such as aircraft safety and mass transfer, but it is very hard to be globally understood due to its fine structure. The subcritical Richardson number is commonly used as an indicator for KHI. This study uses the ERA5 as the latest reanalysis product from the ECMWF as well as a comprehensive data set of HRRS radiosonde soundings to globally characterize the distribution of low Richardson numbers as a proxy for the occurrence of KHI, for the years 2017 to 2022.

Vertical wind shears are considerably underestimated at almost all heights and over all climate zones by the ERA5 reanalysis, compared to the HRRS. It is noteworthy that vertical wind shear in the ERA5 reanalysis at heights of 10–15 km a.g.l. is dramatically underestimated by around 7.65 m/s/km, especially at middle latitudes. However, the spatial distribution of the ERA5 shear exhibits a statistically significant
positive correlation with the HVRRS shear. As a result, the ERA5-determined occurrence frequency of $Ri<1/4$ is significantly underestimated. In addition, it is weak correlated with HVRRS-determined ones at most heights and over most climate zones. However, the vertical resolution of ERA5 reanalysis sharply decreases with altitude, which is not comparable with HVRRS. Thus, to match with ERA5 reanalysis at specified height intervals, the HVRRS was vertically interpolated with resolutions spanning from 100-m to 400-m. Even at a comparable resolution, vertical wind shear is underestimated by around 50%, leading to a considerable underestimation in $OF(Ri<1/4)$, compared to radiosondes.

Interestingly, the ERA5-determined occurrence frequency of $Ri<1$ is generally consistent with the frequency of $Ri<1/4$ obtained from HVRRS, in terms of magnitude and temporal variation. Rather than $Ri<1/4$, we argue that the threshold value of $Ri<1$ could be more proper when using ERA5 reanalysis for KHI study, especially in the middle and upper troposphere over midlatitude and subtropic regions in the Northern/Southern Hemisphere, where a high consistency between HVRRS and ERA5 has been found in terms of $OF(Ri<Rit)$ magnitude. In other words, under a similar occurrence frequency, the identification of vertical segments with $Ri<1$ in ERA5 is equitable with identification of vertical segments with $Ri<1/4$ using HVRRS. It is worth highlighting that HVRRS experiences a 200-m vertical moving average procedure to inhabit measurement noises and turbulence fluctuations. Without this procedure, the threshold $Ri$ for the ERA5 reanalysis would even higher than 1.

The climatology of $OF(Ri<Rit)$ exhibits significant seasonal cycles over all latitudes. A poleward decrease can be clearly identified in the middle and upper troposphere. In addition, over mountainous area, complex terrain may locally enhance low-level $OF(Ri<Rit)$. Moreover, it is immediately obvious that the both $OF(Ri<Rit)$ and $OF(0<Ri<Rit)$ in the middle and upper troposphere of the Niño 3 region is largely enhanced probably by the tropical convective heating.

Moreover, both $OF(Ri<Rit)$ and $OF(0<Ri<Rit)$ exhibit close relationship with GW activities and background mean flow. For instance, large $OF(Ri<Rit)$ (≥10%) favors intensive GW activities energy larger than 10 J/kg and strong or mean flow stronger than
Over complex terrains, the orographic GW breaking could locally enhance \( \text{OF}(R_i < R_t) \). Those findings are valuable for pointing out the performance of the ERA5 reanalysis in terms of resolving low Richardson numbers as a proxy for KHI, in comparison with a near-global high-resolution radiosonde measurement. In addition, the spatial-temporal variability of \( \text{OF}(R_i < R_t) \) over different climate zones from near-ground up to 30 km is quantitatively characterized by ERA5 and HVRRS, which could provide new insights that increase our understanding of the fine structure of upper air.

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Competing interests

The contact author has declared that neither they nor their co-authors have any competing interests.

Data availability

The dataset can be accessed at ECMWF (2022).

Author contributions

JZ conceptualized this study. JS carried out the analysis with comments from other co-authors. JZ wrote the original manuscript. WW, SZ, TY, WD provided useful suggestions for the study. All authors contributed to the improvement of paper.

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https://doi.org/10.1038/s41467-023-39039-7, 2023.


Sharman, R. D., and Pearson, J. M: Prediction of energy dissipation rates for aviation


Table 1. Comparisons of mean wind shears between HVRRS and ERA5 reanalysis at heights of 0–2 km a.g.s.l. (a), 10–15 km a.g.s.l. (b), and 20–25 km a.g.s.l. (c).

(a) Wind shear at 0–2 km a.g.s.l. (m/s/km)

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(b) Wind shear at 10–15 km a.g.s.l. (m/s/km)

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Table 2. The occurrence rate of low $R_i$ at 0.8–1.3 km a.g.s.l. (a), 2.2–3.2 km a.g.s.l. (b), 6–15 km a.g.s.l. (c), and 20–21 km a.g.s.l. (d). The critical $R_i$ ($R_{it}$) is 1/4 for radiosonde, and it increases from 1/4 to 2 for ERA5 reanalysis. Note that HVRRS data were vertically resampled to 100-m, 200-m, 300-m, and 400-m at these four height intervals to match with the ERA5 reanalysis. In addition, the moving average number in Eq.(1) is 0. RS stands for radiosonde.

(a) Frequency of low $R_i$ at 0.8–1.3 km a.g.s.l. (%) / Vertical resolution of RS is 100-m

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Table 3. Vertical wind shears at 0.8–1.3 km a.g.l. (a), 2.2–3.2 km a.g.l. (b), 6–15 km a.g.l. (c), and 20–21 km a.g.l. (d). Note that HVRSS data was vertically resampled to 100-m, 200-m, 300-m, and 400-m at these four height intervals to match with the ERA5 reanalysis. RS stands for radiosonde.
### (a) Wind shear at 0.8–1.3 km a.g.s.l. (m/s/km) / Vertical resolution of RS is 100-m

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<td>11.892</td>
<td>11.298</td>
<td>11.51</td>
<td>13.325</td>
<td>13.06</td>
<td>13.85</td>
</tr>
</tbody>
</table>

### (b) Wind shear at 2.2–3.2 km a.g.s.l. (m/s/km) / Vertical resolution of RS is 200-m

<table>
<thead>
<tr>
<th></th>
<th>Polar (NH)</th>
<th>Midlatitude (NH)</th>
<th>Subtropics (NH)</th>
<th>Tropics (SH)</th>
<th>Subtropics (SH)</th>
<th>Midlatitude (SH)</th>
<th>Polar (SH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERA5</td>
<td>3.703</td>
<td>4.504</td>
<td>5.254</td>
<td>4.676</td>
<td>5.444</td>
<td>4.734</td>
<td>4.204</td>
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</tbody>
</table>

### (c) Wind shear at 6–15 km a.g.s.l. (m/s/km) / Vertical resolution of RS is 300-m

<table>
<thead>
<tr>
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<th>Polar (NH)</th>
<th>Midlatitude (NH)</th>
<th>Subtropics (NH)</th>
<th>Tropics (SH)</th>
<th>Subtropics (SH)</th>
<th>Midlatitude (SH)</th>
<th>Polar (SH)</th>
</tr>
</thead>
</table>

### (d) Wind shear at 20–21 km a.g.s.l. (m/s/km) / Vertical resolution of RS is 400-m

<table>
<thead>
<tr>
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<th>Polar (NH)</th>
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<th>Subtropics (NH)</th>
<th>Tropics (SH)</th>
<th>Subtropics (SH)</th>
<th>Midlatitude (SH)</th>
<th>Polar (SH)</th>
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</thead>
<tbody>
<tr>
<td>ERA5</td>
<td>3.994</td>
<td>3.854</td>
<td>4.802</td>
<td>5.634</td>
<td>4.73</td>
<td>3.64</td>
<td>2.98</td>
</tr>
</tbody>
</table>
Table 4. Similar to Tab.1 but for the occurrence frequency of $R_i < R_{it}$. Note that $R_{it}$ is indicated by $R_i < 1/4$ in radiosonde, but it is identified with 1 in ERA5 reanalysis.

(a) $OF(R_i < R_{it})$ at 0–2 km a.g.l. (%)

<table>
<thead>
<tr>
<th></th>
<th>Polar (NH)</th>
<th>Midlatitude (NH)</th>
<th>Subtropics (NH)</th>
<th>Tropics (NH)</th>
<th>Subtropics (SH)</th>
<th>Midlatitude (SH)</th>
<th>Polar (SH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ERA5</td>
<td>28.024</td>
<td>41.262</td>
<td>40.364</td>
<td>40.143</td>
<td>47.450</td>
<td>42.920</td>
<td>27.59</td>
</tr>
<tr>
<td></td>
<td>94</td>
<td>85</td>
<td>20</td>
<td>7.22</td>
<td>11.56</td>
<td>0.06</td>
<td>0.46</td>
</tr>
</tbody>
</table>

(b) $OF(R_i < R_{it})$ at 10–15 km a.g.l. (%)

<table>
<thead>
<tr>
<th></th>
<th>Polar</th>
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<th>Subtropics</th>
<th>Tropics</th>
<th>Subtropics</th>
<th>Midlatitude</th>
<th>Polar</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVRRS</td>
<td>0.512</td>
<td>2.052</td>
<td>5.214</td>
<td>11.112</td>
<td>6.040</td>
<td>1.535</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.62</td>
<td>6.86</td>
<td>17.03</td>
<td>7.15</td>
<td>1.67</td>
<td>0.28</td>
</tr>
<tr>
<td>ERA5</td>
<td>0.44</td>
<td>2.62</td>
<td>6.86</td>
<td>17.03</td>
<td>7.15</td>
<td>1.67</td>
<td>0.28</td>
</tr>
</tbody>
</table>

(c) $OF(R_i < R_{it})$ at 20–25 km a.g.l. (%)

<table>
<thead>
<tr>
<th></th>
<th>Polar</th>
<th>Midlatitude</th>
<th>Subtropics</th>
<th>Tropics</th>
<th>Subtropics</th>
<th>Midlatitude</th>
<th>Polar</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVRRS</td>
<td>0.452</td>
<td>0.489</td>
<td>0.425</td>
<td>0.51</td>
<td>0.384</td>
<td>0.67</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.06</td>
<td>0.07</td>
<td>0.04</td>
<td>0.11</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>ERA5</td>
<td>0.06</td>
<td>0.07</td>
<td>0.04</td>
<td>0.11</td>
<td>0.06</td>
<td>0.06</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Figure 1. The averaged occurrence frequencies of $Ri<1/4$ at heights of 0–2 km a.s.l. (a) and 10–15 km a.s.l. (b), with vertical resolutions ranging 10-m to 100-m and moving point numbers increasing from 0 to 50. The error bars correspond to the standard deviation. The metrics are counted based on all radiosonde profiles during years 2017–2022.
Figure 2. The spatial distribution of mean wind shear in ERA5 reanalysis at heights of 0–2 km a.g.l. (a) and 10–15 km a.g.l. (c), where the areas with a near-surface pressure lower than 800 hPa are masked with white. The overlaid colored circles represent the result in HVRRS at the same height levels. Each data point represents a vertically
averaged value of the wind shear at one radiosonde station during the whole study period. Density plots (b, d) show the correlation between wind shears in HVRRS and ERA5 reanalysis. The ERA5 derived wind shears are spatially and temporally collocated with those of HVRRS. In addition, the red lines represent a least-squared linear regression, and the star superscripts indicate that values are statistically significant ($p<0.05$).
Figure 3. Monthly mean wind shears during years 2017–2022 in HVRRS (a–g) and ERA5 reanalysis (h–n) at different climate zones. The ERA5 derived wind shears are spatially and temporally collocated with those of HVRRS. NH=Northern Hemisphere; SH=Southern Hemisphere.
Figure 4. The monthly occurrence frequency of $Ri<1/4$ at Corpus Christi station (27.77°N, −97.5°W) in HVRRS (a) and ERA5 reanalysis (b). Note that the contour curves in (a) and (b) concern the mean horizontal wind speed, and that the ERA5 derived quantities are spatially and temporally collocated with those of HVRRS.
Figure 5. The annual cycles of the occurrence frequency of $R_i < 1/4$ in different climate zones at 10–15 km a.g.l. The red and blue boxes represent the frequencies in HVRRS and ERA5 reanalysis, respectively. The ERA5 derived $R_i$ is spatially and temporally collocated with that of HVRRS. NH, Northern Hemisphere; SH, Southern Hemisphere.
Figure 6. The altitude variation of the occurrence frequency of Ri below certain thresholds (0.25, 0.5, 1, 1.5, and 2) in ERA5 reanalysis in various climate zones. The ERA5 derived Ri is spatially and temporally collocated with that of HVRRS. The occurrences of Ri<1/4 in HVRRS are overlapped with red lines.
Figure 7. The correlation coefficients between monthly averaged occurrence frequency of $R_i < 1/4$ in the HVRRS and the monthly occurrence frequency of $R_i$ below certain thresholds (0.25, 0.5, 1, 1.5, and 2) in ERA5 reanalysis. The ERA5 derived $R_i$ is spatially and temporally collocated with that of HVRRS. The coefficients in various climate zones are estimated in an increment of 2 km.
Figure 8. The percentage of $OF(R_i<0)$ relative to $OF(R_i<R_{it})$ in HVRRS (red) and ERA5 reanalysis (blue).
Figure 9. The spatial distribution of the mean $OF(R_i < R_{it})$ in ERA5 reanalysis at 0–2 km a.g.l. (a) and 5–10 km a.g.l. (b), 10–15 km a.g.l. (c), and 20–25 km a.g.l.
(d). Note that $R_i$ is set to 1.
Figure 10. The monthly averaged $OF(R_{i<}\leq R_{i})$ in the HVRRS (a–g) and ERA5 reanalysis (h–n) in seven climate zones. NH=Northern Hemisphere; SH=Southern Hemisphere.
Figure 11. The seasonal averaged \( OF(Ri<Rit) \) in the HVRRS (a–g) and ERA5 reanalysis (h–m) in seven climate zones. MAM, March–April–May; JJA, June–July–August; SON, September–October–November; DJF, December–January–February. NH=Northern Hemisphere; SH=Southern Hemisphere.

Figure 12. The association of HVRRS-determined \( OF(Ri<Rit) \) with different standard deviations of orography (dimensionless). (a), (b), and (c) are for height ranges of 1–2 km, 3–4 km, and 5–6 km a.g.l., respectively. The correlation coefficients between \( OF(Ri<Rit) \) and standard derivation of orography are marked in the top right corner, where the star superscripts indicate that values are statistically significant (\( p<0.05 \)).
Figure 1. The monthly averaged $O(\text{Ri} < \text{Rit})$ in ERA5 reanalysis over the Niño 3 region ($5^\circ$ N–$5^\circ$ S, $150^\circ$ W–$90^\circ$ W). The blue and red shadings in time axis indicate the time periods with El Niño and La Niña events, respectively.
Figure 14. Geographical distribution of mean tropospheric GW total energy obtained from the HVRRS (a). The latitudinal variation of mean energy in a grid cell of 5° latitude (b). The joint distribution of $OF(R_i<\tilde{R_i})$ with GW energy and wind shear (c). The $OF(R_i<\tilde{R_i})$ and wind shear are derived from individual HVRRS profiles and vertically averaged over the tropospheric segment that is used for GW study. The numerical number in (c) indicates the matched profile number in each grid, using a bin size of 2 J/kg along the x axis and 2 m/s/km along the y axis.
Figure 15. Joint distribution of HVRRS-derived wind speed, wind shear, and OFF(Ri<Rit), with a bin size of 5 m/s along the x axis and 5 m/s/km along the y axis.

Note that all the relationship is based on the mean result of individual profiles at heights of 10–15 km a.g.l. The number indicates the matched profile number in each grid.