

The paper suggests non-linear tidal interactions:

*“may be caused by the nonlinear interaction between tidal waves and ..”*

I recommend a fundamental shift of how the mechanisms behind the shifts of stratospheric winds is understood. Much of what I will briefly describe is published in **Mathematical Geoenergy** (Wiley/AGU, 2018).

First, we assume that the semi-annual cycle in the upper stratosphere is a result of the nodal cycle about the Earth’s ecliptic axis. This results in a semi-annual cycle and not annual cycle due to the symmetry of the hemispheres. In terms of a heuristic mathematical construct, this can be formulated by the multiplication of an annual sinusoidal cycle *convoluted* with a semi-annual delta function impulse train phased according to the (+/-) pairs of solstice or equinox events – a positive (+) for one seasonal event and negative (-) for the complementary event. From this, a rudimentary square wave time-series is generated, with a semi-annual period resulting from the positive excursions pairing to create a positive and similar for the multiplication of the negative excursions. This is adequate to empirically describe the SAO of the upper stratosphere, and identify it with a *forcing* and not a *resonant* condition.

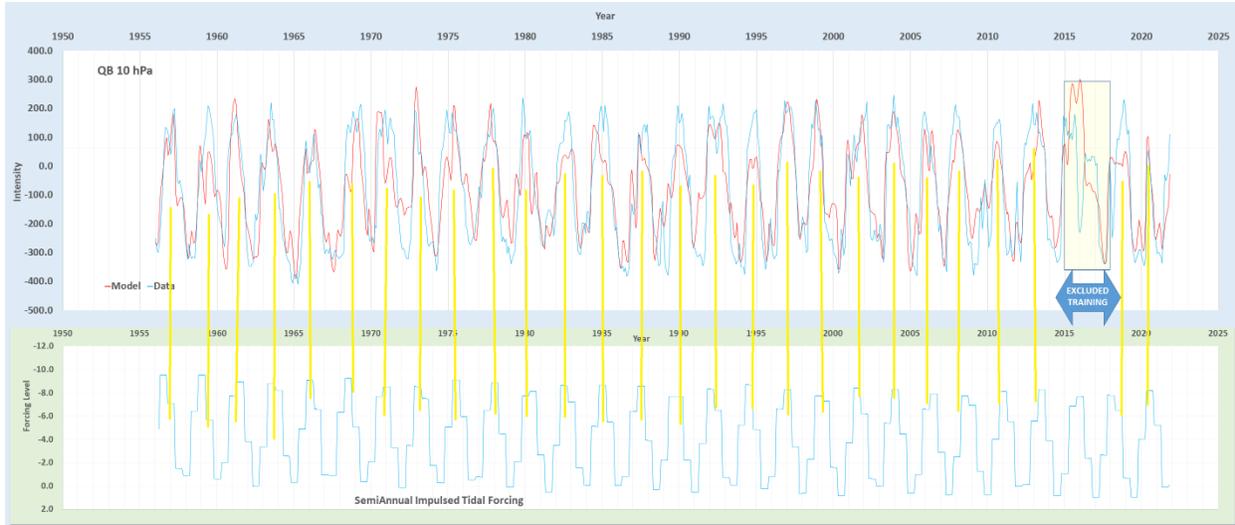
Next, consider that at lower stratospheric altitudes, the QBO cycle of ~28 months takes over. The idea is that a similar nodal construct can be applied but instead of applying only an annual nodal cycling to the convolution, we also add in the nodal lunar tidal cycle. Now we note the important realization that the 0-wavenumber symmetry of the QBO behavior demands that the draconic or nodal lunar cycle of 27.2122 days must be applied to model a *global* effect (not the longitudinally dependent 27.3216 day cycle associated with *regional* tides). This is adequate to empirically describe the QBO of the lower stratosphere, and identify it with a tidal *forcing* where the density is greater and thus more susceptible to gravitational wave energy.

Of course, this hypothesis is completely dependent on the timing of the draconic cycle agreeing with the empirical observation of QBO cycling. The predicted frequency for the multiplication of a draconic cycle *convoluted* with a semi-annual delta function impulse train is calculated as

$$365.25 \text{ modulo } 27.2122 = 0.422 / \text{year}$$

or 2.368 year period due to physical aliasing of the waveforms (see Mathematical Geoenergy cited above).

This indeed matches well the empirically observed cycling of QBO as shown in the time-series plot below, where all the excursions pair one-to-one with observations, including potentially resolving the issue of the perturbation of 2016.



The question as to why this correlation was missed by atmospheric scientists over the years is difficult to determine. Certainly Richard Lindzen considered the possibility, as the cited quotes below reveal.

*"For oscillations of tidal periods the nature of the forcing is clear"*

Planetary waves on beta-planes, RS Lindzen - Monthly Weather Review, 1967

*"it is unlikely that lunar periods could be produced by anything other than the lunar tidal potential"*

Effects of mean winds and horizontal temperature gradients on solar and lunar semidiurnal tides in the atmosphere

RS Lindzen, S Hong - Journal of the Atmospheric Sciences, 1974

I can only offer again that conventional tidal analysis (for predicting king tides, etc) operates at a local or regional level, where the 27.3216 day lunar synodic (or tropical) cycle is operational. For this particular lunar cycle, modulo arithmetic would generate an aliased  $\sim 2.7$  year period, which is not close enough to match the long-term QBO periodicity observed. Yet, for conventional tidal analysis, the draconic tidal factor never appears in any analyses, since globally synchronized tides would never be considered, and also importantly, the modulo arithmetic of impulse driven signals at the edge of metastability is also not applied. This means that two critical assumptions – (1) nodal lunar cycling and (2) modulo aliasing – need to be considered, which in retrospect could have easily been overlooked as together they are a necessary condition. A third assumption, that solutions of Laplace's Tidal Equations as applied to the equatorial waveguide can provide the non-linear shaping to allow model fitting to the family of QBO time-series is also described in Mathematical Geoenergy.

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The intention of this comment is to provide alternative explanations via geophysics. A tidal approach is much more plausible and parsimonious than attempting to apply variations in solar output via sunspot activity, which the authors of the paper under review consider. The elegance of transitioning from the semi-annual oscillation of the upper stratosphere to the lunar-modified oscillation of the denser lower density cannot be easily refuted. If Lindzen had the insight to realize the connection in the late 1960's much more effort could have been applied to model the behavior and apply it to other aspects of climate.

