Response to reviewer 2

We sincerely appreciate your valuable comments and suggestions on our manuscript.

We have carefully deliberated on your suggestions and seriously revised our manuscript. Following is our one-by one response to your comments.

Specific comments:

1. The realistic atmosphere is a nonlinear system. However, the nonlinearity can induce spurious harmonic components that cause energy spreading and result in the spectra making little physical sense (Huang et al. (1998), https://doi.org/10.1098/rspa.1998.0193). The summation of the Fourier components fitting the nonlinear and non-stationary nature of data generates artificial harmonics, which misleads the true energy distribution in the frequency or wavenumber domain. In my opinion, the authors mistook the artificial harmonics for the nature signal of the possible nonlinear wave-wave interaction.
   The reviewer recommends that the authors should derive the Hilbert and marginal spectra of the wind data and prove the authenticity of the Fourier-based spectra as shown in the manuscript. The HHT is a method for studying the nonlinear wave-wave interaction and energy transfer. The authors should prove if the spectral peaks are natural or not before investigating the physical mechanism.

   In order to confirm the new spectral components in the upper and lower sidebands are excited through the nonlinearity rather than artificial harmonics, we construct linearly superimposed wind fields to directly compare their spectra with those from the radar observation.
   The first wind field is the summation of AO, SAO, DT and SDT, as follows,

$$u_1 = 12\sin(\frac{2\pi}{T_1}t) + 6\sin(\frac{2\pi}{T_2}t) + 18\sin(\frac{2\pi}{T_3}t) + 10\sin(\frac{2\pi}{T_4}t)$$

   where T_1, T_2, T_3 and T_4 are set to be the periods of AO, SAO, DT and SDT, and their amplitudes are chosen to be 12, 6, 18 and 10 ms^{-1}, respectively. The total duration of 9 years is taken to be the same as the radar observation period.
   In the second wind field, the AO magnitude is changed to be linearly modulated with time relative to that in the first wind field, which is expressed as,

$$u_2 = (0.0003t + 1)\sin(\frac{2\pi}{T_1}t) + 6\sin(\frac{2\pi}{T_2}t) + 18\sin(\frac{2\pi}{T_3}t) + 10\sin(\frac{2\pi}{T_4}t).$$

This means that the AO amplitude increases from 1 ms^{-1} to about 24.7 ms^{-1} during 9 years. The SAO, DT and SDT are consistent with those in the first wind field.

Figure R1 (only presented in Response) shows the constructed two wind fields, together with the zonal wind from the radar observation at 90 km.

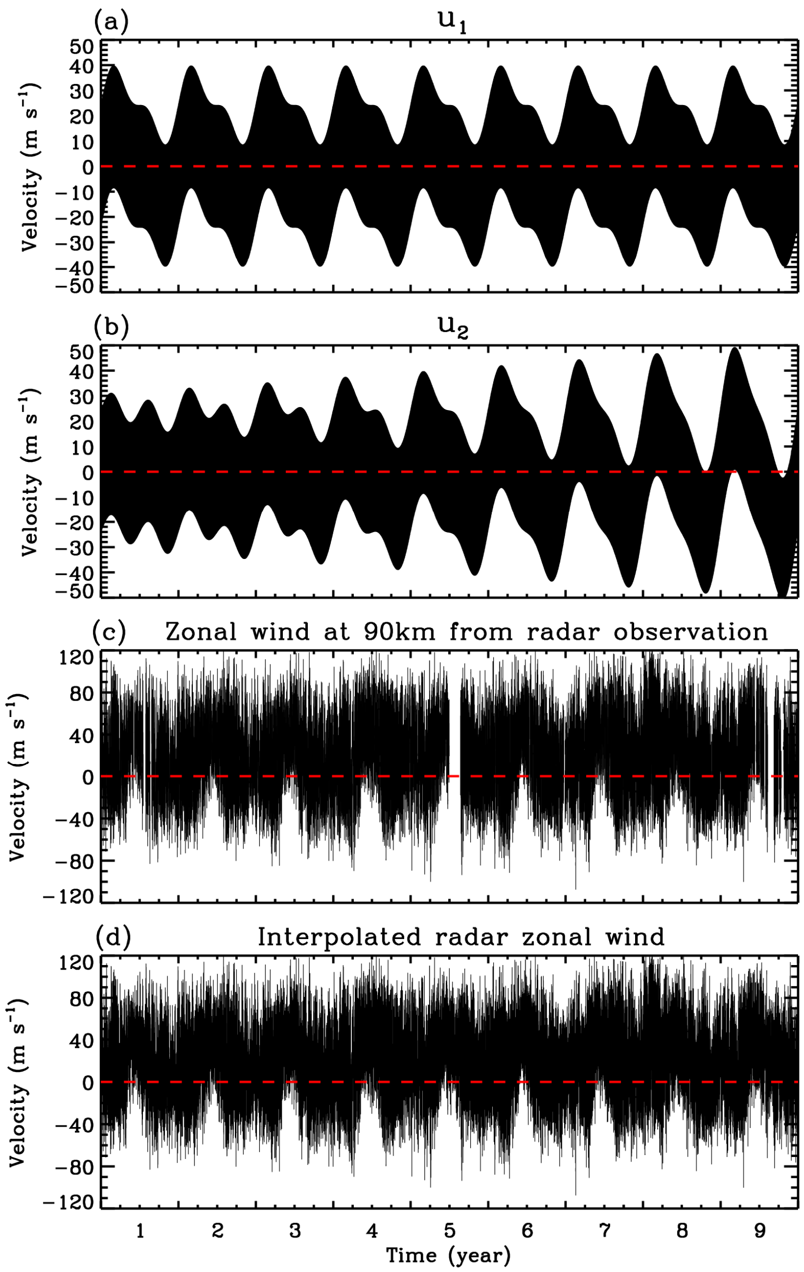

Figure R1. Constructed (a) first and (b) second wind fields. (c) Observed zonal wind at 90 km and (d) interpolated zonal wind.

Firstly, we discuss the Lomb-Scargle spectrum. Figure R2 presents the Lomb-Scargle spectra of the constructed winds and observed wind. As you pointed out, the first and second columns in Figure R2 demonstrates that there are spurious spectral components in the Lomb-Scargle spectrum analysis due to using the fitting. Nevertheless, the resolution between the spurious spectra is 1/(4 x 9 year) because of a 4-times oversampling is applied in the Lomb-Scargle spectrum analysis, moreover, the intensity of these spurious spectra in the sidebands decreases rapidly as their deviation from the dominant DT and SDT spectra. In this case, the spurious spectra at the sum and difference frequencies are extremely weak, which is not impacted by the modulated AO. These spectra at sum and difference frequencies at the first and second column are different from those of the observed wind at the third column in Figure 2R. In the observed wind, the spurious spectra arise but are obviously weaker than the new spectra excited in the interaction.

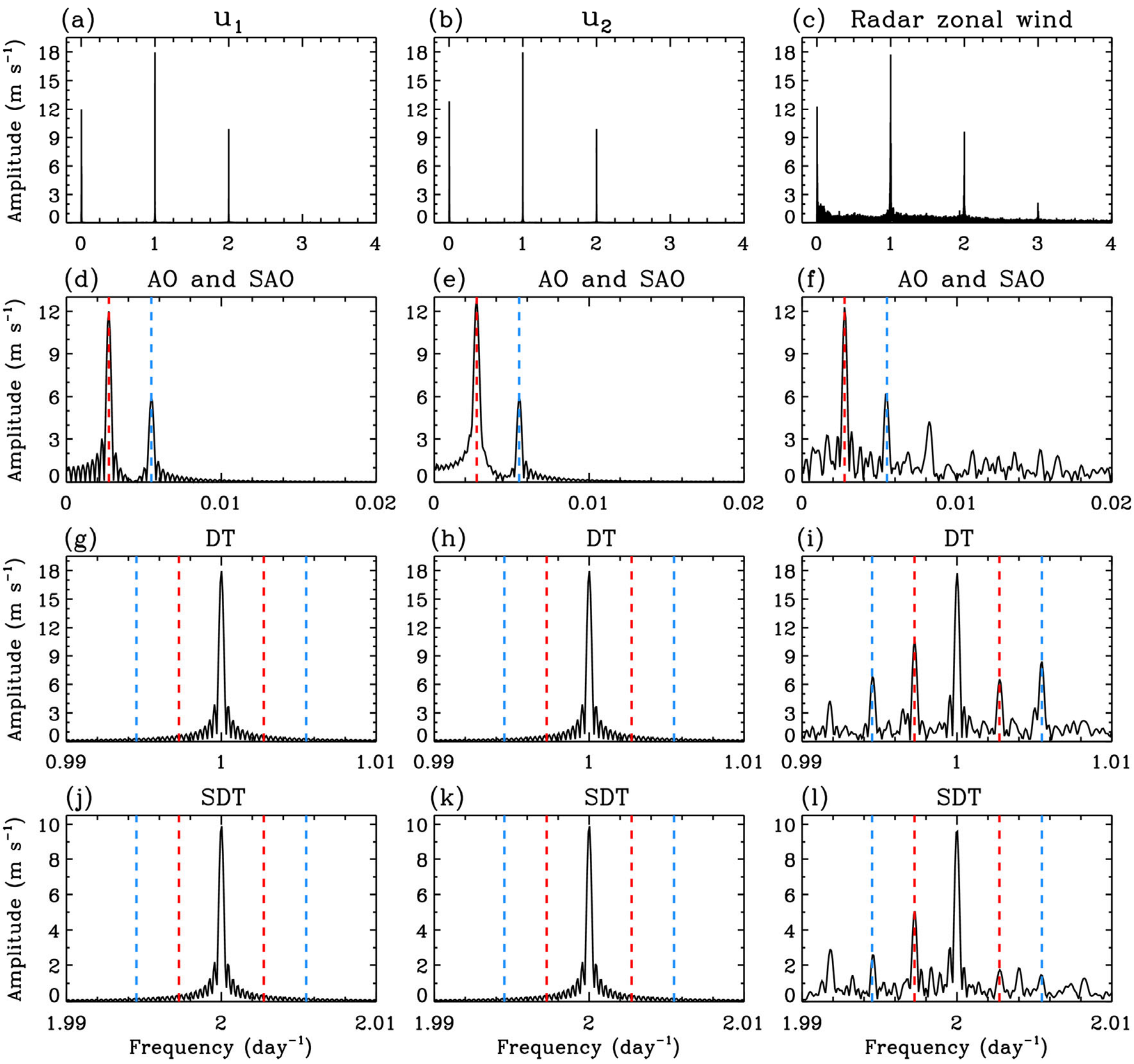

Figure R2. Lomb-Scargle spectra of constructed (left column) first and (mid column) second wind fields and (right column) zonal wind from the radar observations for 9 years. The red (blue) dashed vertical line is marked at the frequency of the AO (SAO) at the second row, and the sum and difference frequencies between the DT and the AO (SAO) at the third row and between the SDT and the AO (SAO) at the fourth row.

Next, we discuss the Fourier spectrum. The Fourier transform requires a continuous dataset, thus the zonal wind from the radar observation is linearly interpolated for the missing data. For missing observation less than 6 hours, the linear interpolation is directly carried out. For longer missing data but smaller than 3 days, the data at the same time in the two days (before and after the 3 days) are utilized in the interpolation. For missing observation larger than 3 days, the data on the same dates in the two years (before and after the missing observation) are used for interpolating. The interpolated zonal wind at 90 km is presented in Figure 1R(d).

Figure R3 shows the Fourier spectra of the constructed two wind fields and the interpolated zonal wind at 90 km. It is interesting that there is not significant spurious spectrum in the constructed two wind fields. The Fourier spectra of AO, SAO, DT and SDT in the interpolated zonal wind are in good agreement with their Lomb-Scargle spectra in the observed raw wind due possibly to the few missing data. Meanwhile, the new spectra at the sum and difference frequencies in the interpolated zonal wind are consistent with the Lomb-Scargle spectra in the observed raw wind, but do not occur in the linearly superimposed wind fields that we constructed.

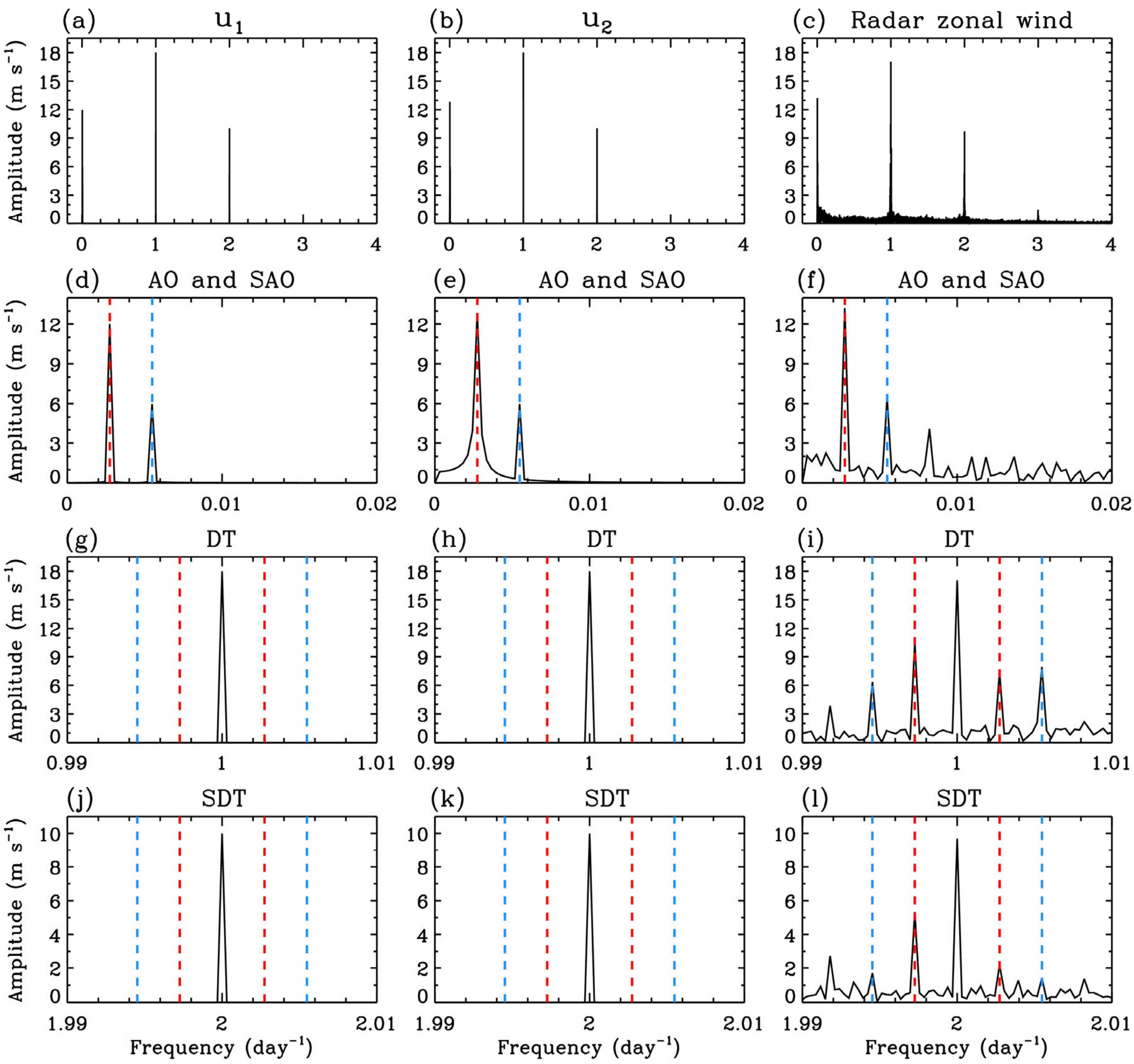

Figure R3. Same as Figure R2 but for Fourier transform spectra.

Finally, we present a discussion of two-dimensional Fourier spectrum. Similarly, we construct two two-dimensional wind fields with a linear superposition of AO, SAO and DT, as follows,

$$u_3 = 12\sin(\frac{2\pi}{T_1}t) + 6\sin(\frac{2\pi}{T_2}t) + 18\sin(\frac{2\pi}{T_3}t - n\theta)$$

$$u_4 = (0.0003t + 1)\sin(\frac{2\pi}{T_1}t) + 6\sin(\frac{2\pi}{T_2}t) + 18\sin(\frac{2\pi}{T_3}t - n\theta)$$

where T_1, T_2 and T_3 are the periods of AO, SAO and DT, respectively; the longitude θ is between 0° and 360°; and the zonal wavenumber n of DT is taken as -1. In the wind field u_4, a modulated AO amplitude replaces the constant amplitude in the wind field u_3.

Figure R4 shows the wavenumber-frequency spectra of these two-dimensional wind field, together with the wavenumber-frequency spectrum of the zonal wind in our manuscript. Similar to the one-dimensional Fourier spectra in Figure R3, there is not significant spurious spectrum in the two-dimensional Fourier spectra of the linearly superimposed wind fields, however, the spectral components which meet the sum and difference resonant conditions of wavenumber and frequency occur in the zonal wind in our manuscript.

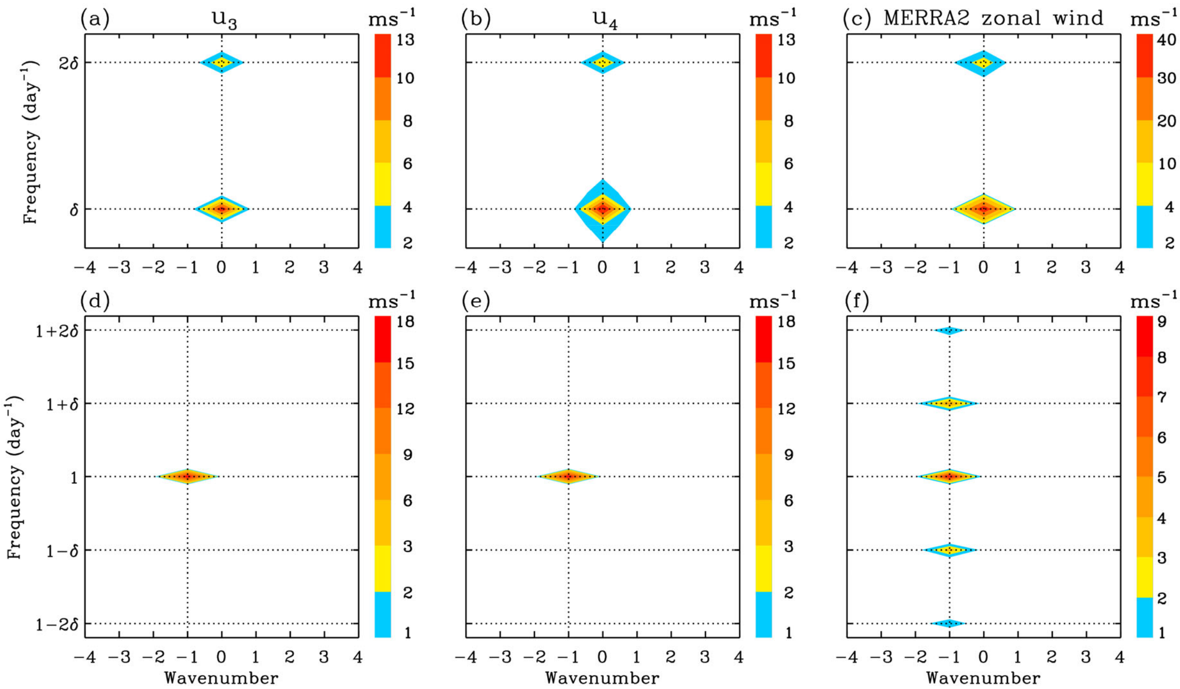

Figure R4. Frequency-wavenumber spectra of (left and mid columns) constructed two-dimensional wind fields and (right column) zonal wind in our manuscript. The sign δ is the frequency of AO (1/365 day^{-1}). The negative wavenumber represents the westward phase progression.

Based on the comparison and analysis above, the artificial harmonics are induced in the Lomb-Scargle spectrum indeed, and these weak spurious spectra decrease rapidly as they are far away from the dominant spectral peak, whereas, the new sum and difference spectra in our manuscript are significantly stronger than these artificial harmonics. In the one- and two-dimensional Fourier spectra of linearly superimposed wind fields, the spurious spectra do not occur, and the new sum and difference spectral components do not occur yet, which is obviously different from the results in our manuscript.

In particular, at some heights, the new components have the amplitudes of 12.5 and 18.5 ms^{-1} in the zonal and meridional wind, and are stronger than the DT and SDT themselves, as shown in Figures

4(b) and 5(b) in our manuscript. Such strong new spectral components should not be the artificial harmonics, and cannot occur in the linearly superimposed wind yet, therefore, these new spectral components in our manuscript should be generated through the nonlinear interaction.

2. Note that this study shows only the spectra of the data. Investigation of the physical mechanism is missing. Does the interaction of the tidal modes and the atmospheric background circulation cause the nonlinear process? How's the atmospheric gravity waves dump their momentum energy into the wind flow? Does the QBO driven by tropospheric activities also play a role in the mechanism of the nonlinear interaction? The QBO is highly nonlinear and nonstationary because it varies significantly during the study period from 2012 to 2021. Investigating the mechanism in detail is also required.

The physical mechanism of nonlinear interaction originates from the nonlinear terms in the fluid dynamic equations. For example, the energy equation can be written as follows,

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z} - T(\gamma-1)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) + \frac{Q}{c_V}$$

where T is the atmospheric temperature; u, v, w are the zonal, meridional and vertical components of wind, respectively; $\gamma = c_p / c_V$, and c_V and c_p are the specific heats at constant volume and pressure, respectively; Q is the change in atmospheric heat per mass and per time. If we consider the effects of the Earth's rotation and revolution movement, Q includes the diurnal variation Q_1 and annual variation Q_2. As energy sources, Q_1 and Q_2 can excite the diurnal tide and AO in the atmospheric temperature, respectively. According to the momentum equations, the atmospheric velocity is followed to be changed due to the temporally and spatially changed temperature, and similarly, the oscillations occur in the atmospheric density and pressure from the continuity and state equations. This means that the Earth's rotation and revolution induce the diurnal tide and AO in the atmospheric parameters.

Let's take the energy equation as an example again. All the other terms except the last heat source one on the right-hand side of the equation are the nonlinear terms, and these nonlinear product terms can induce the high-order harmonics in the temperature. Similarly, the high-order harmonics are generated in the wind velocity, density and pressure. The nonlinear terms in the dynamic equations are the intrinsic mechanism of high-order harmonic generation. Generally, the harmonic magnitudes decrease with their order number increasing, as shown in previous study of interaction between gravity waves (Huang et al., JGR, 2013). Hence, there exist high-order modes of DT in the atmosphere, and the SDT is strong but the 8- and 6-h tidal modes are weak. The new components with the sum and difference frequencies between DT/SDT and AO/SAO are also excited through this mechanism, in other words, these new components are the nonlinear results between the atmospheric diurnal and annual variabilities originating from the rotation and revolution movement of the Earth in nature.

According to your suggestion, we add the corresponding description in the revision as follows,
"Similar to the interaction of waves in the atmosphere, the interaction between the tides and AO/SAO comes from the nonlinear advection terms in the dynamic equations. Differential absorption of powerful solar radiation is the basic driver of atmospheric motion, and the rotation and revolution movement of the Earth leads to the diurnal and annual variabilities in the atmosphere. In this case, the diurnal and annual periods are stable in the atmosphere, and then the new components are exactly located at their sum and difference frequencies. When considering the seasonality of the tidal amplitudes, this means that there are also their sum and difference spectral components in the spectrum analysis. The investigation (not presented) indicates that the annual and semiannual modulations in the tidal amplitudes are weak, and the modulation can deviate from the period of a year or half a year due possibly to the changes in water vapor, ozone and atmospheric temperature and motion conditions. Hence, these new components originate mainly from the nonlinear dynamic response in the atmosphere to the rotation and revolution movement of the Earth."