Referee #01 Response

“Sources of Concentric Gravity Waves Generated by a Moving Mesoscale Convective System in Southern Brazil” by Nyassor et al.

The authors thank the reviewers for their insightful and constructive comments, corrections and suggestions. We implemented the comments, corrections and suggestions into a revised version of the manuscript. Please find our answers to the questions of the reviewers below, accompanied by the blue marked-up texts in the manuscript version. All major changes made due to the comments of RC 1 and RC 2 are highlighted in indigo.

Comment on acp-2022-267

Anonymous Referee #1

Referee comment on “Sources of Concentric Gravity Waves Generated by a Moving Mesoscale Convective System in Southern Brazil” by Prosper Kwamla Nyassor et al., Atmos. Chem. Phys. Discuss., https://doi.org/10.5194/acp-2022-267-RC1, 2022

In their manuscript “Sources of Concentric Gravity Waves Generated by a Moving Mesoscale Convective System in Southern Brazil”, the authors combine measurements from multiple instruments at multiple altitudes with ray tracing models to connect gravity wave observations in the OH layer with their sources at the tropopause. Single cores of a moving mesoscale convective system are identified as most likely sources of the concentric gravity waves observed in the OH layer.

This work is quite impressive from the amount of data used and the analysis performed. However, I would suggest to better explain the reasoning behind all the measurements taken and the analysis performed. Both the introduction and the summary/conclusion section are missing clear scientific research questions motivating the work. At the moment the paper seems a bit like a data dump without a clear motivation behind. This impression could possibly be overcome by refining the introduction and conclusion. Additionally, I would suggest to revise all the sections/figures for their necessity to support the findings and possibly remove some of them.

Author response: We thank Referee #01 for the revision of the manuscript and constructive suggestions. We addressed the individual comments below.

A detailed list of minor comments is given below.
Comment #01: L24: Better: “in the tropical troposphere”, as otherwise one might start arguing about the importance of orographic versus convective gravity waves.

Response: There are several known sources of the GWs among which tropospheric convection - severe weather conditions such as thunderstorms are considered to be the most important and natural sources of AGWs in the tropical troposphere. ...... “tropical” is added.

Comment #02: L27: Maybe better: ... mechanism is known to generate concentric gravity waves ... (or remove at least “natural”, you probably do not want to start a discussion about nonnatural GW sources).

Response: Among these three, the mechanical oscillator (overshooting) mechanism is known to be one of the sources of concentric gravity waves (CGWs). ........ “natural” is removed.

Comment #03: L27: Most [...] cases [...] have [...] 

Response: Most tropospheric deep convection - CGWs cases in literature have other associated convection related phenomena, such as hailstorm (Yue et al., 2009; Vadas et al., 2012), lightning (Yue et al., 2014; Nyassor et al., 2021) and Transient Luminous Events (TLEs) (Boccippio et al., 1995; São Sabbas and Sentman, 2003; Sentman et al., 2003; Pasko et al., 2012) which are used as a measure of the severity of the thunderstorm. ..... “has” has been changed to “have”.

Comment #04: L38: ... imgaes ... have been used ...

Response: Regarding the observation of tropospheric convective sources of GWs/CGWs, infrared images of cloud top brightness temperatures (CTBT) from satellite imagery have been used (Vadas et al., 2009a; Yue et al., 2009; Azeem et al., 2015; Takahashi et al., 2018; Figueiredo et al., 2018; Nyassor et al., 2021). ... “has” has been changed to “have”.

Comment #05: L42: ray tracing models.

Response: Similar works by Vadas et al. (2009a), Vadas et al. (2012), Xu et al. (2015) and Nyassor et al. (2021) used ray tracing models to relate observed CGWs in OH emission altitude ~87 km to overshooting top in CTBT images captured by the Geostationary Operational Environmental Satellites (GOES). .... “model” has been changed to “models”.

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**Introduction:** You are nicely describing what was done before and what you are doing, but you are missing to describe the difference and provide a reasoning for your study. Why do we need this paper? What does it bring new to the world?

**Response:** The introduction has been modified to explain the new things the paper bring in to the field.

**Comment #06:** L101-103: This is a hypothesis which will be confirmed later. Maybe rephrase: According to previous studies (. . . ), the appearance of concentric structures in the airglow hints to: 1) point-like convective overshooting of the tropopause and 2) weak intervening background wind.

**Response:** According to previous studies (Vadas and Fritts, 2009; Vadas et al., 2009b; Yue et al., 2009), the appearance of the concentric structures in the airglow suggests; 1) point-like convective overshooting of the tropopause and 2) weak intervening background wind. .... The entire sentences has been rephrase.

**Comment #07:** L105: One parenthesis too much.

**Response:** The extra parenthesis has been removed.

**Table 1:** What is the exact definition of the propagation direction? Degree measured clockwise from North? And how do you define the propagation direction of a concentric outward propagating wave? Is only the value of the centre fit shown for the propagation direction?

**Question a.** What is the exact definition of the propagation direction?

For CGWs, they are expected to propagate in all directions. For this study, since we estimated the wave parameters along a specific direction, we only obtain one value. However, knowing that CGW propagates in all direction, we considered several propagations on the first visible concentric wavefront, then we estimate the wave parameters. Under weak wind condition, the parameters are expected to be similar, which is the case for these events. So, to give a general characteristics on the parameters of the wave, we compute the average for all the parameters obtained from each direction we considered. Hence, the propagation direction presented in Table is chosen arbitrarily for the ray tracing since the values obtained from all directions considered are similar. Based on your comment, we have explain this in the main text.
Question b. Degree measured clockwise from North?

Yes, the propagation direction is measured in degree clockwise from the North.

Question c. And how do you define the propagation direction of a concentric outward propagating wave?

As mentioned in the response to the “a.”, we determined the propagation direction for a single direction that we are computing the wave parameters. The propagation direction is determined as follows:

Conti...:

As shown in Figure above, we apply discrete Fourier transform (Equation 1) to the regions in the selected time series images (labelled as t1, t2, t3, t4, t5).

\[ F(k, l) = \sum_{x=0}^{m-1} \sum_{y=1}^{n-1} e^{-i2\pi \left( \frac{nx}{m} + \frac{ly}{n} \right)} f(x, y) \]  

Where \( F(k, l) \) is the Fourier transform of the function \( f(x, y) \), \( k, l \), are the zonal and meridional wavenumbers, \( M \times N \) is the dimensions of the analyzed image.

Next the cross-spectrum, \( C(k, l) \) giving by Equation 2 was then estimated for each two successive time series images represented by \( f(x, y) \) and \( g(x, y) \).

\[ C(k, l) = f(k, l)g^*(k, l) \]  

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in which $C(k,l)$ is the cross spectrum between two successive time series and $f(k,l)$ and $g(k,l)$ represent the discrete Fourier transform, respectively. $g^*(k,l)$ represent the complex conjugate of $g(k,l)$ is the complex conjugate. The cross-spectrum was then average over the $n-1$ computed spectra.

We then computed the amplitude of the cross spectrum expressed by the modulus of the cross-spectrum (Equation 3),

$$|C(k,l)|$$

and the phase of the cross-spectrum express as

$$\varphi(k,l) = \arctg\left\{\frac{\text{Im}[C(k,l)]}{\text{Re}[C(k,l)]}\right\}, \quad -\pi \leq \varphi \leq \pi. \tag{4}$$

The determination of wave parameters ($\lambda_H, \tau, c_H, \phi$) using cross-spectrum had been described by Maekawa (2000).

From the cross-spectrum:

i. The zonal ($k$) and meridional ($l$) wavenumbers of gravity waves are the $k$ and $l$ corresponding to the maximum amplitude of the cross-spectrum;

ii. Using the zonal ($k$) and meridional ($l$) wavenumbers, and the phase ($\varphi(k,l)$), the observed phase velocity is given by:

$$c_H = \frac{1}{\sqrt{k^2 + l^2}} \times \frac{\Delta \varphi(k,l)}{360^\circ} \times \frac{1}{\Delta t}, \tag{5}$$

where $\Delta t$ is the time difference between two successive images

iii. The horizontal wavelength $\lambda_H$ of the wave was determined by

$$\lambda_H = \frac{1}{\sqrt{k^2 + l^2}}, \tag{6}$$

iv. Next, the observed period of the gravity wave is estimated using:

$$\tau_H = \frac{\lambda_H}{c_H}, \tag{7}$$

v. Finally, the propagation direction of the wave can be determined from

$$\phi = \tan^{-1}\left[\frac{l(l_{\text{max}})}{k(k_{\text{max}})}\right], \tag{8}$$

where $k_{\text{max}}$ and $l_{\text{max}}$ are the maximum wavenumbers in the $x$ and $y$ directions.
Continuation:

d. Is only the value of the centre fit shown for the propagation direction?

Yes, it is. However, we would like to mention that the center fit is an approximation, considering zero wind, but for the only ray path shown is for only a chosen value as described in (a).

Figure 2: (+ description): Maybe do this as an example on top of one of the plots from Figure 1, so one can better understand what you are doing. Maybe explain the crest/trough part already at point 1. Makes it easier to follow. In 2: The 3 circles have to have the same radius, no? In 5: Why so complicated? Why not directly measure from P to where one of the lines from 3 crosses the black circle? Are all these steps done manually for each case or is there an automated software?

(a.) The 3 circles have to have the same radius, no?

Yes, the 3 circles in this case have the same radius.

(b.) In 5: Why so complicated? Why not directly measure from P to where one of the lines from 3 crosses the black circle?

It appears a bit complicated here because, we are trying to give details on our approach and also because we are using three circles in this case, care is taken in order to accurately verify the computation.

(c.) Are all these steps done manually for each case or is there an automated software?

Yes, please, we have developed a software to automatically run the computation. We only need the three initial positions of the three (3) centers as inputs.

Based on your comment the description here has been modified to simplify version in the main text as shown below.
Chapter 2.1: Ray tracing part: How is the vertical wavelength/wave number determined, which is necessary for the raytracing? All 4 stopping criteria seem a bit arbitrary. Can you explain in more detail why you chose these four stopping criteria? What is the physical reasoning behind each of them?

• How is the vertical wavelength/wave number determined, which is necessary for the raytracing?

The vertical wavelength/wave number was determined along the ray path at each step. However, Equation 3 (dispersion relation) in the main text has some conditions imposed. This restriction causes the dispersion equation to reduce to

\[
\omega_{tr}^2 = \frac{k_B^2 N^2}{m^2 + k_H^2 + 1/4H^2},
\]

(9)

when dissipation is negligible (Gossard and Hooke, 1975) below the turbopause. Equation 3 is the dispersion relation for a GW in the presence of molecular viscosity and thermal diffusivity.

All 4 stopping criteria seem a bit arbitrary. Can you explain in more detail why you chose these four stopping criteria? What is the physical reasoning behind each of them?

• Since a GW propagates at the group velocity, this ray tracing is constrained to permit GWs propagating slower than the speed of sound, \(c_g \leq 0.9c_s\), where the factor 0.9 is arbitrarily chosen. Here \(c_g = \sqrt{c_{gx}^2 + c_{gy}^2 + c_{gz}^2}\) is the group velocity in the wave propagation direction. This condition is set to remove spectrum of acoustic waves.

• When a GW encounters a region in the atmosphere where the horizontal phase velocity is exactly equal to the horizontal wind speed, the GW approaches a critical level or absorption level. Physically, the vertical propagation of the wave becomes very slow, thereby causing the wave not to propagate horizontally because \(m \to \infty\). In this condition, the intrinsic frequency of the wave approaches zero in this region causing the wave to be rapidly absorbed by the atmosphere. Therefore, for the ray tracing to calculate the wave trajectory, it is necessary for the intrinsic frequency of GW to be greater than zero (\(\omega_{tr} > 0\)).
Chapter 2.1:

- As GWs propagate higher into the thermosphere, molecular viscosity and thermal diffusivity become an important dissipative process owing to the decrease in density with altitude. This causes an increase in the momentum flux of GWs in the lower thermosphere until it reaches a maximum value and then begins to decrease rapidly with increasing altitude. This, therefore, implies as GWs attain their maximum momentum flux, they tend to dissipate. Hence, it is necessary for the momentum flux along a GW ray path must satisfy $R_m > 10^{-15} R_0$. $R_m$ is the momentum flux at each altitude and $R_0$ is the momentum flux at the reference altitude. The factor $10^{-15}$ was arbitrarily chosen.

- The vertical wavelength needs to be smaller than the viscosity scale, to ensure that the viscosity will not change too much in time and altitude, that is, $|\lambda_z| < \frac{2\pi}{\frac{\nu}{\mu/\rho}}$. Here, $\nu = \mu/\rho$ is kinematic viscosity with $\mu$ being molecular viscosity and $\rho$ the density (Vadas, 2007). This condition is necessary because GWs with these characteristics must satisfy the imposed simplifications (slowly varying parameters) to obtain the dispersion relation.

- Based on your comments, this aspect of the manuscript have been modified to include this comment.

The advantage of this ray-tracing method (variable background wind over this large altitude range) should be made clear somewhere in the text.

This model is used to study the propagation of GWs relative to variable background wind in the atmosphere and to determine the possible source location. The propagation of GWs in the presence of background wind enables the investigations of the effect of the wind on the wave.

Comment #08: L186ff: Please reorder for better readability:

Response: "Adapting Equation 3, the overshooting top height ($OT_{\text{Height}}$) from Griffin et al. (2016), the overshooting tops (OT) were estimated using the brightness temperature (BT) of the OT, the tropopause height and temperature" has been modified to:

Adapting Equation 3 from Griffin et al. (2016), the overshooting top height ($OT_{\text{Height}}$) is estimated using the brightness temperature (BT) and the lapse rate (LR) of the overshooting top (OT), the tropopause height and temperature.
\[ OT_{Height} = H_{Trop} + \frac{OT_{BT} - T_{Trop}}{OT_{LR}} \]  \hspace{1cm} (10)

Here \( H_{Trop} \) is the tropopause height, \( OT_{BT} \) is the brightness temperature of the OT, \( T_{Trop} \) is the tropopause temperature and \( OT_{LR} \) is the OT lapse rate.

The cloud top brightness temperature was obtained from the Advanced Baseline Imager (ABI), which is an imaging radiometer of GOES-R satellite. The ABI has 16 different spectral bands, including two visible channels, four near-infrared channels, and 10 infrared channels with a spatial resolution of 0.5-2 km. Among the weather and climate products of these channels, the CTBT product is derived from the 11, 12 and 13.3 \( \mu \text{m} \) infrared observations.

The OT lapse rate was estimated using the radiosonde profile and the CTBT. The OT lapse rate for the days considered in the determination of the tropopause temperature and altitude were averaged and was found to be -7.35 Kkm\(^{-1}\).

Comment #09: L160: Why do you state this here, as you are not tracing any waves forward to the thermosphere?

Response: We mentioned it here just to emphasize the fact that these two conditions are not the most important stopping conditions within the altitude range considered.

Comment #10: [. . .] regions have been used to identify [. . .]

Response: L220: “ [. . .] regions has been used to identify [. . .]” has been changed to “ [. . .] regions have been used to identify [. . .]”.

Comment #11: L221 & 222: I am confused by the use of references in this sentence. Why are you referencing other papers for things you have done here in your paper?

Response: There references were giving because, we intend to give credence to previous work. Base on the comments, the sentence on L221 & 222 has been modified to:

The coldest regions has been used to identify convective overshooting tops (Bedka et al., 2010; Jurkovic et al., 2015) and the possible source of the CGWs (Nyassor et al., 2021 and reference therein).
Comment #12: **L231:** Maybe better “deployed”

Response: We have changed the word “employed” to “use”.

Figure 6: The description of the different colours and vectors belongs to the caption, not the text.

Response: The regions where wave propagation is not permitted (forbidden region) due to the characteristics of the wind in the troposphere correspond to the wind from 0-17 km (red rings). Between the tropopause and mesopause (light blue rings), the wind between 18 and 87 is presented, whereas above 87 to 100 km, the wind are shown by the green rings.

The description of the different colours and vectors has been included in the caption as:

**Caption description:** The wind characteristics from 0-17 km is represented by red rings, from 18-87 km by light blue rings and above 87 km by green rings.

Comment #13: **L258f:** If this criterion is not applicable to the cases you show, why do you introduce it?

Response: We have modified this part of the text, taking into consideration your comment.

The waves observed here have phase speed sufficiently large that the winds below do not introduce major distortions in the wave fronts and that they hence can be still recognized as concentric wave structures.

Comment #14: **L284:** Maybe use “shown” instead of “demonstrated”?

Response: “demonstrated” has been changed to “shown”

Comment #15: How is the CAPE calculated?

The CAPE can be estimated using

\[
CAPE = \int_{p_f}^{p_n} R_d(T_{vp} - T_{ve})dInp
\]  

(11)
where \( T_{vp} \) is the virtual temperature of a lifted parcel moving upward moist adiabatically from the level of free convection to the level of neutral buoyancy, \( T_{ve} \) is the virtual temperature of the environment, \( R_d \) is the specific gas constant for dry air, \( p_f \) is the pressure at the level of free convection, and \( p_n \) is the pressure at the level of neutral buoyancy.

Source: https://glossary.ametsoc.org/wiki/Convective_available_potential_energy

Comment #16: L290ff: Is this an observation from your data or from Kim et al.? I would suggest to rephrase: It was observed in previous studies that the presence of deep convection with colder cloud tops brightness temperatures further decreases the tropopause temperature (Kim et al., 2018). In our data this also happens between 12:00 UT on September 30, 2019, and 12:00 UT on October 3, 2019, when the cloud top temperature ranges between \(-40^\circ C\) to \(-90^\circ C\).

Response: Is this an observation from your data or from Kim et al.?

Yes, this is an observation form the data we used in this work, the temperature profile from radiosonde and radio occultation, and cloud top images from GOES-16.

Your suggestion has been considered in the main text.

Comment #17: Reorder Figures 10 & 11 according to their first mentioning in the text.

Response: Due to the comment of the RC1, this entire section has been modified.

Comment #18: L315ff: Please rephrase. The first and second sentence seem to be in contrast to each other. Were all four cores there or only core 3 & 4?

As mentioned in the previous comment, the entire section on the tracking of the convective cores in space and time have been re-written due to the comments and suggestions of RC1.

Comment #19: L344 - 355: Please explain better where you get the diameter of the plume at the tropopause from (31 km you state, but not where you extracted this value from).

Response: The explanation of the estimate of the plume diameter is given below. This is the new/modified version of the previous explanation.
This radius is the same as the radius of the first visible concentric structure in the OH images. The vertical dotted line indicates the height of the cone. The red horizontal solid line indicates the diameter of the tropopause, and the black and white vertical slanted dashed lines extending vertically at either side of the dotted line are used to demarcate the propagation of the CGWs above the tropopause after the overshooting of the tropopause. To determine the tropopause diameter, the concept of a conical propagation configuration of CGWs was used Vadas et al. (2009b); Nyassor et al. (2021). The base of the cone with a radius (same as the radius of the CGWs) of 154 km was set at an altitude of 87 km. We then followed the slant heights of the cone, thereby determining the radius at each kilometer until we reached the vertex of the cone. The vertex of the black and white dash line-slanted path is above the tropopause. The radius at the tropopause (i.e., the red horizontal line in the right panel of Figure 10) is ∼31 km (diameter of ∼62 km).

To approximate the diameter of the tropopause, the diameter of the convective plume was considered. According to Vadas et al. (2009a, 2012), a typical diameter of a convective plume is 15-20 km. So, we take into account the dome-like protrusion shooting from the highest overshooting point to the anvil of the cloud. Then, we set a threshold of 70°C and constructed a circle with a radius 31 km around the pixel with the coldest CTBT. The 70°C threshold was set because it was observed that other protrusion do originate from CTBT within this BT range within every 10 min. So, to restrict the selection of OTs to the maximum spatial resolution of 2 km plus ∼4 km (two extra pixel), we set the diameter of the overshooting region to be the diameter of the plume at the tropopause. The determination of the plume at the tropopause is important because it is used approximate possible regions within which overshootings are most likely to occur.

Chapter 3.4: This chapter seems a bit like unnecessary information to me. Please either remove or explain why it is necessary for your findings (e.g. mention it in the summary section).

Response: This events are presented in the current work to give emphasize to the severity of the convective activity before, during, and after the wave events presented.

Comment #20: L441: Is signifies the correct word here?

Response: I guess “indicates” would be the correction word here. So “signifies” has been changed to “indicates”.

Comment #21: L482f: spectra (plural) – > were
Response: The entire Section 3.5 has been removed.

Figure 15: Please use a more intuitive colour scale for a continuous (not around 0 centred) range. See also Crameri, Shephard, Heron (2020) for hints on good colour scale usage.

Response: To L482 & Figure 15.

As a result of the comments and suggestions in RC1, the entire Section 3.5 has been removed.

Comment #22: L524: *Rossby*

Response “Rosby” has been changed to “Rossby”

Figure A1: For better comparability, the colour scale (extend) for the upper plots should be kept constant. Additionally, please add an x-axis. Otherwise the points in the lower panels are really hard to interpret. Is the tropopause here defined by thy laps rate?

Response: Thanks very much. Please, the modified plot is shown below:

Figure A1:

- Is the tropopause here defined by thy laps rate?

Yes, please. We estimated the tropopause using both the cold point criterion and confirmed it using the lapse rate criterion.
Comment #23: Please check citation formatting throughout the whole manuscript.
Response: The citation formatting had been checked throughout the manuscript and the necessary correction had been implemented.