We thank the reviewer for the comments on the manuscript. Below, we have replied to the review and have detailed the corresponding edits that we have made to the manuscript. We have listed out the reviewer comments in *black italic* and the replies in blue.

RC1: Dr. Daven Henze

Overall, the manuscript revisions address nearly all of the questions raised by myself and other reviewers, which were mostly issues of clarification, and I appreciate their efforts and explanations.

I do have one outstanding point / question: — the authors assert that y - K'x' = y - Kx ~N(0,So), i.e. the measurements follow a normal distribution with covariance So. Can they provide a plot to show this? Normality here is helpful as the framework after the x to x' transformation appears Gaussian and traditional equations apply for linear estimates of the posterior error at the solution, and the Ak. It is still a bit contradictory though to say that x is log normally distributed, K linearly well represents the forward model, and yet that y - Kx is normally distributed. It's possibly very close though. So, certainly it is easy to check — can they make a histogram of y - Kx, or even just y, and show that it is normally distributed? I know with some remote sensing datasets there will be a subset of y that is actually negative, consistent with a normal distribution. This can pose a bit of a quandary though if one filters the negative data, because then the distribution is no longer Gaussian. So I wonder if the authors encounter this at all, and, if they do, how it was dealt with.

Regardless, this is a rather small point of discussion, given that the authors recognize the limitations of the analytical expressions themselves and incorporate ensemble information into their error estimates. I'm only belaboring this point as it applies to a methodology this group is using for multiple studies. I trust they can check / report the skewness of their data and make updates to the manuscript as appropriate without further need for review.

We thank the reviewer for the suggestion. We have added a figure (Fig. S1) in the Supporting Information to show that $\boldsymbol{y} - \mathbf{K}\hat{\boldsymbol{x}}$ is normally distributed, and denoted it on Page 7 lines 298-300 (underline part added):

'As $\boldsymbol{x}' - \boldsymbol{x}'_{\boldsymbol{a}} \sim N(0, \mathbf{S}'_{\mathbf{a}})$ and $\boldsymbol{y} - \mathbf{K}'\boldsymbol{x}' = \boldsymbol{y} - \mathbf{K}\boldsymbol{x} \sim N(0, \mathbf{S}_{\mathbf{o}})$ (Fig. S1), both the prior and observational errors are Gaussian with zero mean; there is a non-linearity relationship between $\boldsymbol{x}'$ and $\boldsymbol{y}$ that are linked by $\mathbf{K}'$.'

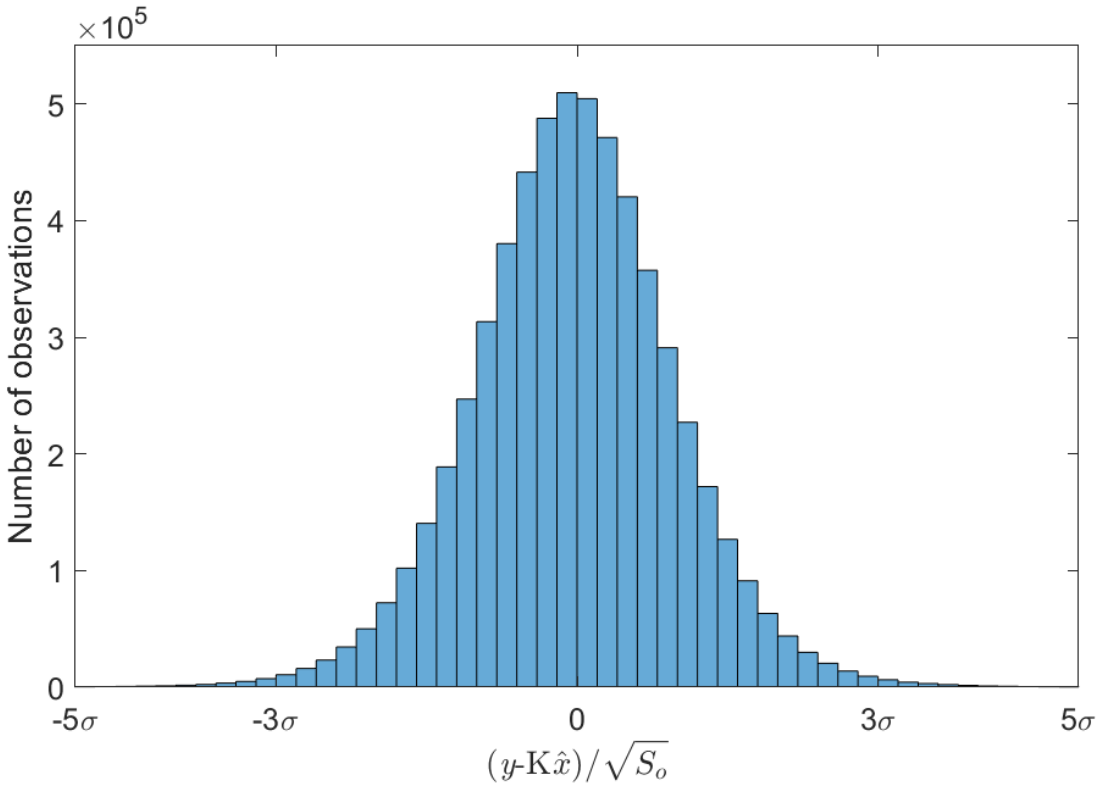

Figure S1. The histogram distribution of $\boldsymbol{y - K\hat{x}}$.