Responses to the first reviewer's comments

We appreciate your effort and time in reviewing our manuscript as well as minor comments. We have revise the manuscript, accordingly. Please note that the reviewer's comments are written in bold font, and our responses are written using a regular font.

The present version of the manuscript is much clearer and self-consistent. There are still some minor revisions needed, however the overall message is ok

1. In my initial response, I noted that the original Matsuno paper explicitly holds N constant, however this paper allows it to vary in the vertical. The reviewers cited two other studies that also make this assumption in their response. I realize that this paper isn't the first paper to grossly violate the conditions under which Matsuno derived his index of refraction. My point is that if you are already taking the theory well outside of its original application, you may as well use the Weinberger et al 2021 definition of the index of refraction which includes more physical processes without being any less physically inconsistent.

I suspect the results won't be very much different, however some discussion of this point is needed.

 \checkmark Our response:

Thanks for the suggestion. In the form of the refractive index used in equation 12, N varies in the vertical direction and hence is sensitive to its changes near the tropopause and therefore this form of refractive index is suitable for the current study.

As suggested, the following text is added to the manuscript :

It is worthwhile to mention that the index of refraction has different forms. Weinberger (2021) has evaluated the ability of four different versions of the index of refraction to capture the upward wave propagation from the troposphere to the stratosphere. In particular, they show that the vertical gradients in buoyancy frequency near the tropopause is critical for understanding the upward wave propagation from the troposphere. In the form of the refractive index used in equation 12 the buoyancy frequency varies in the vertical direction and hence is sensitive to its changes near the tropopause and therefore is suitable for the current study.

Minor comments:

line 87: Whether CMIP5/6 underestimate the HT effect or not was discussed by Rao et al 2020. They found that if one accounts for the lack of downward propagation to the lowermost stratosphere and the particular QBO regimes simulated by each model, there isn't an underestimate. However there is an underestimate if one uses a simple QBO definition based on fixed pressure levels.

2. the stippling on figure 4 and 6 still look incorrect to me. Particularly noticeable are the bottom row.

✓ Our response:

To make sure that the calculations including the stippling points are corrected, we checked the codes. They are correct.

The text is modified as the reviewer suggested:

"Most Coupled Model Intercomparison Project 6 (CMIP6) models

underestimate the HT relationship in the present-day climate (Elsbury2021) although this depends on the QBO definition and the particular QBO regimes simulated by each model (Rao2020b)."

line 179 "tropics and lower latitudes": should "lower" be replaced by "higher"

 $\checkmark~$ Our response:

Thanks for identifying it. It is corrected in the revised version.