Responses to Paul Pukite:

Community response to Global zonal wind variations and responses to solar activity, and QBO, ENSO during 2002–2019 (Xiao Liu, Jiyao Xu, Jia Yue, and Vania F. Andrioli)

Attached is an alternate explanation for the periodicity of stratospheric winds. My observation, independent of the acceptability of the paper under review, is that more progress can be made only after refuting the most plausible explanations.

I recommend a fundamental shift of how the mechanisms behind the shifts of stratospheric winds is understood. Much of what I will briefly describe is published in Mathematical Geoenergy (Wiley/AGU, 2018).

Response: Thanks for your recommendation on the point of the mechanisms behind shifts of stratospheric winds. Especially the SAO in the upper stratosphere and QBO in the lower stratosphere. We are very interested in your new insight on the points of QBO, SAO, ENSO., etc. Following your suggestion, we have tried to get the main idea and method of reproducing these oscillations from related references. Such as: (1) the published book (Pukite et al., 2019), (2) some comments on the web: https://geoenergymath.com/tag/qbo/, (3) the model of QBO and ENSO on github: https://github.com/pukpr/GeoEnergyMath/, (4) interactive discussions on the forum: https://forum.azimuthproject.org/discussion/comment/22250/#Comment_22250, (4) many rounds of communications through email.

Pukite, P., Coyne, D., & Challou, D. (2019). Mathematical Geoenergy: Discovery, Depletion, and Renewal (Vol. 241). John Wiley & Sons.

Now, we can reproduce the SAO and QBO according to your idea and detailed explanations through email. See below for the detailed procedure of reproducing QBO wind. It should be noted that there is still room to improve the reproduced QBO wind by us as compared the optimal QBO wind reproduced by you.

First, we assume that the semi-annual cycle in the upper stratosphere is a result of the nodal cycle about the Earth's ecliptic axis. This results in a semi-annual cycle and not annual cycle due to the symmetry of the hemispheres. In terms of a heuristic mathematical construct, this can be formulated by the multiplication of **an annual sinusoidal cycle** convoluted with **a semi-annual delta function impulse train phased according to the (+/-) pairs of solstice or equinox events** – a positive (+) for one seasonal event and negative (-) for the complementary event. From this, a rudimentary square wave time-series is generated, with a semi-annual period resulting from the positive excursions pairing to create a positive and similar for the multiplication of the negative excursions. This is adequate to empirically describe the SAO of the upper stratosphere and identify it with a forcing and not a resonant condition.

Next, consider that at lower stratospheric altitudes, the QBO cycle of ~28 months takes over. The idea is that a similar nodal construct can be applied but instead of applying only an annual nodal cycling to the convolution, we also add in the nodal lunar tidal cycle. Now we note the important realization that the 0-wavenumber symmetry of the QBO behavior demands that the draconic or nodal lunar cycle of 27.2122 days must be applied to model a global effect (not the longitudinally dependent 27.3216 days cycle associated with regional tides). This is adequate to empirically describe the QBO of the lower stratosphere and identify it with a tidal forcing where the density is greater and thus more susceptible to gravitational wave energy.

Of course, this hypothesis is completely dependent on the timing of the draconic cycle agreeing with the empirical observation of QBO cycling. The predicted frequency for the multiplication of a draconic cycle convoluted with a semi-annual delta function impulse train is calculated as

365.25 modulo 27.2122 = 0.422 / year

or 2.368 years period due to physical aliasing of the waveforms (see Mathematical Geoenergy cited above). This indeed matches well the empirically observed cycling of QBO as shown in the time-series plot below, where all the excursions' pair one-to-one with observations, including potentially resolving the issue of the perturbation of 2016.

Responses: Following your comments here and our discussions through email, we performed the following four steps to realize aliased signals through the input-signals of annual cycle, lunar cycle, annual impulse.

First step: a semi-annual signal is constructed through the aliasing of an annual sinusoidal cycle and annual impulse. An artificial QBO signal is constructed through the aliasing of an lunar cycle and annual impulse. Here the dense sampling frequency is used.

An annual sinusoidal cycle is expressed as:

$$AO_{signal}(t_i) = A_{AO}\cos(2\pi t_i + \theta_{AO}). \quad (1)$$

Here A_{AO} and θ_{AO} are the amplitude and phase. The sampling frequency (F_{sampl}) is set to be 360 per year. The sampling time is $t_i = i/F_{sampl}$ and has unit of year. The index i is integer and is in the range of N years ($[0, N \cdot F_{sampl}]$). The sampling frequency is much higher than that used in the monthly QBO data and is used only for illustration purpose. Below, we will show that the sampling frequency can be set to be 12 (monthly data) for QBO signal. The black dots in Figure 1(a) shows the annual cycle in an interval of 16 years.

An annual impulse is expressed as a periodic Gaussian series ($x_i = t_i\%1$, % mean modulo):

$$AO_{impulse}(t_i) = \exp\left[-\frac{x_i^2}{2\sigma^2}\right] + \exp\left[-\frac{(x_i-1.0)^2}{2\sigma^2}\right] - \exp\left[-\frac{(x_i-0.5)^2}{2\sigma^2}\right]. \quad (2)$$

The series of annual impulse is shown as red dots in Figure 1(a).

Then, a semi-annual signal can be obtained through element-wise multiplication,

$$SAO_{signal}(t_i) = AO_{signal}(t_i) \times AO_{impulse}(t_i). \quad (3)$$

The $SAO_{signal}(t_i)$ is shown as black dots in Figure 1(b) and has clear semi-annual cycles.

In a same manner, we replace annual sinusoidal cycle with a lunar monthly sinusoidal wave with period of $\tau_L = 27.2122$ days to construct an artificial QBO signal. The lunar monthly sinusoidal wave is expressed as,

$$L_{signal}(t_i) = A_L \cos(2\pi F_L t_i + \theta_L). \quad (4)$$

Here A_L and θ_L are the amplitude and phase of lunar monthly sinusoidal wave. $F_L = \tau_Y/\tau_L$ is the frequency of lunar cycle with unit of cycles per year. The constant $\tau_Y = 365.2412384$ days is the period of one year corresponding to the sun's complete seasonal cycle. Now, an element-wise multiplication is performed on between $L_{signal}(t_i)$ and $AO_{impulse}(t_i)$. The multiplication result is shown as black dots in Figure 1(c) and exhibits an aliased ~2.3-year cycle.

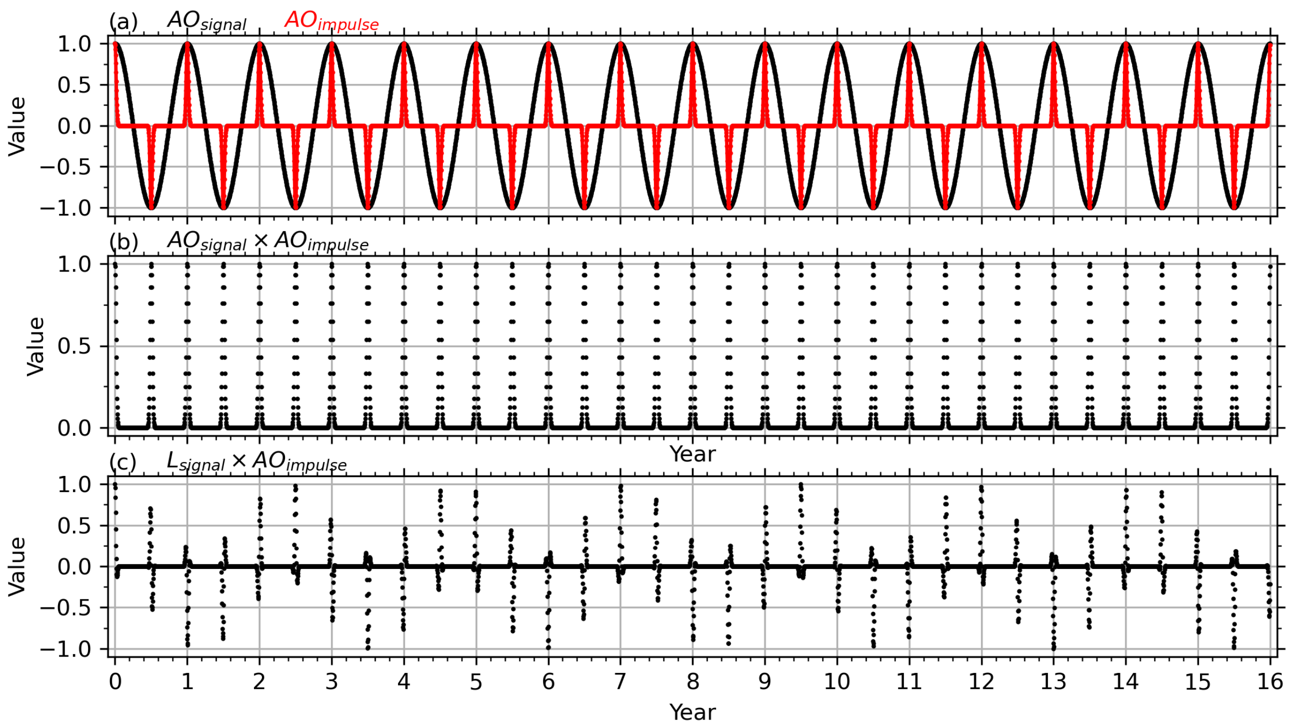

Figure 1: Temporal series of the annual sinusoidal cycle (AO_{signal}, black dots in a), annual impulse ($AO_{impulse}$, red dots in a), the resulted semi-annual cycle through multiplication between AO_{signal} and $AO_{impulse}$ (b), and the resulted QBO cycle through multiplication between L_{signal} and $AO_{impulse}$ (c).

Second step: an artificial QBO signal through a lunar cycle and annual impulse. Here the monthly sampling frequency is used. The "square off" QBO signal is obtained through the "sample-and-hold" integration.

The dense sampling frequency in Figure 1 cannot be applicated to realistic QBO data, who has a sampling frequency of one month. Figure 2(a) shows the lunar monthly cycle with sampling frequencies of 360 per year (black, labeled with $L_{signal}(dense)$) and 12 per year (red, labeled with $L_{signal}(month)$). The lunar monthly cycle is accurately represented when the sampling frequency is 360 per year but is hard to be read. However, the lunar monthly cycle cannot be represented properly when the sampling frequency is 12 pers, which is lower than the minimum frequency required by the Nyquist sampling theory.

For the sampling frequency of 12 per year, the element-wise multiplication between $L_{signal}(month)$ and $AO_{impulse}$ (red in Figure 2b) results in signal with period of ~2.3-year (black dot in Figure 2b). The signal of ~2.3-year changes to be a "square off" temporal series (Figure 2c) after performing a special integrating process (i.e., sample-and-hold). The sample-and-hold integration means that a value of signal for that month is sampled, and then held constant until the next impulse, which is then added to the value.

It seems that the periodicity of the "square off" temporal series (Figure 2c) is more readable than the black dots series shown in Figure 2(b).

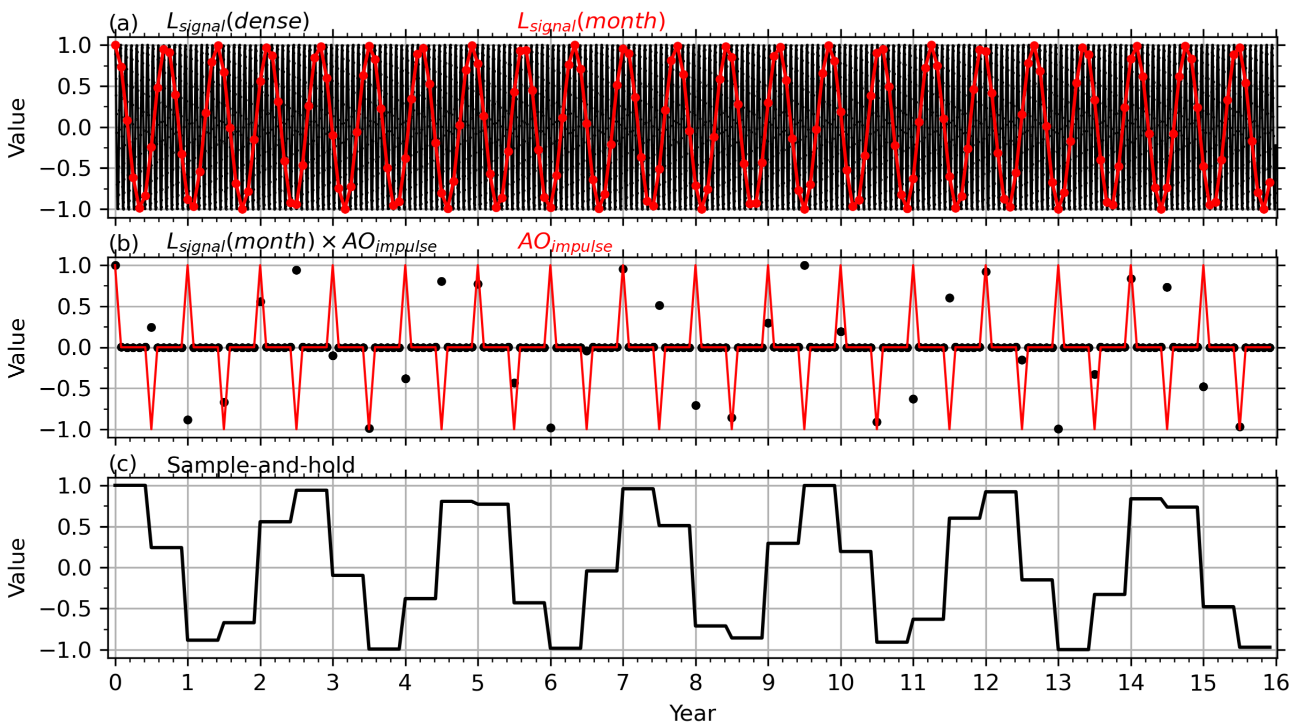

Figure 2: Temporal series of the lunar monthly sinusoidal cycles for the sampling frequency of 360 per year ($L_{signal}(dense)$, black in a) and 12 per year ($L_{signal}(month)$, red in a), annual impulse ($AO_{impulse}$, red in b) and the multiplication result (black in b), and the sample-and-hold integrating results (black in c) of the multiplication results shown as black dots in b.

Third step: the QBO signal is derived through aliasing of lunar cycle and seasonal cycles.

We follow the Equation 12.17-12.22 of the book edited by Pukite et al. (2019) to understand the modulo aliasing. The known lunar tidal forcing signal is periodic signal (i.e., Equation 4 and rewriting here):

$$L_{signal}(t_i) = A_L \cos(2\pi F_L t_i + \theta_L). \quad (5)$$

Here A_L and θ_L are the amplitude and phase of lunar monthly sinusoidal wave. $F_L = \tau_Y/\tau_L$ is the frequency of lunar cycle with unit of cycles per year. The constant $\tau_Y = 365.2412384$ days is the period of one year corresponding to the sun's complete seasonal cycle. The seasonal signal is likely a strong periodic delta function, which peaks at a specific time of the year. This can be approximated by a Fourier series of frequency i with unit of per year,

$$s(t) = \sum_{i=1}^{n} a_i \sin(2\pi t i + \theta_i). \quad (6)$$

Here, a_i and θ_i are, respectively, the amplitude and phase of the seasonal signal with frequency of i. Specifically, $i = 1$ corresponds to the annual cycle, $i = 2$ corresponds to the semi-annual cycle, etc.

The combination of the lunar cycle $L(t)$ amplified by a strongly cyclically peaked seasonal signal $s(t)$:

$$f(t) = s(t)L(t) = k \sum_{i=1}^{n} a_i \sin(2\pi f_L t + \phi) \sin(2\pi t i + \theta_i). \quad (7)$$

$$f(t) = \frac{k}{2} \sum_{i=1}^{n} a_i (\cos[2\pi(f_L - i)t + (\phi - \theta_i)] - \cos[2\pi(f_L + i)t + (\phi + \theta_i)]), \quad (8)$$

$$f(t) = \frac{k}{2} \sum_{i=1}^{n} a_i \cos[2\pi(f_L - i)t + (\phi - \theta_i)] - \frac{k}{2} \sum_{i=1}^{n} a_i \cos[2\pi(f_L + i)t + (\phi + \theta_i)]. \quad (9)$$

Equation (9) includes both the lower-frequency difference terms and higher-frequency additive terms. The lower-frequency terms are,

$$f_{LF}(t) = \frac{k}{2} \sum_{i=1}^{n} a_i \cos[2\pi f_i t + (\phi - \theta_i)]. \quad (10)$$

Here, $f_i = f_L - i$ is the frequency of difference aliasing, this can be used to construct the aliasing frequency listed in Table 11.1 of Pukite et al. (2019).

Table 1. The modulo aliasing frequencies and periods

Lunar frequency (f_L, 1/year)	Seasonal frequency (i, 1/year)	Frequency of difference aliasing (f_i, 1/year)	Period of difference aliasing (year)
13.422	10	3.422	0.292
13.422	11	2.422	0.412
13.422	12	1.422	0.703
13.422	13	0.422	2.368
13.422	14	-0.578	-1.730
13.422	15	-1.578	-0.634
13.422	16	-2.578	-0.388

Fourth step: Predictors used to reproduce the realistic QBO signal.

The predictors, which are used to fit the realistic QBO signal, can be constructed through aliasing of lunar cycle (cosine and sinusoidal form (Figure 3a)) and impulse with frequencies 10/12, 11/12, 12/12, 13/12, 14/12, 15/12, and 16/12 (red lines in Figures 3b-h). All predictors (black and blue in Figures 3b-h) have the "square off" form, which are obtained through element-wise multiplication of lunar cycle and impulse, and then through the "sample-and-hold" integration.

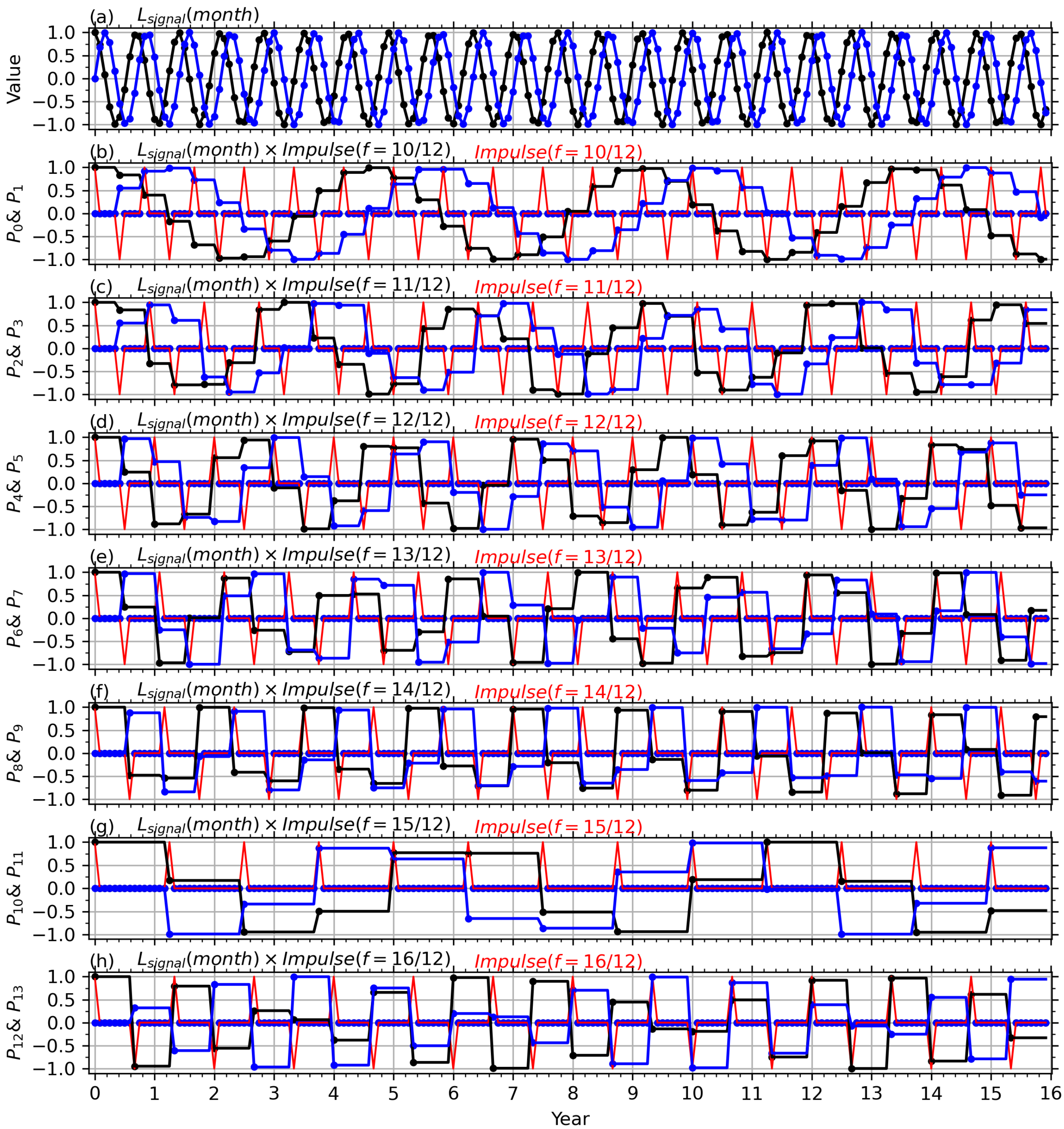

Figure 3: Lunar cycles in cosine (black in a) and sinusoidal form (blue in a) and predictors (black and blue in b-h).

Using the predictors shown in Figure 3, multiple linear regression (MLR) is used to fit the QBO wind at 30 hPa (black line in Figure 4a). The fitting and predicting results are shown as red line in Figure 4a. We note that the MLR model is trained in the interval of shaded region. The dominant contributions are the annual impulses (P_4 and P_5 in Figure 3d), that have largest regression coefficients. In a same manner, we show in Figure 5 the results of QBO wind at 10 hPa.

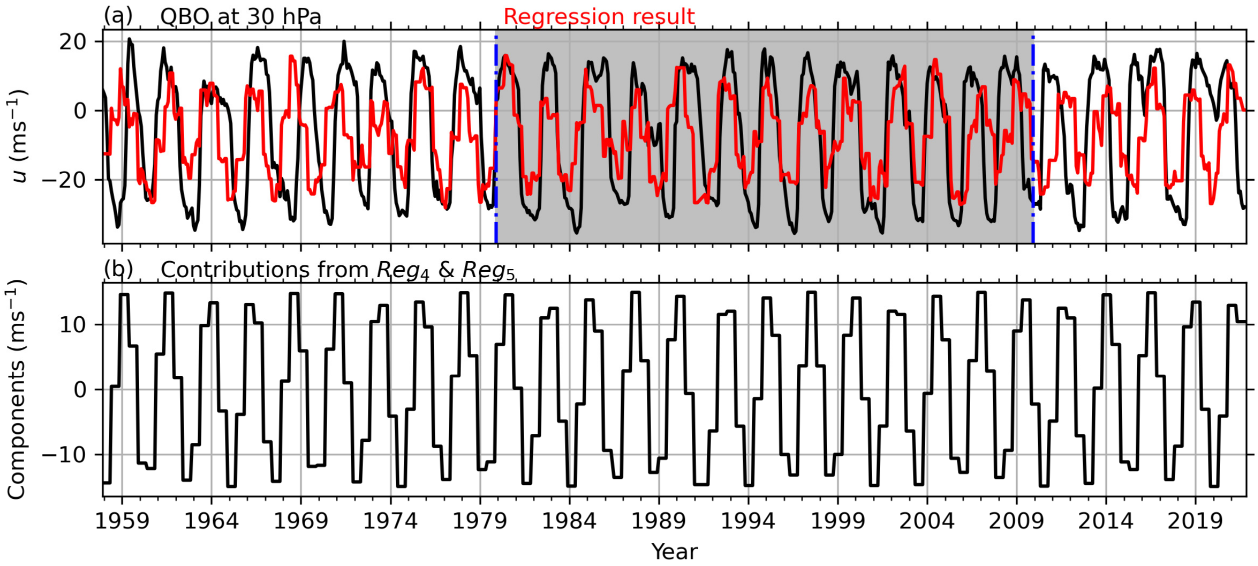

Figure 4: QBO wind at 30 hPa (black in a) and its MLR result (red in a). Also shown in b is the dominant contributions from the P_4 and P_5 in Figure 3(d).

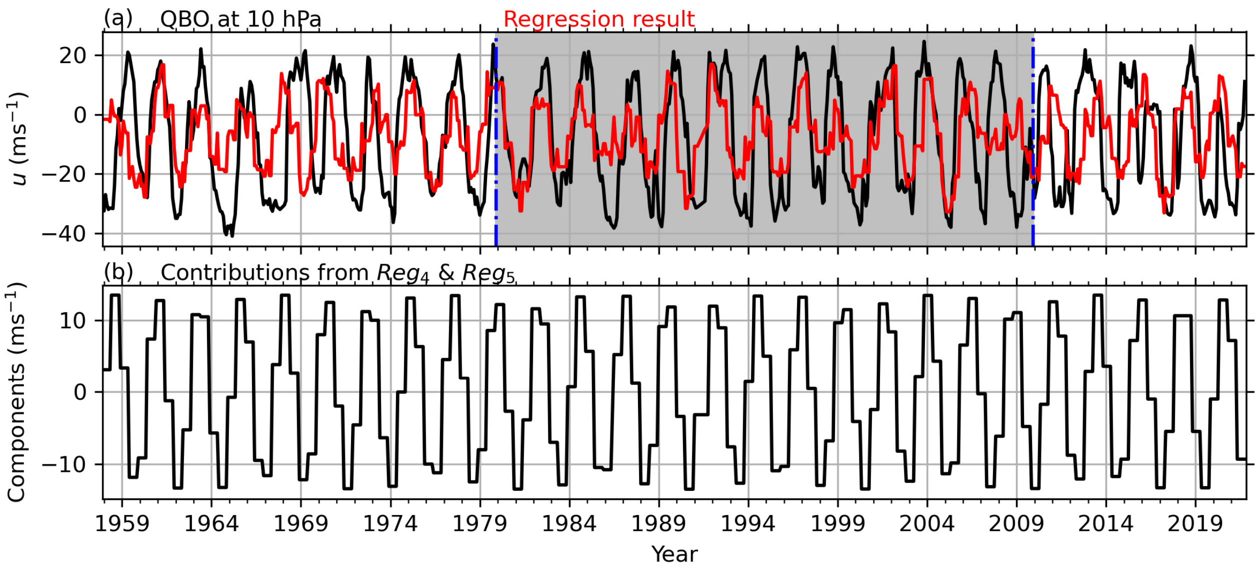

Figure 5: Same caption as Figure 4 but for the QBO wind at 10 hPa.

Using the aliased frequency shown in Table 1 and the sample-and-hold integration, Prof. Pukite has reproduced the QBO signal shown in Figure 5. This indicates that the lunar cycle, modulo aliasing with seasonal signal would generate an QBO signal.

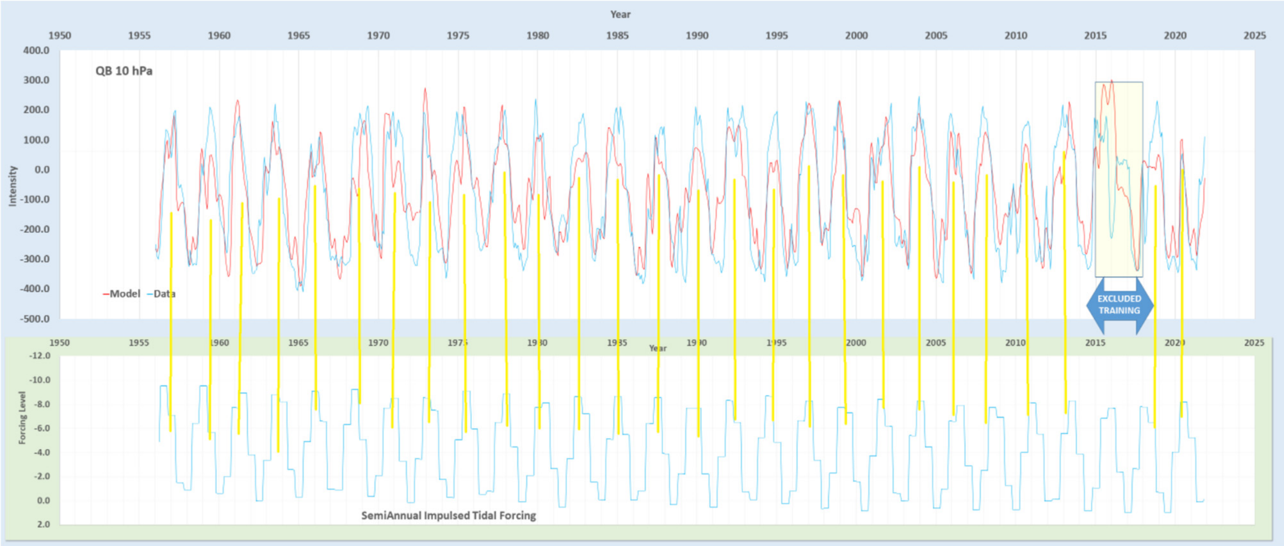

Figure 6: Provide by Prof. Paule R. Pukite, who reproduced QBO signal by using the aliased frequencies shown in Table 1 with variable initial phases.

Comparisons between Figures 5 and 6, we can see that fitting results by Prof. Pukite are better than those shown in Figure 5. This is because that Prof. Pukite included a gradient-descent algorithm to optimize correlation coefficient model fitting. The additional optimization improved the fitting to a higher quality.

The question as to why this correlation was missed by atmospheric scientists over the years is difficult to determine. Certainly, Richard Lindzen considered the possibility, as the cited quotes below reveal.

I can only offer again that conventional tidal analysis (for predicting king tides, etc.) operates at a local or regional level, where the 27.3216 days lunar synodic (or tropical) cycle is operational. For this particular lunar cycle, modulo arithmetic would generate an aliased ~2.7 years period, which is not close enough to match the long-term QBO periodicity observed. Yet, for conventional tidal analysis, the draconic tidal factor never appears in any analyses, since globally synchronized tides would never be considered, and also importantly, the modulo arithmetic of impulse driven signals at the edge of metastability is also not applied. This means that two critical assumptions – (1) nodal lunar cycling and (2) modulo aliasing – need to be considered, which in retrospect could have easily been overlooked as together they are a necessary condition. A third assumption, that solutions of Laplace's Tidal Equations as applied to the equatorial waveguide can provide the non-linear shaping to allow model fitting to the family of QBO time-series is also described in Mathematical Geoenergy.

"For oscillations of tidal periods the nature of the forcing is clear"
Lindzen, R. D. (1967). Planetary waves on beta planes. Monthly Weather Review, 95(7), 441–

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"It is unlikely that lunar periods could be produced by anything other than the lunar tidal potential"

Lindzen, R. S., & Hong, S.-S. (1974). Effects of mean winds and horizontal temperature gradients on solar and lunar semidiurnal tides in the atmosphere. Journal of the Atmospheric Sciences, 31(5), 1421–1446.

The intention of this comment is to provide alternative explanations via geophysics. A tidal approach is much more plausible and parsimonious than attempting to apply variations in solar output via sunspot activity, which the authors of the paper under review consider. The elegance of transitioning from the semi-annual oscillation of the upper stratosphere to the lunar-modified oscillation of the denser lower density cannot be easily refuted. If Lindzen had the insight to realize the connection in the late 1960's much more effort could have been applied to model the behavior and apply it to other aspects of climate.

Responses: Your comments are valuable for us to understand the mechanism of QBO in the lower stratosphere. We believe that you provide a new insight on this point, which has been added in the Introduction as "Recently, Pukite et al. (2018) proposed that the QBO signal in the stratosphere can be generated by the modulo aliasing between nodal lunar cycle (27.2122 days) and seasonal impulse signals. Especially, the modulo aliasing between the lunar cycle and an annual impulse results in a signal with periods of 2.3 years, which is close the QBO periods of 2–3 years".