Influences of an Entrainment-Mixing Parameterization on Numerical Simulations of Cumulus and Stratocumulus Clouds

Xiaoqi Xu[1,2,4], Chunsong Lu[1*], Yangang Liu[3*], Shi Luo[1,5], Xin Zhou[3], Satoshi Endo[3], Lei Zhu[1], Yuan Wang[1]

1. Key Laboratory for Aerosol-Cloud-Precipitation of China Meteorological Administration/Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters (CIC-FEMD), Nanjing University of Information Science & Technology, Nanjing, China

2. Nanjing Joint Institute for Atmospheric Sciences, Nanjing, China

3. Environmental and Climate Sciences Department, Brookhaven National Laboratory, Upton, US

4. Key Laboratory of Transportation Meteorology, CMA, Nanjing, China

5. College of Aviation Meteorology, Civil Aviation Flight University of China, Guanghan, China

* Correspondence: luchunsong110@gmail.com; lyg@bnl.gov

Abstract

18   Different entrainment-mixing processes can occur in clouds; however, a

19   homogeneous mixing mechanism is often implicitly assumed in most commonly used

20   microphysics schemes. Here, we first present a new entrainment-mixing

21   parameterization that uses the grid-mean relative humidity without requiring the

22   relative humidity of the entrained air. Then, the parameterization is implemented in a

23   microphysics scheme in a large eddy simulation model, and sensitivity experiments are

24   conducted to compare the new parameterization with the default homogeneous

25   entrainment-mixing parameterization. The results indicate that the new entrainment-

26   mixing parameterization has a larger impact on the number concentration, volume-

27   mean radius, and cloud optical depth in the stratocumulus case than in the cumulus case.

28   This is because inhomogeneous and homogeneous mixing mechanisms dominate in the

29   stratocumulus and cumulus cases, respectively, which is mainly due to the larger

30   turbulence dissipation rate in the cumulus case. Because stratocumulus clouds break up

31   during the dissipation stage to form cumulus clouds, the effects of this new entrainment-

32   mixing parameterization during the stratocumulus dissipation stage are between those

33   during the stratocumulus mature stage and the cumulus case. A large aerosol

34   concentration can enhance the effects of this new entrainment-mixing parameterization

35   by decreasing the cloud droplet size and evaporation time scale. The results of this new

36   entrainment-mixing parameterization with grid-mean relative humidity are validated by

37   use of a different entrainment-mixing parameterization that uses parameterized

38   entrained air properties. This study sheds new light on the improvement of entrainment-

39   mixing parameterizations in models.

41   **1. Introduction**

42   The process of entrainment and subsequent mixing between clouds and their

43   environment is one of the most uncertain processes in cloud physics, which is thought

44   to be crucial to many outstanding issues, including warm-rain initiation and subsequent

45   precipitation characteristics, cloud-climate feedback, and evaluating the indirect effects

46   of aerosol (Paluch and Baumgardner, 1989; Yum, 1998; Ackerman et al., 2004; Kim et

47   al., 2008; Huang et al., 2008; Del Genio and Wu, 2010; Lu et al., 2011; Lu et al., 2014;

48   Kumar et al., 2013; Zheng and Rosenfeld, 2015; Fan et al., 2016; Gao et al., 2020; Gao

49   et al., 2021; Zhu et al., 2021; Xu et al., 2021; Kumar et al., 2013; Yang et al., 2016;

50   Yang et al., 2021). The most well-studied concepts are homogeneous/inhomogeneous

51   entrainment-mixing mechanisms. During homogeneous mixing, all droplets experience

52   evaporation, and no droplet is evaporated completely. During extremely

53   inhomogeneous mixing, some droplets near the entrained air evaporate completely,

54   while the remaining droplets maintain their original sizes. If the situation is somewhere

55   between these two extreme scenarios, an inhomogeneous mixing process occurs. Some

56   studies suggest that homogeneous mixing is likely to be typical (Jensen et al., 1985;

57   Burnet and Brenguier, 2007; Lehmann et al., 2009), whereas others have claimed that

58   extremely inhomogeneous scenario is dominant (Pawlowska et al., 2000; Burnet and

59   Brenguier, 2007; Haman et al., 2007; Freud et al., 2008; Freud et al., 2011). Different

60   mechanisms can be undistinguishable when the relative humidity in the entrained air is

61   high (Gerber et al., 2008).

62   Some sensitivity studies assuming homogeneous or extremely inhomogeneous

63   mixing have found that different mixing mechanisms can significantly influence the

64   microphysics and radiative properties of clouds (Lasher-Trapp et al., 2005; Grabowski,

65   2006; Chosson et al., 2007; Slawinska et al., 2008). For example, Grabowski (2006)

66   used a cloud-resolving model and found that the amount of solar energy reaching the

67   surface in the pristine case, assuming the homogeneous mixing scenario, is the same as

68   in the polluted case with extremely inhomogeneous mixing. This result was verified by

69   Slawinska et al. (2008) using a large-eddy simulation (LES) model. Although the

70   influence of different mixing mechanisms in simulations is lower when two-moment

71   microphysics schemes are used (Hill et al., 2009; Grabowski and Morrison, 2011;

72   Slawinska et al., 2012; Xu et al., 2020), Hill et al. (2009) also claimed that there are

73   still many uncertainties in the entrainment-mixing process, and the effect of different

74   mixing mechanisms can be more important over the entire cloud life-cycle.

75   In recent years, methods have been developed to describe general entrainment-

76   mixing processes, with homogeneous and extremely inhomogeneous scenarios as

77   special cases (Andrejczuk et al., 2006; Andrejczuk et al., 2009; Lehmann et al., 2009;

78   Lu et al., 2011). Hoffmann et al. (2019) and Hoffmann and Feingold (2019) conducted

79   LES at the subgrid-scale with turbulent mixing, using a linear eddy model. Andrejczuk

80 et al. (2009) used the results of direct numerical simulation (DNS) to establish a

81 relationship between instantaneous microphysical properties and Damköhler number

82 (D_a, Burnet and Brenguier, 2007), and developed a parameterization of the entrainment-

83 mixing process. Lu et al. (2013) developed a parameterization of the entrainment-

84 mixing process based on the relationship between the homogeneous mixing degree (ψ)

85 and transition scale number (N_L) in the explicit mixing parcel model (EMPM), as well

86 as aircraft observation data. Gao et al. (2018) investigated how ψ is related to D_a and

87 N_L in a DNS, to improve the parameterization of the entrainment-mixing process. Luo

88 et al. (2020) simulated more than 12,000 cases with EMPM by changing a variety of

89 parameters affecting entrainment-mixing processes and developed a parameterization

90 that improved the one proposed by Lu et al. (2013).

91   Although several entrainment-mixing parametrizations have been proposed, to the

92 best of our knowledge, only one study (Jarecka et al., 2013) has coupled an entrainment-

93 mixing parameterization with cloud microphysics to consider the change in cloud

94 droplet concentration during the entrainment-mixing process. Jarecka et al. (2013)

95 applied an entrainment-mixing parameterization, in terms of the Damköhler number, to

96 a two-moment microphysics scheme and found small impacts of entrainment-mixing

97 parameterization in shallow cumulus clouds. To further explore the influences of

98 entrainment-mixing processes, this study first modifies the entrainment-mixing

99 parameterization in terms of the transition scale number proposed by Luo et al. (2020)

100 to couple it more easily with microphysics schemes. The parameterization is then

101   implemented in the two-moment Thompson aerosol-aware scheme (Thompson and

102   Eidhammer, 2014). Finally, the effects of parameterization on the physical properties

103   of clouds are examined in both cumulus and stratocumulus clouds.

104   The rest of this paper is organized as follows: Section 2 describes the new

105   entrainment-mixing parameterization, simulated cases, and modelling setup. The major

106   results are presented and discussed in Section 3. The influences of the new entrainment-

107   mixing parameterization on cloud physics and the underlying mechanisms are

108   examined, and the effects of turbulence dissipation rate (ε) and aerosol concentration

109   are also discussed. Some concluding remarks are presented in Section 4.

110

111   **2. Parameterization, simulated cases, and modeling setup**

112   **2.1 The new entrainment-mixing parameterization**

113   According to Morrison and Grabowski (2008), the effect of the entrainment-

114   mixing process on cloud microphysical properties can be expressed as follows:

$$115 \qquad\qquad N_{c} = N_{c0} \left(\frac{q_{c}}{q_{c0}} \right)^{\alpha}, \qquad\qquad (1)$$

116   where N_c and N_{c0} are the cloud droplet number concentrations after and before the

117   evaporation process, respectively, and q_c and q_{c0} represent the corresponding cloud

118   water mixing ratios. It is noteworthy that when a new saturation is achieved after

119   evaporation, q_c is determined by q_{c0}, relative humidity (RH), air pressure, and

120   temperature. The parameter α can be pre-set to any value between 0 (homogeneous

121   mixing) and 1 (extremely inhomogeneous mixing) to represent a different degree of

122 subgrid-scale mixing homogeneity. In this study, instead of specifying α as a

123 predetermined constant, here it is determined through expressions (Lu et al., 2013; Luo

124 et al., 2020)

$$\alpha = 1 - \psi, \tag{2a}$$

$$\psi = c \exp(a N_{\mathrm{L}}^{b}). \tag{2b}$$

127 where a, b and c are the three fitting parameters (Luo et al., 2020). The dimensionless

128 number N_{L} is a dynamical measure of the degree of subgrid-scale mixing homogeneity

129 (Lu et al., 2011) defined by

$$N_{\mathrm{L}} = \frac{L^{*}}{\eta}, \tag{3a}$$

$$\eta = \left(v^{3} / \varepsilon \right)^{1/4}, \tag{3b}$$

$$L^{*} = \varepsilon^{1/2} \tau_{\mathrm{evap}}^{3/2}, \tag{3c}$$

133 where L^{*} is the transition length (Lehmann et al., 2009), η is the Kolmogorov microscale,

134 v is the kinematic viscosity; ε is calculated from the subgrid turbulent kinetic energy

135 (Deardorff, 1980):

$$\varepsilon = C E^{3/2} / L, \tag{4}$$

137 where $C = 0.70$ is an empirical constant, E is the subgrid turbulent kinetic energy, and

138 L is the model grid size. The evaporation time scale (τ_{evap}) is defined as the time taken

139 for droplets to evaporate completely in an unsaturated environment, and is calculated

140 as

$$\tau_{\mathrm{evap}} = -\frac{r^{2}}{2 A S_{\mathrm{e}}}, \tag{5a}$$

142
$$A = \cfrac{1}{\left[\left(\cfrac{L_h}{R_v T} - 1 \right) \cfrac{L_h \rho_L}{KT} + \cfrac{\rho_L R_v T}{D e_s(T)} \right]},$$
(5b)

143 where r is the volume-mean radius of cloud droplets, A is a function of pressure and

144 temperature, S_e is the supersaturation (RH-1) of entrained air, L_h is the latent heat, R_v is

145 the specific gas constant for water vapour, T is air temperature, ρ_L is the density of liquid

146 water, K is the coefficient of thermal conductivity of air, D is the diffusion coefficient

147 of water vapour in the air, and $e_s(T)$ is the saturation vapour pressure over a plane water

148 surface at temperature T.

149   Unfortunately, S_e in Equation (5a) is generally unavailable in atmospheric models,

150 including LES models. Thus, the entrainment-mixing parameterization developed by

151 Luo et al. (2020) based on the properties of entrained air cannot be used directly. To

152 solve this problem, we modify the entrainment-mixing parameterization of Luo et al.

153 (2020) by replacing S_e with the domain-mean RH in the EMPM, after entrainment but

154 before evaporation, based on 12,218 cases:

155
$$\psi = 107.19 \exp(-1.99 N_L^{-0.29}).$$
(6)

156 Figure 1 shows the fitting results of the modified new entrainment-mixing

157 parameterization. Compared to the parametrization proposed by Luo et al. (2020), the

158 modified parameterization has similar ψ-N_L distributions, but with a larger N_L for the

159 same ψ, because the EMPM domain-mean RH is larger than the entrained air RH. With

160 this modification, N_L, ψ, and thus the effect of the entrainment-mixing processes on

161 droplet concentration can be directly calculated using the LES grid-mean RH. It is

162   important to note that the parameterization does not mean that the entrained air RH is

163   equal to that of the LES grid-mean RH. It is also worth noting that a wide range of ε,

164   S_e, and fraction of entrained air (f) are taken into account when establishing the

165   parameterization with the EMPM. The details of the EMPM simulations and related

166   calculations are provided by Luo et al. (2020).

167

168   **2.2 LES model, simulation cases, and modelling setup**

169   The LES model is built by applying the large-scale forcing module presented in

170   Endo et al. (2015) to the Weather Research and Forecasting (WRF) model tailored for

171   solar irradiance forecasting (WRF-Solar, Hacker et al., 2016; Haupt et al., 2016). The

172   large-scale forcing data (VARANAL) used in this process is derived from the

173   constrained variational analysis (CVA) approach developed by Zhang et al. (2001) and

174   provided by the U.S. Department of Energy's Atmospheric Radiation Measurement

175   Program (www.arm.gov). The modified entrainment-mixing parameterization is

176   implemented in the two-moment Thompson aerosol-aware scheme (Thompson and

177   Eidhammer, 2014).

178   To investigate the behaviours of the new entrainment-mixing parameterization in

179   different cloud types, cumulus and stratocumulus cases are simulated. For both the

180   cumulus and stratocumulus cases, the horizontal resolution of the model is 100 m × 100

181   m with a domain area of 14.4 km × 14.4 km. The vertical direction is divided into 225

182   layers with a resolution of 30 m.

183   For each cloud case, ψ is first set to 1 for the *default* experiment because most LES

184   models assume a homogeneous entrainment-mixing mechanism. The simulation with

185   the new entrainment-mixing parameterization (Equations (1-6)) is hereafter referred to

186   as *new*. First, N_L is diagnosed for each grid, and ψ is then calculated using Equation (6).

187   Finally, the variation in N_c during entrainment-mixing is obtained using Equations (1)

188   and (2a). To examine the influence of the aerosol number concentration on the

189   entrainment-mixing process, we conduct the numerical experiments *default_10* and

190   *new_10* by multiplying the initial aerosol number concentrations, for the *default* and

191   *new* models, respectively, by a factor of 10. Thus, four sets of numerical experiments

192   are conducted for both the cumulus and stratocumulus cases; the names of the

193   experiments and corresponding descriptions are summarized in Table 1.

194

195   **3. Results**

196   **3.1 Cumulus case**

197   For the cumulus case, the simulation starts at 12:00 UTC on 11 June 2016 and

198   ends at 03:00 UTC on 12 June 2016 with an output interval of 10 min and spin-times

199   of 3 h. To demonstrate the utility of the model, Figure 2 compares the temporal

200   evolution of the observed and simulated cloud fraction (a) and solar irradiance (b) from

201   the *default* experiment. Grid points with q_c larger than 0.01 g/kg are defined as "cloudy

202   areas". Also shown for comparison is observational data with a one-hour temporal

203   resolution, which is provided by the LES Atmospheric Radiation Measurement

204   Symbiotic Simulation and Observation (LASSO) campaign (Gustafson et al., 2020).

205   The observations show that the cloud forms at 12:00 UTC on 11 June and dissipates

206   completely by 01:00 UTC on 12 June with a maximum cloud fraction of 0.47 at 16:00

207   UTC on 11 June. Considering the difference between the solar irradiances obtained

208   from point measurements and the value representing the simulation domain, the

209   observed solar irradiance at the Southern Great Plains (SGP) Central Facility are

210   compared with the results of central grid point in simulation (Figure 2(b)). Evidently,

211   although the results of simulation do not fluctuate as much as the observations, the

212   model captures the general behaviours of both cloud fraction and solar irradiance. The

213   general agreement between the simulations and observations lends credence to using

214   the model in further study.

215   Figure 3 shows the evolution of the microphysical and optical properties of clouds

216   in the cloudy areas of all simulation experiments, including q_c, N_c, droplet volume-

217   mean radius (r_v), cloud water path (CWP), and cloud optical depth (τ). To visually and

218   simultaneously compare the change in cloud droplet concentration under different

219   aerosol concentrations, the maximum cloud droplet concentration (N_{cmax}) from *default*

220   is used to normalize N_c in *default* and *new*, while N_{cmax} from *new_10* is used to

221   normalize N_c in *default_10* and *new_10*. The CWP is calculated as:

$$\mathrm{CWP} = \int_0^H \rho_a q_c(z) dz, \tag{7}$$

223   where ρ_a is the air density, $q_c(z)$ is the cloud water mixing ratio at each height (z), and

224   H is the cloud thickness. The optical depth τ is estimated with

225
$$\tau = \frac{3}{2} \frac{1}{\rho_w} \int_0^H \frac{\rho_a q_c(z)}{r_e(z)} dz, \tag{8}$$

226 where ρ_w is the water density and $r_e(z)$ is the effective radius of the cloud droplets at

227 each height (z). The time-averaged values of these physical properties of the clouds are

228 listed in Table 2 for convenience.

229   For the low aerosol number concentration, the simulations with the new

230 entrainment-mixing parameterization have smaller N_c (35.53 cm^{-3}) and larger r_v (13.29

231 μm) than the default homogeneous simulation (35.78 cm^{-3} for N_c and 13.27 μm for r_v

232 in *default*). However, comparing *new* to *default*, the relative changes in N_c, r_v, and τ are

233 very small. When the aerosol concentration increases ten-fold (*default_10* and *new_10*),

234 q_c, CWP, and τ increase according to the aerosol indirect effect (Peng et al., 2002; Wang

235 et al., 2019; Li et al., 2011; Wang et al., 2011). Meanwhile, r_v decreases significantly

236 owing to the larger cloud number concentration. The effects of the new entrainment-

237 mixing parameterization also increase, for example, the change in N_c increases from –

238 0.70% (*new* compared to *default*) to –2.74% (*new_10* compared to *default_10*), r_v

239 increases from +0.15% to +0.57%, and τ from –0.38% to –0.58%; the reasons for these

240 changes are discussed later. These small changes are similar to those identified in

241 previous cumulus studies (Jarecka et al., 2013; Hoffmann et al., 2019).

242

243 **3.2 Stratocumulus case**

244   The stratocumulus case is simulated from 9:00 UTC on 19 April 2009 to 03:00

245 UTC on 20 April 2009; the first three hours are set to be spin-up times. We examine the

246   stratocumulus region of the cloud base at ~2.1 km and the cloud top at ~2.3 km (cloud

247   thickness of ~200 m). Figure 4 shows the time series of the domain-averaged cloud

248   fraction and total downward irradiance at the central point in the observation and the

249   *default* experiment from 12:00 UTC to 24:00 UTC. Similar to the cumulus case, the

250   simulations compare favourably with the observations, which further reinforces the

251   utility of the LES model. The observed data show that the cloud fraction increases with

252   time and peaks at 16:00 UTC. The simulated cloud fraction has a value of 1 before

253   16:00 UTC, fluctuates from 16:00 UTC to 21:00 UTC, and decreases sharply after

254   21:00 UTC. This period can be divided into three stages, namely the mature stage, pre-

255   dissipation stage, and dissipation stage.

256   As with the cumulus case, the temporal evolutions of the physical properties (q_c,

257   N_c, r_v, CWP, and τ) of the clouds are shown in Figure 5. In contrast to the oscillating

258   changes exhibited by the physical quantities in the cumulus case (Figure 3), the physical

259   properties in the stratocumulus case exhibit a mostly smooth temporal evolution.

260   Furthermore, *default* and *new* exhibit clear distinctions during the early periods, but

261   these differences decrease during the dissipation stage. This is also the case with

262   *default_10* and *new_10*.

263   To compare the different behaviours of the simulation experiments at different

264   stages, the results at the mature and dissipation stages are analysed in detail. The mean

265   values of the main microphysical and optical properties of the clouds are summarised

266   in Table 3. As expected, the cloud microphysical and optical properties at the mature

stage are all larger than those at the dissipation stage. The effects of the new

entrainment-mixing parametrization are also more significant at the mature stage.

Compared to *default*, the *new* model results in a 7.36% smaller N_c, 3.20% larger r_v, and

5.98% smaller τ during the mature stage. During the dissipation stage, the changes in

N_c, r_v, and τ are –4.76%, +2.12 %, and –2.56%, respectively. The largest influence of

the new entrainment-mixing parametrization occurs during the mature stage when the

aerosol concentration is ten times greater. The differences in N_c, r_v, and τ between

new_10 and *default_10* are –9.69%, +3.88%, and –5.85%, respectively, averaged over

the mature stage. These differences are much larger than those reported by Hill et al.

(2009) who found that assuming extremely inhomogeneous mixing has a negligible

effect on stratocumulus simulations. Our results also prove the speculation of Hill et al.

(2009) that the mixing process might play an important role when the stratocumulus is

thin (~200 m in this study). Furthermore, implementing the new entrainment-mixing

parameterization has similar effects on cloud properties to those described by Hoffmann

and Feingold (2019) who used the linear eddy model to represent subgrid-scale

turbulent mixing. Note that stratocumulus clouds occur in most regions around the

world and are important contributors to the surface radiation budget (Wood, 2012;

Zheng et al., 2016; Wang et al., 2021; Wang and Feingold, 2009). Stratocumulus clouds

dominate in some regions and occur over 60% of the time as vast long-lived sheets,

such as the *semi-permanent subtropical marine stratocumulus sheets* (Wood, 2012). In

these regions, a nearly 6% decrease in τ, caused by the new entrainment-mixing

288   parameterization is expected to have significant effects on the simulation of regional

289   radiative properties and climate change.

290   The averaged influences of the new entrainment-mixing parametrization over all

291   the simulation periods are also examined (Table 4). Quantitatively, the effect of the new

292   entrainment-mixing parameterization is much greater on stratocumulus clouds than on

293   cumulus clouds. Compared to *default*, *new* has an average change of –6.20% in N_c,

294   +2.01% in r_v, and –3.23% in τ. When the aerosol concentration increases ten-fold, the

295   differences in N_c, r_v, and τ between *default_10* and *new_10* are −9.00%, +3.16%, and

296   −4.14%, respectively. These differences are larger than the largest changes in the

297   cumulus case.

298

299   **3.3 Mechanisms of the effects of the new entrainment-mixing parameterization**

300   The different effects of the new entrainment-mixing parameterization on different

301   types of clouds and even on different stages of stratocumulus clouds are likely be related

302   to variations in the dominant mixing mechanism. To confirm this, we calculate the

303   average ψ at all grid points experiencing evaporation, the proportion of inhomogeneous

304   mixing grid points to all grid points experiencing evaporation, and the average ψ at the

305   inhomogeneous mixing grid points in *new* and *new_10* (Table 5) for the cumulus case,

306   and mature and dissipation stages in the stratocumulus case.

307   For the cumulus case, simulations exhibit large ψ and a small proportion of

308   inhomogeneous mixing, indicating that homogeneous mixing is the dominant

309 entrainment-mixing mechanism (Luo et al., 2020; Lu et al., 2013). Correspondingly,

310 the influences of the new entrainment-mixing parameterization on the cloud physical

311 properties are not significant, as shown in Figure 3 and Table 2. The *new_10* model

312 exhibit a smaller average ψ and a larger proportion of inhomogeneous mixing than *new*,

313 which results in larger changes in cloud physics, as mentioned in Section 3.1.

314   For the stratocumulus case, Table 5 shows the average ψ at all grid points

315 experiencing evaporation, the proportion of inhomogeneous mixing grid points to all

316 grid points experiencing evaporation, and the average ψ at the inhomogeneous mixing

317 grid points during the two stages. The mature stage always has a smaller ψ but a larger

318 proportion of inhomogeneous mixing than the dissipation stage. The inhomogeneous

319 mixing process dominates the mature stage in *new*, because more than 60% of the grid

320 points experience inhomogeneous mixing. The inhomogeneous mixing process is more

321 dominant in *new_10*, because less than 3% of the cloudy grid points experience a

322 homogeneous mixing process during the mature stage, which explains why *new_10* has

323 the largest influence when implementing the new entrainment-mixing parametrization.

324 Meanwhile, the average ψ in both stages is smaller than that in the cumulus case for the

325 same simulation configuration. Thus, the effects of the new entrainment-mixing

326 parameterization are more significant for stratocumulus than for cumulus clouds,

327 especially at the mature stage. It is noted that the average ψ and the proportion of

328 inhomogeneous mixing at the dissipation stage of *new* in the stratocumulus case are

329 very close to the results of *new_10* in the cumulus case. This is because the cloud

330   fraction decreases sharply during the dissipation stage; the stratocumulus clouds break

331   up and produces cumulus clouds with small cloud droplet radius.

332

333   **3.4 The effects of dissipation rate and aerosol concentration on the entrainment-**

334   **mixing process**

335   Previous studies have shown the notable effects of the dissipation rate and aerosol

336   concentration on the entrainment-mixing process. For example, Luo et al. (2020)

337   changed ε from 10^{-5} m^2 s^{-3} to 10^{-2} m^2 s^{-3} and noted huge differences in the corresponding

338   ψ. Small et al. (2013) compared aircraft observations with different background

339   concentrations and found that higher pollution flights tended to slightly more

340   inhomogeneous mixing; Jarecka et al. (2013) also showed various homogeneities of

341   subgrid mixing when aerosol concentration increases ten-fold. To explain the different

342   behaviours of different simulations with the new entrainment-mixing parameterization,

343   the influences of ε and aerosol concentration are examined. Figure 6 shows the

344   probability distribution functions (PDFs) of ε, r_v, τ_{evap}, and N_L for cloud grids

345   experiencing entrainment-mixing processes in *new* and *new_10* for the cumulus and

346   stratocumulus cases, respectively. The PDFs from the mature and dissipation stages of

347   the stratocumulus case are shown in Figure 7.

348

349   **3.4.1 Dissipation rate**

350   According to Equation (3), N_L is a function of $\varepsilon^{3/4}$; hence, the PDF of ε directly

351 affects N_L and further results in different ψ. For the cumulus case, the mean ε of 0.0043

352 m^2 s^{-3} in *new* is similar to those obtained for cumulus clouds in previous studies (e.g.

353 Lu et al., 2016; Hoffmann et al., 2019). As shown in Figure 1, cloud grids experience a

354 homogeneous mixing process if N_L is larger than ~10^5, the limited distribution of N_L

355 values less than 10^5 in *new* results in a very small number of cloud grid points

356 undergoing inhomogeneous mixing process. Even at the cloud grid points that undergo

357 inhomogeneous mixing, the average ψ is large (98.62%), because most of the N_L values

358 are larger than 10^3. Therefore, the cloud properties in *new* are close to those in *default*.

359   For the stratocumulus case, the mean value of ε (2.9×10^{-4} m^2 s^{-3} in *new*) is an order

360 of magnitude less than those in the cumulus case. Therefore, compared with the

361 cumulus case, N_L is reduced in the stratocumulus case, while the peak value of *new*

362 almost reaches the criterion of inhomogeneous mixing (~10^5). For the two stages of

363 stratocumulus clouds, ε is an order of magnitude smaller, but r_v was larger (Figure 7)

364 during the mature stage than during the dissipation stage. According to Equation (5a),

365 droplets with smaller r_v are more prone to complete evaporation and have a smaller τ_{evap}.

366 The combination of smaller ε and larger r_v results in a smaller N_L (Equation (3)). This

367 is the reason for the new entrainment-mixing parametrization having more significant

368 effects during the mature stage than during the dissipation stage. In addition, the

369 similarity of the ε and r_v values during the dissipation stage of the stratocumulus case

370 in *new*, compared to the cumulus case in *new_10* (Figures 6a and 6b), explains the

371 similar average ψ values of these scenarios and the proportion of inhomogeneous

372   mixing (Table 5).

373   Therefore, the distribution of ε has a vital impact on the influence of the new

374   entrainment-mixing parameterization. Smaller values of ε result in the new

375   entrainment-mixing parameterization having a more significant influence. Moreover,

376   the r_v in the stratocumulus case is smaller than that in the cumulus case, which is also

377   conducive to a more inhomogeneous mixing process. These are the reasons why the

378   implementation of the new entrainment-mixing parameterization has a larger influence

379   in the stratocumulus case than in the cumulus case, when compared to a homogeneous

380   mixing mechanism.

381

382   **3.4.2 Aerosol concentration**

383   The aerosol concentration affects the entrainment-mixing process by decreasing

384   the cloud droplet radius. As r_v decreases, the distributions of τ_{evap} in *new_10* moves to

385   a smaller overall value, while the mean value is an order of magnitude smaller than that

386   in *new*, which causes a much smaller N_L because N_L is proportional to $\tau_{evap}^{3/2}$ (Equations

387   (3a) and (3b)). The larger percentage of smaller N_L values indicates that in *new_10*,

388   more grid points undergo an inhomogeneous mixing process, and the proportion of such

389   grid points is much larger than in the *new* model (Table 5). Therefore, compared to *new*,

390   *new_10* exhibit a smaller ψ and the effects of the new entrainment-mixing

391   parameterization on cloud properties are more obvious, for both the cumulus and

392   stratocumulus cases.

393

3.5 Verification by the simulations with a different parameterization using entrained air relative humidity

In the above simulations, the new entrainment-mixing parameterization is based on the grid-mean RH. This section serves to verify these simulations using the the entrainment-mixing parameterization proposed by Luo et al. (2020)

$$\psi = 107.96\exp(-0.95N_{\mathrm{L}}^{-0.35}), \tag{9}$$

which was developed using the entrained air RH in the EMPM. This parameterization needs the entrained air RH within each grid in WRF, which is estimated following Grabowski (2007) and Jarecka et al. (2009). Briefly, assuming that RH mixes linearly when the dry air entrains into the cloud, then entrained air RH can be simply calculated by

$$RH_{\mathrm{entrained}} = \frac{RH_{\mathrm{grid}} - (1-f)RH_{\mathrm{cloud}}}{f}, \tag{10}$$

where the subscripts *entrained*, *cloud*, and *grid* indicate the RH of the entrained, cloudy, and grid point air, respectively. In Equation (10), although the cloudy air RH is approximately 100% and grid-mean RH is predicted in the model, the entrained air fraction f needs to be further parameterized. To obtain f at 100 m, a parameterization of f is developed based on the simulations for both the cumulus and stratocumulus cases with a higher resolution of 10 m; the other configurations are the same as those in the experiment *default*. The 10 m-resolution simulation results are then averaged to the resolution of 100 m. Following Xu and Randall (1996), "1 - f " can be fitted by the

414   function

$$1 - f = RH_{\text{grid}}^{\gamma} \left[1 - \exp\left(-\beta q_{\text{c}} \right) \right], \tag{11}$$

416   where γ and β are empirical parameters. Figure 8 shows that the parameterization with

417    $\gamma = 8.72$ and $\beta = 1.47 \times 10^4$ can well reproduce the simulated values of "$1 - f$", with the

418   correlation coefficient of 0.89 and significant level p-value < 0.01.

419   Equations (9-11) are applied in the simulations for both the cumulus and

420   stratocumulus cases with different aerosol background (hereafter *new_f* and

421   *new_f_10*). Same as Figures 3 and 5, the temporal evolutions of the cloud physical

422   properties (q_{c}, N_{c}, r_{v}, CWP, and τ) in *default*, *default_10*, *new_f*, and *new_f_10* are

423   shown in Figures 9 and 10. The results are similar to Figures 3 and 5. The mean values

424   of these properties of *new_f* and *new_f_10* for the cumulus and stratocumulus cases

425   are also shown in Table 6, the results of *new* and *new_10* are also shown in the

426   parentheses for the convenience of comparison. The results of *new_f* and *new* are very

427   similar, with the maximum difference being no more than 1%, so are the results of

428   tenfold aerosol background. Such a close agreement suggests that the results of the

429   new entrainment-mixing parametrization with grid-mean RH are reliable.

430   It is worth noting that instead of parameterizing *f*, Jarecka et al. (2009) and

431   Jarecka et al. (2013) added an equation to predict *f* for each grid. In principle, this is a

432   good choice if this method is available in models.

433

434

435

4. Concluding remarks

The entrainment-mixing process near cloud edges has important effects on cloud microphysics, but the most commonly used microphysics schemes simply assume one extreme mechanism, that is, homogeneous entrainment-mixing. This study first improves the entrainment-mixing parameterization proposed by Luo et al. (2020), which connects the homogeneous mixing degree and transition scale number to estimate the homogeneity of the subgrid mixing process and its impact on the droplet number concentration. The improved parameterization uses grid-mean relative humidity and can be implemented directly into microphysics schemes; there is no need to know the relative humidity of the entrained air. Second, the modified entrainment-mixing parameterization is implemented in the two-moment Thompson aerosol-aware scheme of the LES version of WRF-Solar, to examine its effects on the microphysical and optical properties of cumulus and stratocumulus clouds. Third, several sensitivity experiments are conducted to investigate the effects of the new entrainment-mixing parameterization under different conditions of turbulence dissipation rate and aerosol number concentration. The results of implementing the new entrainment-mixing parameterization are finally verified by the results using entrained air properties.

Unlike the commonly assumed homogeneous mixing scenario, the new entrainment-mixing parameterization produces a smaller cloud droplet number concentration and larger cloud droplet radius, with the degree of difference depending on cloud types and stages. Sensitivity tests show that in the cumulus case, the largest

457 average influence of the new entrainment-mixing parameterization occurs under a high

458 aerosol background, but results in only a 2.74% decrease in cloud droplet number

459 concentration and a 0.57% increase in cloud droplet volume-mean radius. The changes

460 become even smaller with a low aerosol background because of the larger cloud droplet

461 radius. In contrast, the new entrainment-mixing parameterization has a larger influence

462 on the microphysical and optical properties of stratocumulus clouds, especially under a

463 high aerosol background and during the mature stage, with a cloud fraction equal to 1.

464 The largest changes resulting from the new entrainment-mixing parameterization are –

465 9.69%, +3.88%, and –5.85%, for cloud number concentration, cloud droplet volume-

466 mean radius, and cloud optical depth, respectively. The new entrainment-mixing

467 parameterization has less of an influence on the dissipation stage than on the mature

468 stage of the stratocumulus case, but affects this case more than the cumulus case.

469   The varying effects of the new entrainment-mixing parameterization are caused

470 by variations in the dominant entrainment-mixing mechanism between different cloud

471 types and stages. Compared to the cumulus case, the stratocumulus case has a much

472 smaller homogeneous mixing degree and a larger proportion of inhomogeneous mixing

473 grid points, especially during the mature stage, which indicates that the inhomogeneous

474 mixing mechanism dominates in the stratocumulus case, while the homogeneous

475 mixing mechanism dominates in the cumulus case. As mentioned above, the changes

476 in physical properties of stratocumulus clouds in the dissipation stage are between those

477 in the mature stage and those of the cumulus case; this is because stratocumulus clouds

478 dissipate sharply to form small cumulus clouds, and the degree of homogeneous mixing

479 during the dissipation stage is therefore between that which occurs during the mature

480 stage and the cumulus case.

481   Sensitivity studies show that turbulence dissipation rate and aerosol concentration

482 can have notable effects on the subgrid homogeneity of the mixing process. A larger

483 dissipation rate can accelerate the mixing process, which results in a larger transition

484 scale number and homogeneous mixing degree; and therefore, a mostly homogenous

485 mixing mechanism. This is why the cumulus case exhibit smaller changes than the

486 stratocumulus case after the new entrainment-mixing parameterization is implemented.

487 Larger aerosol number concentrations cause a smaller cloud droplet radius. Smaller

488 droplets evaporate more easily, which leads to a smaller transition scale number and a

489 smaller homogeneous mixing degree. Thus, the entrainment-mixing mechanism tends

490 to be inhomogeneous. Therefore, a larger aerosol number concentration increases the

491 influence of the new entrainment-mixing parameterization in both the cumulus and

492 stratocumulus cases.

493   The influences of implementing the new entrainment-mixing parameterization

494 with grid-mean relative humidity have been verified by simulations with entrained air

495 properties. The entrained air properties are obtained and calculated from simulations

496 with a finer resolution (10 m). Sensitivity tests show similar cloud microphysical and

497 optical properties in the two different methods, which suggests that the new

498 entrainment-mixing parameterization with grid-mean relative humidity is convincing.

499   Note that the new entrainment-mixing parameterization could be more important

500   in the models if the relative humidity near the cloud is more accurately simulated,

501   because numerical diffusion may spuriously humidify the entrained air (Hoffmann and

502   Feingold, 2019). The artificially increased relative humidity limits the influences of the

503   new entrainment-mixing parameterization, because homogeneous and inhomogeneous

504   entrainment-mixing processes are close to each other under conditions of high relative

505   humidity.

506

507   **Author contributions.** XX, CL and YL designed the experiments. XX carried out the

508   experiments and conducted the data analysis with contributions from all coauthors. XX,

509   CL, XZ, and SE developed the model code. XX prepared the paper with help from CL,

510   YL, YW, SL, and LZ.

511

512   **Competing interests.** The authors declare that they have no conflict of interest.

513

514   **Acknowledgements.** This research is supported by the National Key Research and

515   Development Program of China (2017YFA0604001), the National Natural Science

516   Foundation of China (41822504, 42175099, 42027804, 41975181, 42075073). Liu,

517   Zhou, and Endo are supported by the U.S. Department of Energy's Office of Energy

518   Efficiency and Renewable Energy (EERE) under Solar Energy Technologies Office

519   (SETO) Award Number 33504, and Office of Science Biological and Environmental

520 Research as part of the Atmospheric Systems Research (ASR) Program. Brookhaven

521 National Laboratory is operated by Battelle for the U.S. Department of Energy under

522 Contract DE-SC00112704. The large-scale forcing data used in this paper can be

523 downloaded from the U.S. Department of Energy's Atmospheric Radiation

524 Measurement Program with https://adc.arm.gov/discovery/#/results. The LASSO data

525 can be downloaded from https://archive.arm.gov/lassobrowser. The views expressed

526 herein do not necessarily represent the views of the U.S. Department of Energy or the

527 United States Government.

528

529 **Reference**

530 Ackerman, A. S., Kirkpatrick, M. P., Stevens, D. E., and Toon, O. B.: The impact of
531 humidity above stratiform clouds on indirect aerosol climate forcing, Nature, 432,
532 1014-1017, 2004.
533 Andrejczuk, M., Grabowski, W. W., Malinowski, S. P., and Smolarkiewicz, P. K.:
534 Numerical simulation of cloud-clear air interfacial mixing: Effects on cloud
535 microphysics, J. Atmos. Sci., 63, 3204-3225, doi:10.1175/JAS3813.1, 2006.
536 Andrejczuk, M., Grabowski, W. W., Malinowski, S. P., and Smolarkiewicz, P. K.:
537 Numerical simulation of cloud–clear air interfacial mixing: Homogeneous versus
538 inhomogeneous mixing, J. Atmos. Sci., 66, 2493-2500, doi:10.1175/2009JAS2956.1,
539 2009.
540 Burnet, F. and Brenguier, J. L.: Observational study of the entrainment-mixing process
541 in warm convective clouds, J. Atmos. Sci., 64, 1995-2011, doi:10.1175/JAS3928.1,
542 2007.
543 Chosson, F., Brenguier, J.-L., and Schüller, L.: Entrainment-mixing and radiative
544 transfer simulation in boundary layer clouds, J. Atmos. Sci., 64, 2670-2682,
545 doi:10.1175/JAS3975.1, 2007.
546 Deardorff, J.: Stratocumulus-capped mixed layers derived from a three-dimensional
547 model, Boundary-Layer Meteorol, 18, 495-527, 10.1007/BF00119502, 1980.
548 Del Genio, A. D. and Wu, J.: The role of entrainment in the diurnal cycle of continental
549 convection, J. Climate, 23, 2722-2738, doi: 10.1175/2009JCLI3340.1, 2010.
550 Endo, S., Fridlind, A. M., Lin, W., Vogelmann, A. M., Toto, T., Ackerman, A. S.,
551 McFarquhar, G. M., Jackson, R. C., Jonsson, H. H., and Liu, Y.: RACORO continental

boundary layer cloud investigations: 2. Large‑eddy simulations of cumulus clouds and evaluation with in situ and ground‑based observations, Journal of Geophysical Research: Atmospheres, 120, 5993-6014, 2015.

Fan, J., Wang, Y., Rosenfeld, D., and Liu, X.: Review of Aerosol–Cloud Interactions: Mechanisms, Significance, and Challenges, Journal of the Atmospheric Sciences, 73, 4221-4252, 10.1175/jas-d-16-0037.1, 2016.

Freud, E., Rosenfeld, D., and Kulkarni, J. R.: Resolving both entrainment-mixing and number of activated CCN in deep convective clouds, Atmos. Chem. Phys., 11, 12887-12900, 2011.

Freud, E., Rosenfeld, D., Andreae, M. O., Costa, A. A., and Artaxo, P.: Robust relations between CCN and the vertical evolution of cloud drop size distribution in deep convective clouds, Atmos. Chem. Phys., 8, 1661-1675, 2008.

Gao, S., Lu, C., Liu, Y., Mei, F., Wang, J., Zhu, L., and Yan, S.: Contrasting Scale Dependence of Entrainment‑Mixing Mechanisms in Stratocumulus Clouds, Geophysical Research Letters, 47, 10.1029/2020gl086970, 2020.

Gao, S., Lu, C., Liu, Y., Yum, S. S., Zhu, J., Zhu, L., Desai, N., Ma, Y., and Wu, S.: Comprehensive quantification of height dependence of entrainment mixing between stratiform cloud top and environment, Atmospheric Chemistry and Physics, 21, 11225-11241, 2021.

Gao, Z., Liu, Y., Li, X., and Lu, C.: Investigation of Turbulent Entrainment-Mixing Processes with a New Particle-Resolved Direct Numerical Simulation Model, J. Geophys. Res., 123, 2194-2214, 2018.

Gerber, H. E., Frick, G. M., Jensen, J. B., and Hudson, J. G.: Entrainment, mixing, and microphysics in trade-wind cumulus, J. Meteorol. Soc. Japan, 86A, 87-106, 2008.

Grabowski, W. W.: Indirect impact of atmospheric aerosols in idealized simulations of convective-radiative quasi equilibrium, J. Climate, 19, 4664-4682, doi:10.1175/JCLI3857.1, 2006.

Grabowski, W. W.: Representation of turbulent mixing and buoyancy reversal in bulk cloud models, Journal of the atmospheric sciences, 64, 3666-3680, 2007.

Grabowski, W. W. and Morrison, H.: Indirect Impact of Atmospheric Aerosols in Idealized Simulations of Convective–Radiative Quasi Equilibrium. Part II: Double-Moment Microphysics, Journal of Climate, 24, 1897-1912, 10.1175/2010jcli3647.1, 2011.

Gustafson, W. I., Vogelmann, A. M., Li, Z., Cheng, X., Dumas, K. K., Endo, S., Johnson, K. L., Krishna, B., Fairless, T., and Xiao, H.: The Large-Eddy Simulation (LES) Atmospheric Radiation Measurement (ARM) Symbiotic Simulation and Observation (LASSO) Activity for Continental Shallow Convection, Bulletin of the American Meteorological Society, 101, E462-E479, 10.1175/bams-d-19-0065.1, 2020.

Hacker, J. P., Jimenez, P. A., Dudhia, J., Haupt, S. E., Ruiz-Arias, J. A., Gueymard, C. A., Thompson, G., Eidhammer, T., and Deng, A.: WRF-Solar: Description and Clear-Sky Assessment of an Augmented NWP Model for Solar Power Prediction, Bulletin of the American Meteorological Society, 97, 1249-1264, 10.1175/bams-d-14-00279.1,

594 2016.

595 Haman, K. E., Malinowski, S. P., Kurowski, M. J., Gerber, H., and Brenguier, J.-L.:
596 Small scale mixing processes at the top of a marine stratocumulus - a case study, Q. J.
597 Roy. Meteor. Soc., 133, 213-226, doi:10.1002/qj.5, 2007.

598 Haupt, S. E., Kosovic, B., Jensen, T., Lee, J., Jimenez, P., Lazo, J., Cowie, J.,
599 McCandless, T., Pearson, J., and Weiner, G.: The SunCast□ solar-power forecasting
600 system: the results of the public-private-academic partnership to advance solar power
601 forecasting, National Center for Atmospheric Research (NCAR), Boulder (CO):
602 Research Applications Laboratory, Weather Systems and Assessment Program (US),
603 2016.

604 Hill, A. A., Feingold, G., and Jiang, H.: The influence of entrainment and mixing
605 assumption on aerosol-cloud interactions in marine stratocumulus, J. Atmos. Sci., 66,
606 1450-1464, 2009.

607 Hoffmann, F. and Feingold, G.: Entrainment and mixing in stratocumulus: Effects of a
608 new explicit subgrid-scale scheme for large-eddy simulations with particle-based
609 microphysics, J. Atmos. SCi., 76, 1955-1973, 10.1175/jas-d-18-0318.1, 2019.

610 Hoffmann, F., Yamaguchi, T., and Feingold, G.: Inhomogeneous mixing in Lagrangian
611 cloud models: Effects on the production of precipitation embryos, Journal of the
612 Atmospheric Sciences, 76, 113-133, 2019.

613 Huang, J., Lee, X., and Patton, E. G.: A modelling study of flux imbalance and the
614 influence of entrainment in the convective boundary layer, Boundary-layer meteorology,
615 127, 273-292, 2008.

616 Jarecka, D., Grabowski, W. W., and Pawlowska, H.: Modeling of Subgrid-Scale Mixing
617 in Large-Eddy Simulation of Shallow Convection, Journal of the Atmospheric Sciences,
618 66, 2125-2133, 10.1175/2009jas2929.1, 2009.

619 Jarecka, D., Grabowski, W. W., Morrison, H., and Pawlowska, H.: Homogeneity of the
620 subgrid-scale turbulent mixing in large-eddy simulation of shallow convection, J.
621 Atmos. Sci., 70, 2751-2767, 2013.

622 Jensen, J. B., Austin, P. H., Baker, M. B., and Blyth, A. M.: Turbulent mixing, spectral
623 evolution and dynamics in a warm cumulus cloud, J. Atmos. Sci., 42, 173-192,
624 doi:10.1175/1520-0469(1985)042<0173:TMSEAD>2.0.CO;2, 1985.

625 Kim, B.-G., Miller, M. A., Schwartz, S. E., Liu, Y., and Min, Q.: The role of adiabaticity
626 in the aerosol first indirect effect, J. Geophys. Res., 113, D05210, doi:
627 10.1029/2007jd008961, 2008.

628 Kumar, B., Schumacher, J., and Shaw, R.: Cloud microphysical effects of turbulent
629 mixing and entrainment, Theor. Comp. Fluid Dyn., 27, 361-376, 2013.

630 Lasher-Trapp, S. G., Cooper, W. A., and Blyth, A. M.: Broadening of droplet size
631 distributions from entrainment and mixing in a cumulus cloud, Q. J. Roy. Meteor. Soc.,
632 131, 195-220, doi:10.1256/qj.03.199, 2005.

633 Lehmann, K., Siebert, H., and Shaw, R. A.: Homogeneous and inhomogeneous mixing
634 in cumulus clouds: dependence on local turbulence structure, J. Atmos. Sci., 66, 3641-
635 3659, doi:10.1175/2009JAS3012.1, 2009.

636 Li, Z., Li, C., Chen, H., Tsay, S. C., Holben, B., Huang, J., Li, B., Maring, H., Qian, Y.,
637 and Shi, G.: East Asian studies of tropospheric aerosols and their impact on regional
638 climate (EAST‐AIRC): An overview, Journal of Geophysical Research: Atmospheres,
639 116, 2011.
640 Lu, C., Liu, Y., and Niu, S.: Examination of turbulent entrainment-mixing mechanisms
641 using a combined approach, J. Geophys. Res., 116, D20207,
642 doi:10.1029/2011JD015944, 2011.
643 Lu, C., Liu, Y., Niu, S., and Endo, S.: Scale dependence of entrainment‐mixing
644 mechanisms in cumulus clouds, Journal of Geophysical Research: Atmospheres, 119,
645 13,877-813,890, 2014.
646 Lu, C., Liu, Y., Niu, S., Krueger, S., and Wagner, T.: Exploring parameterization for
647 turbulent entrainment-mixing processes in clouds, Journal of Geophysical Research:
648 Atmospheres, 118, 185-194, 10.1029/2012jd018464, 2013.
649 Lu, C., Liu, Y., Zhang, G. J., Wu, X., Endo, S., Cao, L., Li, Y., and Guo, X.: Improving
650 parameterization of entrainment rate for shallow convection with aircraft measurements
651 and large eddy simulation, J. Atmos. Sci., 73, 761-773, 10.1175/JAS-D-15-0050.1,
652 2016.
653 Luo, S., Lu, C., Liu, Y., Bian, J., Gao, W., Li, J., Xu, X., Gao, S., Yang, S., and Guo, X.:
654 Parameterizations of Entrainment‐Mixing Mechanisms and Their Effects on Cloud
655 Droplet Spectral Width Based on Numerical Simulations, Journal of Geophysical
656 Research: Atmospheres, 125, e2020JD032972, 2020.
657 Morrison, H. and Grabowski, W. W.: Modeling supersaturation and subgrid-scale
658 mixing with two-moment bulk warm microphysics, J. Atmos. Sci., 65, 792-812,
659 doi:10.1175/2007JAS2374.1, 2008.
660 Paluch, I. R. and Baumgardner, D. G.: Entrainment and fine-scale mixing in a
661 continental convective cloud, J. Atmos. Sci., 46, 261-278, doi:10.1175/1520-
662 0469(1989)046<0261:EAFSMI>2.0.CO;2, 1989.
663 Pawlowska, H., Brenguier, J. L., and Burnet, F.: Microphysical properties of
664 stratocumulus clouds, Atmos. Res., 55, 15-33, 2000.
665 Peng, Y., Lohmann, U., Leaitch, R., Banic, C., and Couture, M.: The cloud albedo-cloud
666 droplet effective radius relationship for clean and polluted clouds from RACE and
667 FIRE.ACE, J. Geophys. Res., 107, AAC 1-1-AAC 1-6, 10.1029/2000JD000281, 2002.
668 Slawinska, J., Grabowski, W. W., Pawlowska, H., and Morrison, H.: Droplet activation
669 and mixing in large-eddy simulation of a shallow cumulus field, J. Atmos. Sci., 69, 444-
670 462, 2012.
671 Slawinska, J., Grabowski, W. W., Pawlowska, H., and Wyszogrodzki, A. A.: Optical
672 properties of shallow convective clouds diagnosed from a bulk-microphysics large-
673 eddy simulation, J. Climate, 21, 1639-1647, 2008.
674 Small, J. D., Chuang, P., and Jonsson, H.: Microphysical imprint of entrainment in
675 warm cumulus, Tellus B, 65, 6647-6662, 2013.
676 Thompson, G. and Eidhammer, T.: A study of aerosol impacts on clouds and
677 precipitation development in a large winter cyclone, Journal of the atmospheric

sciences, 71, 3636-3658, 2014.

Wang, H. and Feingold, G.: Modeling mesoscale cellular structures and drizzle in marine stratocumulus. Part I: Impact of drizzle on the formation and evolution of open cells, Journal of the Atmospheric Sciences, 66, 3237-3256, 2009.

Wang, M., Ghan, S., Ovchinnikov, M., Liu, X., Easter, R., Kassianov, E., Qian, Y., and Morrison, H.: Aerosol indirect effects in a multi-scale aerosol-climate model PNNL-MMF, Atmospheric Chemistry and Physics, 11, 5431-5455, 2011.

Wang, Y., Zhao, C., McFarquhar, G. M., Wu, W., Reeves, M., and Li, J.: Dispersion of Droplet Size Distributions in Supercooled Non‐precipitating Stratocumulus from Aircraft Observations Obtained during the Southern Ocean Cloud Radiation Aerosol Transport Experimental Study, Journal of Geophysical Research: Atmospheres, 126, e2020JD033720, 2021.

Wang, Y., Niu, S., Lv, J., Lu, C., Xu, X., Wang, Y., Ding, J., Zhang, H., Wang, T., and Kang, B.: A new method for distinguishing unactivated particles in cloud condensation nuclei measurements: Implications for aerosol indirect effect evaluation, Geophysical Research Letters, 46, 14185-14194, 2019.

Wood, R.: Review: Stratocumulus Clouds, Monthly Weather Review, 140, 2373-2423, 2012.

Xu, K.-M. and Randall, D. A.: A semiempirical cloudiness parameterization for use in climate models, Journal of the atmospheric sciences, 53, 3084-3102, 1996.

Xu, X., Sun, C., Lu, C., Liu, Y., Zhang, G. J., and Chen, Q.: Factors affecting entrainment rate in deep convective clouds and parameterizations, Journal of Geophysical Research: Atmospheres, 126, e2021JD034881, 2021.

Xu, X., Lu, C., Liu, Y., Gao, W., Wang, Y., Cheng, Y., Luo, S., and Van Weverberg, K.: Effects of cloud liquid‐phase microphysical processes in mixed‐phase cumuli over the Tibetan Plateau, Journal of Geophysical Research: Atmospheres, 125, e2020JD033371, 2020.

Yang, B., Wang, M., Zhang, G. J., Guo, Z., Huang, A., Zhang, Y., and Qian, Y.: Linking Deep and Shallow Convective Mass Fluxes via an Assumed Entrainment Distribution in CAM5‐CLUBB: Parameterization and Simulated Precipitation Variability, Journal of Advances in Modeling Earth Systems, 13, e2020MS002357, 2021.

Yang, F., Shaw, R., and Xue, H.: Conditions for super-adiabatic droplet growth after entrainment mixing, Atmos. Chem. Phys., 16, 9421-9433, 10.5194/acp-2016-94, 2016.

Yum, S.: Cloud droplet spectral broadening in warm clouds: An observational and model study, Dissertation for the Doctoral Degree, University of Nevada, Reno, Nevada, USA, 191 pp., 1998.

Zhang, M. H., Lin, J. L., Cederwall, R. T., Yio, J. J., and Xie, S. C.: Objective analysis of ARM IOP data: Method and sensitivity, Mon. Weather Rev., 129, 295-311, 10.1175/1520-0493(2001)129<0295:OAOAID>2.0.CO;2, 2001.

Zheng, Y. and Rosenfeld, D.: Linear relation between convective cloud base height and updrafts and application to satellite retrievals, Geophysical Research Letters, 42, 6485-6491, 10.1002/2015gl064809, 2015.

720 Zheng, Y., Rosenfeld, D., and Li, Z.: Quantifying cloud base updraft speeds of marine
721 stratocumulus from cloud top radiative cooling, Geophysical Research Letters, 43,
722 11,407-411,413, 2016.
723 Zhu, L., Lu, C., Yan, S., Liu, Y., Zhang, G. J., Mei, F., Zhu, B., Fast, J. D., Matthews,
724 A., and Pekour, M. S.: A New Approach for Simultaneous Estimation of Entrainment
725 and Detrainment Rates in Non‐Precipitating Shallow Cumulus, Geophysical Research
726 Letters, 48, 10.1029/2021gl093817, 2021.

727

728

729   Table 1. Summary of names and corresponding descriptions of the four experiments for

730   each case of cumulus and stratocumulus. The meaning of each symbol for each

731   experiment can be found in the text.

	Entrainment-mixing parameterization	Dissipation rate	Aerosol number concentration
default	$\alpha = 0$	-	default
new	$\alpha = 1 - \psi,$ $\psi = 107.19\exp(-1.99N_{L}^{-0.29}).$	$\varepsilon = CE^{3/2} / L.$	default
default_10	$\alpha = 0$	-	default×10
new_10	$\alpha = 1 - \psi,$ $\psi = 107.19\exp(-1.99N_{L}^{-0.29}).$	$\varepsilon = CE^{3/2} / L.$	default×10

732

733 Table 2. Summary of the case mean values of the key quantities in all the simulations

734 of the cumulus case, containing cloud water mixing ratio (q_c), cloud droplet number

735 concentration (N_c), cloud droplet volume-mean radius (r_v), cloud water path (CWP),

736 and cloud optical depth (τ). The experiments are detailed in Table 1.

	default	*new*	*default_10*	*new_10*
q_c(g/kg)	0.44	0.44	0.56	0.57
N_c(cm^{-3})	35.78	35.53	278.80	271.16
r_v(μm)	13.27	13.29	7.05	7.09
CWP(g/m^2)	142.30	144.25	186.52	187.13
τ	13.07	13.02	31.29	31.11

737

738

739 Table 3. Summary of the case mean values of the key quantities in all the simulations

740 of the stratocumulus case, including cloud water mixing ratio (q_c), cloud droplet number

741 concentration (N_c), cloud droplet volume-mean radius (r_v), cloud water path (CWP),

742 and cloud optical depth (τ). The numbers in and out of the parentheses are the results at

743 the mature and dissipation stages, respectively. The experiments are detailed in Table 1.

	default	*new*	*default_10*	*new_10*
q_c(g/kg)	0.13	0.13	0.16	0.16
	(0.039)	(0.039)	(0.041)	(0.041)
N_c(cm^{-3})	35.74	33.11	256.82	231.93
	(19.76)	(18.82)	(138.74)	(126.90)
r_v(μm)	10.32	10.65	5.15	5.35
	(7.53)	(7.69)	(4.02)	(4.14)
CWP(g/m^2)	41.39	41.78	56.21	57.03
	(2.57)	(2.43)	(2.71)	(2.77)
τ	4.68	4.40	13.17	12.40
	(0.39)	(0.38)	(0.78)	(0.78)

744

745 Table 4. Cloud water mixing ratio (q_c), cloud droplet number concentration (N_c), cloud

746 droplet volume-mean radius (r_v), cloud water path (CWP), cloud optical depth (τ) in all

747 simulations for the entire lifetime of the stratocumulus case. The experiments are

748 detailed in Table 1.

	default	*new*	*default_10*	*new_10*
q_c(g/kg)	0.11	0.11	0.13	0.13
N_c(cm^{-3})	29.98	28.12	223.65	203.50
r_v(μm)	9.38	9.57	5.06	5.22
CWP(g/m^2)	30.78	29.92	42.22	43.13
τ	4.02	3.89	10.39	9.96

749

750

751 Table 5. Homogeneous mixing degree (ψ) at all grid points experiencing evaporation,

752 the proportion of inhomogeneous mixing grid points to all grid points experiencing

753 evaporation, and ψ at the inhomogeneous mixing grid points in the experiments *new*

754 and *new_10* (Table 1) for the cumulus (Cu) and stratocumulus (St) cases. The numbers

755 in and out of the parentheses are the results at the mature and dissipation stages in the

756 stratocumulus (St) case, respectively. The experiments are detailed in Table 1.

	ψ at all grids (%)	Proportion of inhomogeneous mixing grids (%)	ψ at the inhomogeneous mixing grids (%)
new (Cu)	99.93	4.52	98.62
new_10 (Cu)	95.33	25.10	92.96
new (St)	78.56	63.07	71.56
	(94.68)	(40.61)	(89.33)
new_10 (St)	68.20	97.31	65.01
	(88.11)	(73.54)	(84.99)

757

758

759 Table 6. Cloud water mixing ratio (q_c), cloud droplet number concentration (N_c), cloud

760 droplet volume-mean radius (r_v), cloud water path (CWP), cloud optical depth (τ) in

761 *new_f* and *new_f_10* for the cumulus (Cu) and stratocumulus (Sc) cases. The results of

762 *new* and *new_10* in Tables 2 and 4 are shown in the parentheses.

	Cu		Sc	
	new_f	*new_f _10*	*new_f*	*new_f_10*
	(*new*)	(*new_10*)	(*new*)	(*new_10*)
q_c(g/kg)	0.44	0.57	0.11	0.13
	(0.44)	(0.57)	(0.11)	(0.13)
N_c(cm^{-3})	35.52	270.56	28.08	202.99
	(35.53)	(271.16)	(28.12)	(203.50)
r_v(μm)	13.30	7.10	9.60	5.21
	(13.29)	(7.09)	(9.57)	(5.22)
CWP(g/m^2)	143.15	185.95	30.16	43.32
	(144.25)	(187.13)	(29.92)	(43.13)
τ	13.00	31.08	3.89	9.93
	(13.02)	(31.11)	(3.89)	(9.96)

763

764

765

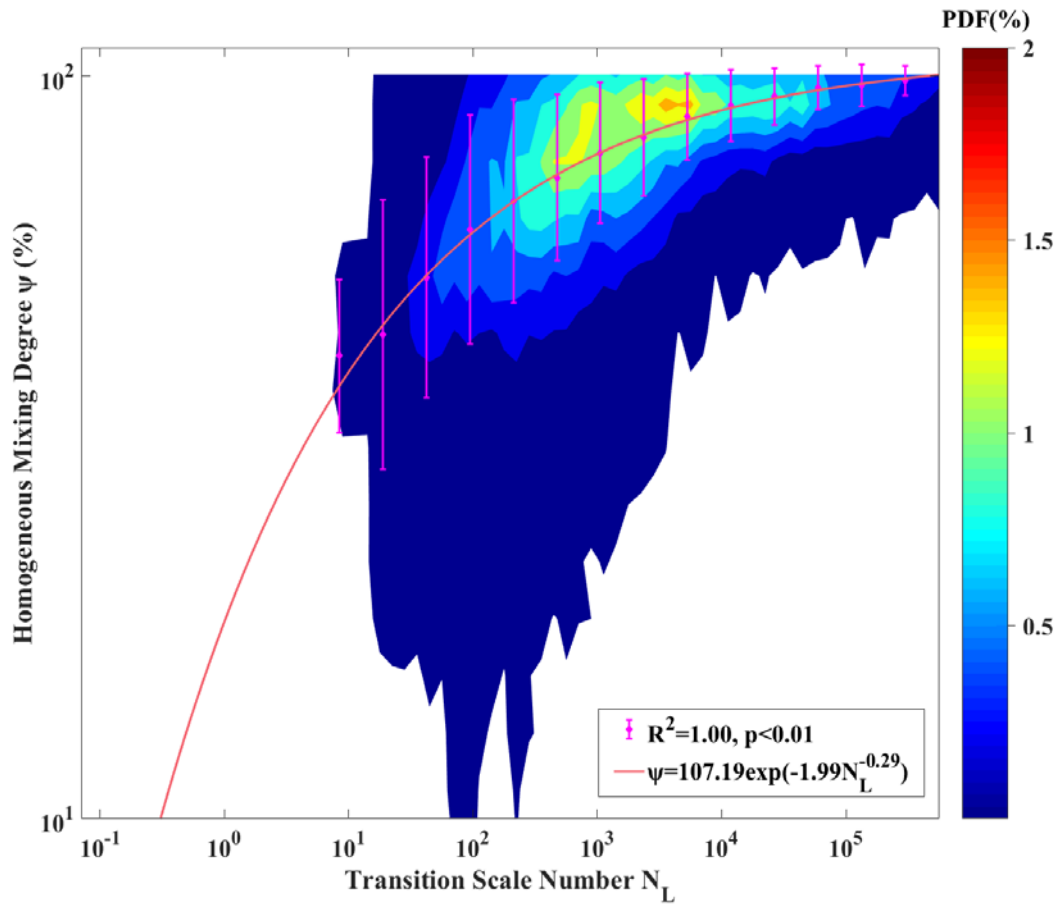



Figure 1. Parameterization of cloud entrainment-mixing mechanisms by relating

homogeneous mixing degree (ψ) to transition scale number (N_L) from EMPM. The

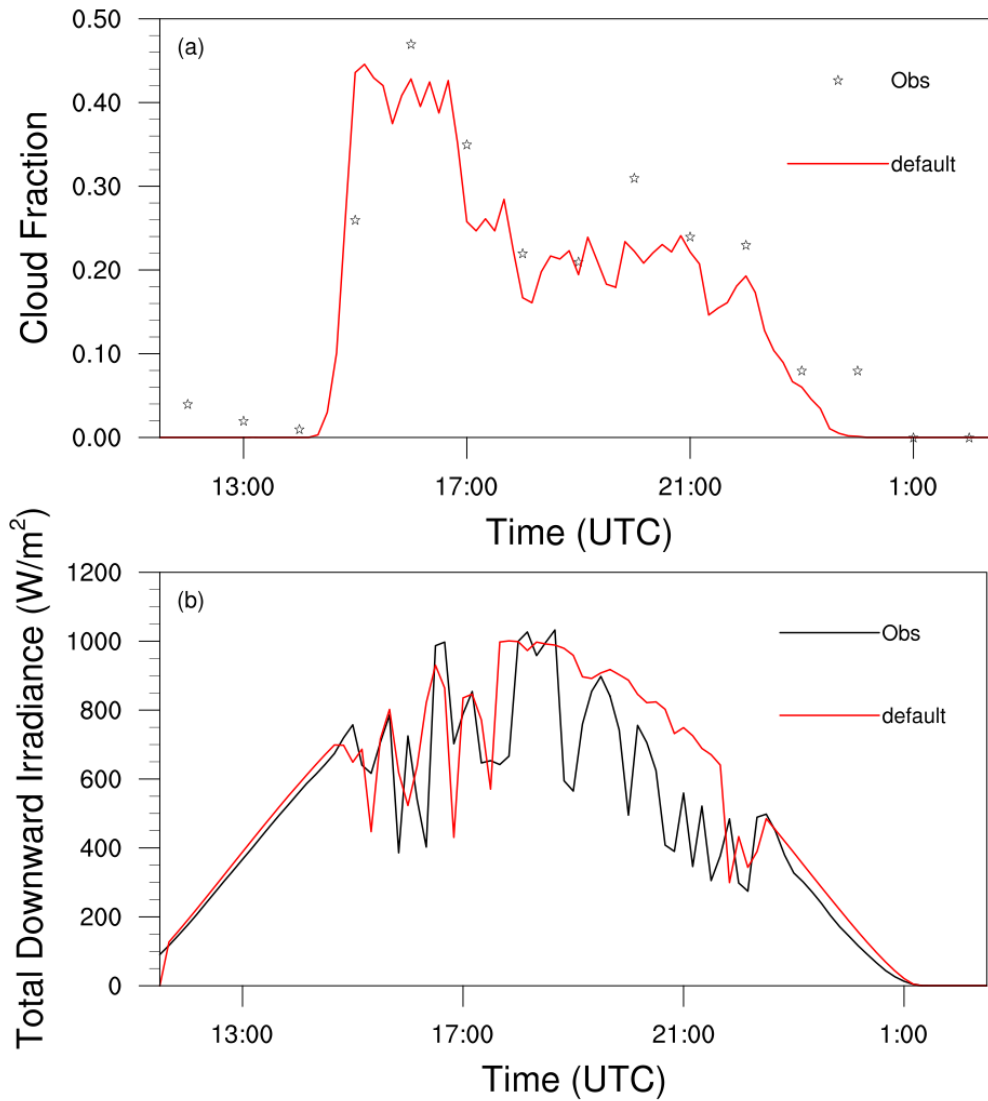
contours represent the joint probability distribution function (PDF) of ψ vs N_L. The

magenta dots and error bars are mean values and standard deviations of ψ in each N_L

bin, respectively. The mean values are fitted using a weighted least squares method with

the number of data points in each N_L bin as the weight. The fitting equation, coefficient

of determination (R^2), and p-value are also given. N_L is calculated by with the domain-

averaged relative humidity after entrainment but before evaporation in the EMPM.

774

775 Figure 2. Time series of (a) domain-averaged cloud fraction and (b) total downward

776 irradiance at the central point from the observation and the *default* experiment in the

777 cumulus case.

778

779

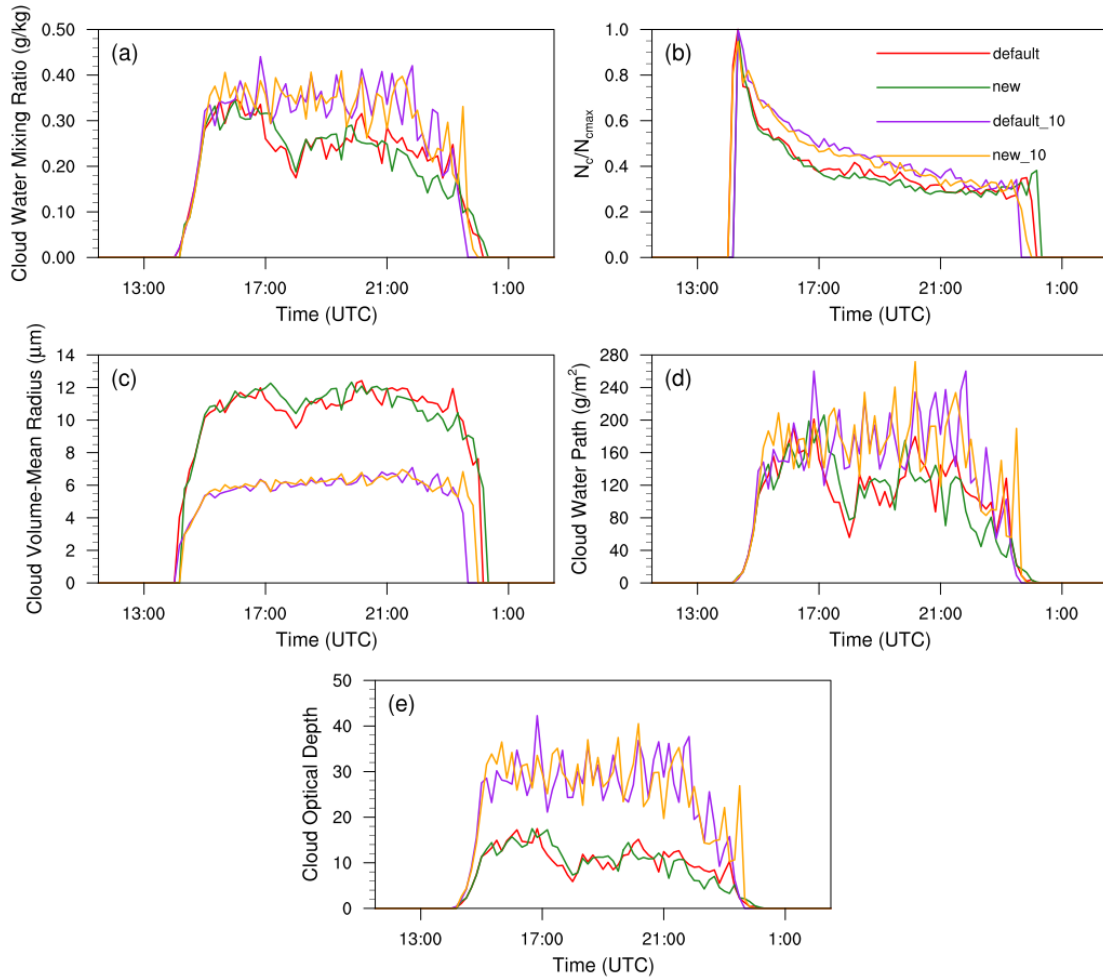



Figure 3. The temporal evolutions of main cloud microphysical and optical properties in all simulation experiments for the cumulus case, including (a) cloud water mixing ratio (q_c) (g/kg), (b) cloud droplet number concentration (N_c) (/cm^3), (c) cloud droplet volume-mean radius (r_v) (μm), (d) cloud water path (CWP) (g/m^2), and (e) cloud optical depth (τ). In (b), N_c in the experiments *default* and *new* are normalized by the maximum cloud droplet concentration (N_{cmax}) from *default*, respectively; N_c in the experiments *default_10* and *new_10* are normalized by N_{cmax} from *default_10*, respectively. The four experiments are detailed in Table 1.

788

789 Figure 4. Time series of (a) domain-averaged cloud fraction and (b) total downward

790 irradiance at the central point from the observation and the *default* experiment in the

791 stratocumulus case.

792

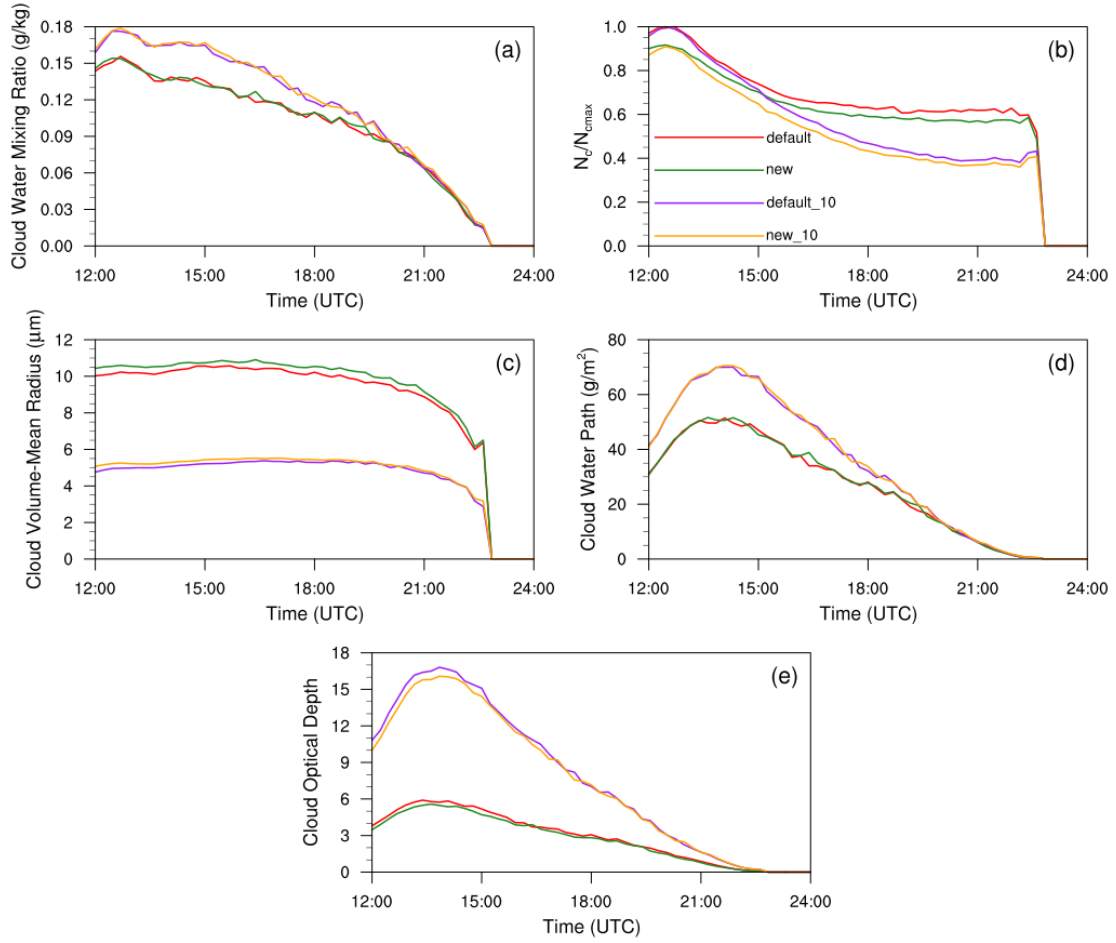



Figure 5. The temporal evolutions of main cloud microphysical and optical properties

in all simulation experiments for the stratocumulus case, including (a) cloud water

mixing ratio (q_c) (g/kg), (b) cloud droplet number concentration (N_c) (/cm^3), (c) cloud

droplet volume-mean radius (r_v) (μm), (d) cloud water path (CWP) (g/m^2), and (e) cloud

optical depth (τ). In (b), N_c in the experiments *default* and *new* are normalized by the

maximum cloud droplet number concentration (N_{cmax}) from *default*, respectively; N_c in

the experiments *default_10* and *new_10* are normalized by N_{cmax} from *default_10*,

respectively. The four experiments are detailed in Table 1.

801

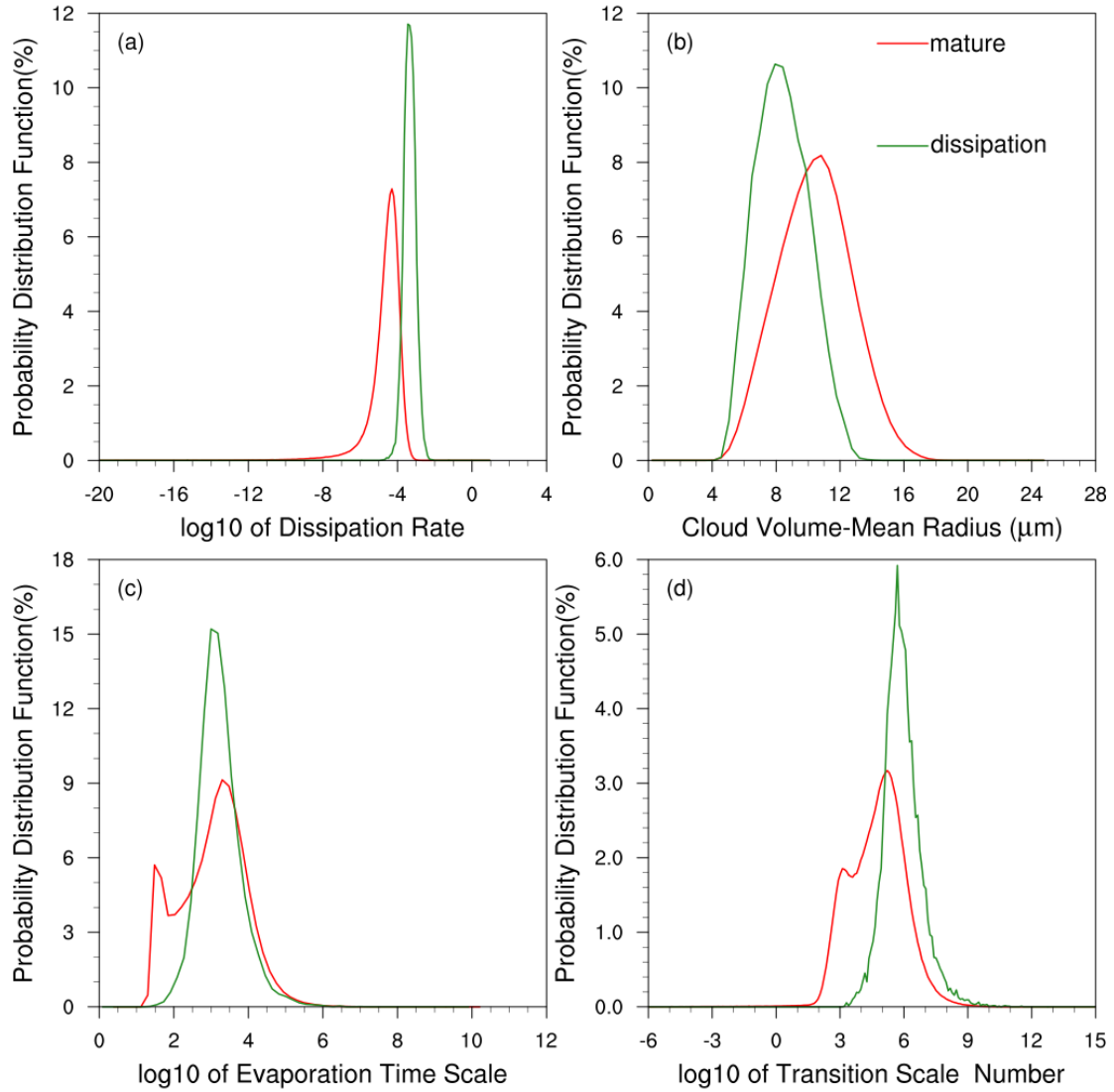
Figure 6. Probability distribution functions (PDFs) of (a) turbulence dissipation rate (ε),

(b) cloud droplet volume-mean radius (r_v), (c) evaporation time scale (τ_{evap}), and (d)

transition scale number (N_L) of cloud grids experiencing the entrainment-mixing

process in the simulations with the new entrainment-mixing parameterization for the

cumulus case (Cu, the solid lines) and the stratocumulus case (St, the dash lines),

respectively. The experiments are detailed in Table 1.

808

809   Figure 7. Probability distribution functions (PDFs) of (a) turbulence dissipation rate (ε),

810   (b) cloud droplet volume-mean radius (r_v), (c) evaporation time scale (τ_{evap}), and (d)

811   transition scale number (N_L) of cloud grids experiencing the entrainment-mixing

812   process at the mature stage from 12:00 UTC to 16:00 UTC (the red lines) and the

813   dissipation stage from 21:00 UTC to 24:00 UTC (the green lines) in *new* for the

814   stratocumulus case. The experiment is detailed in Table 1.

815

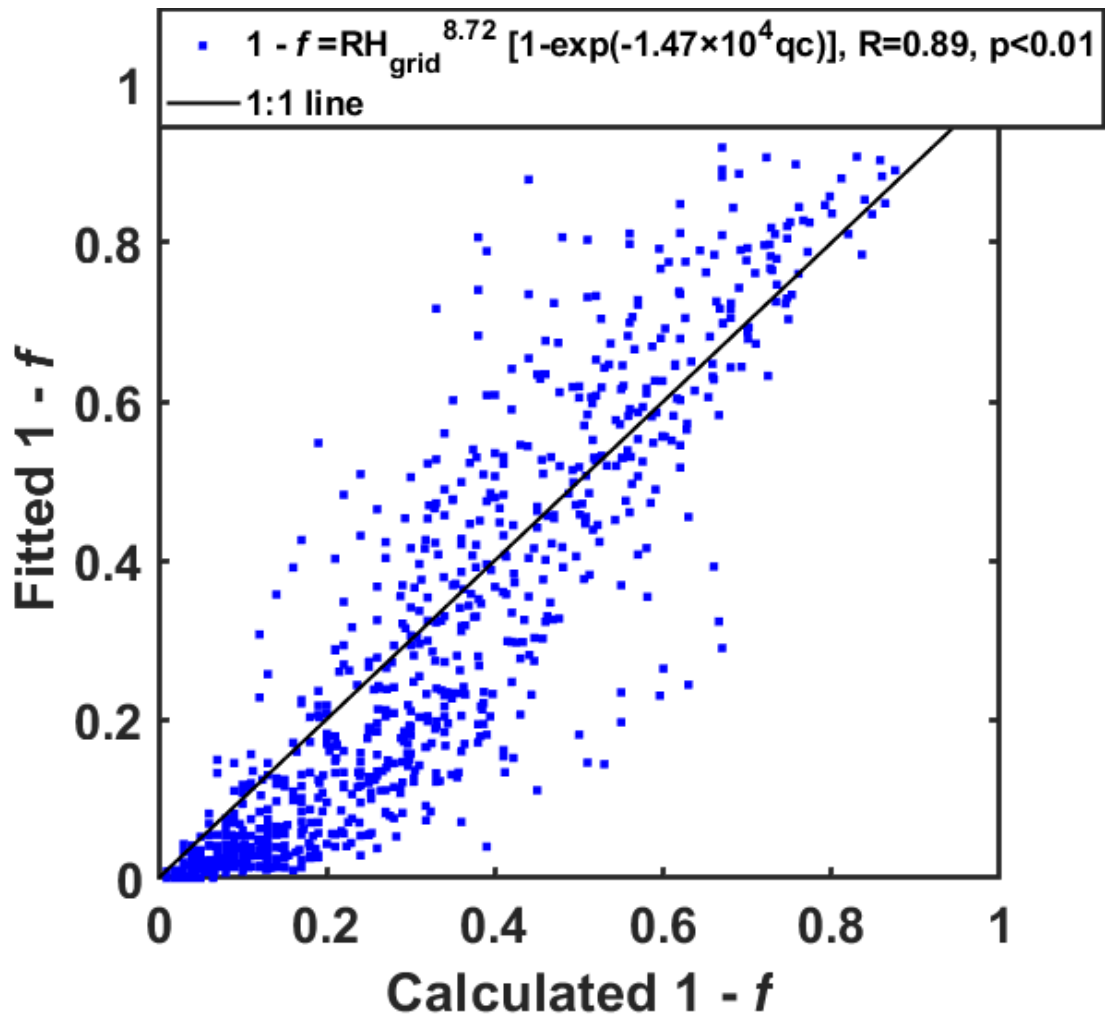
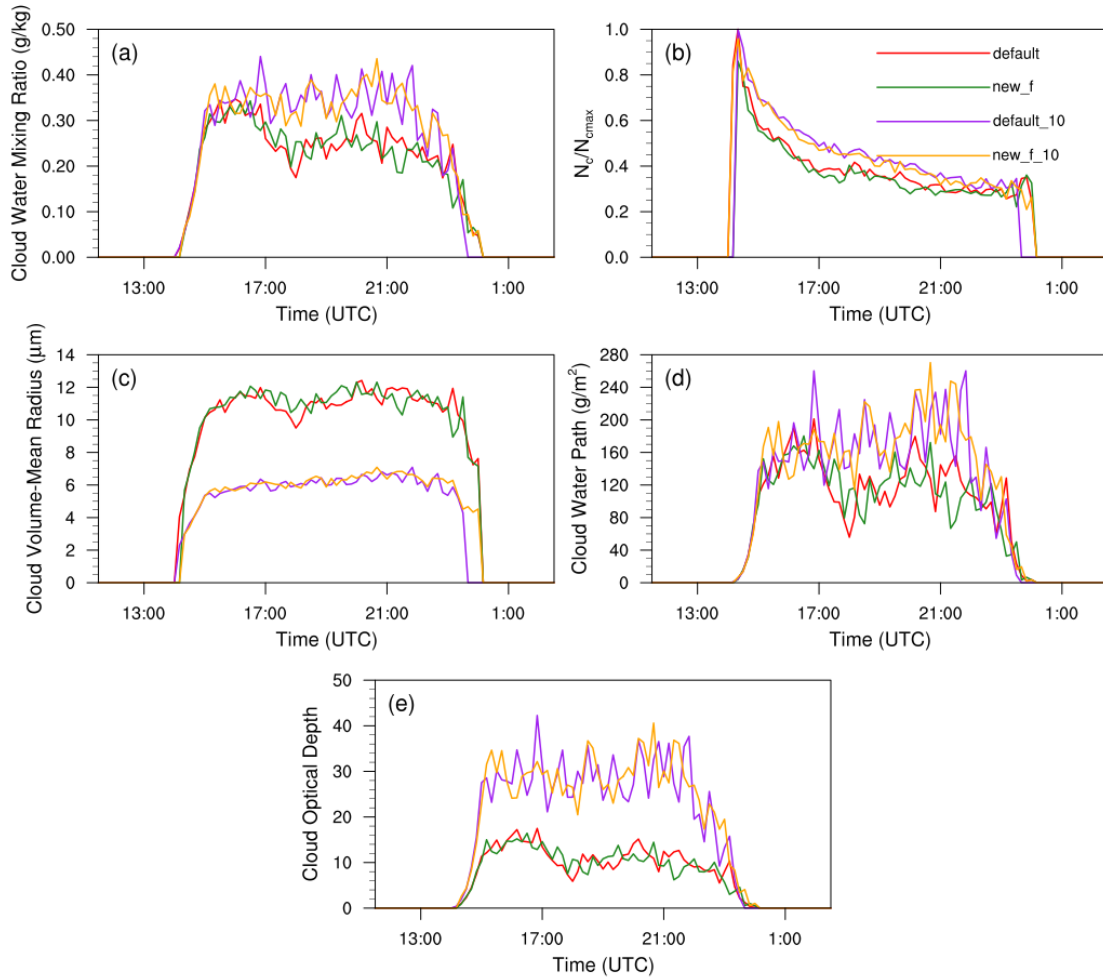



Figure 8. The fitted 1 - *f* as a function of the calculated 1 - *f*. The fitted $1 - f$ is obtained

by the fitting functions with grid-mean relative humidity (RH_{grid}) and cloud water

mixing ratio (q_c). The black line denotes the 1:1 line.

819

820 Figure 9. The temporal evolutions of main cloud microphysical and optical properties

821 in all simulation experiments for the cumulus case, including (a) cloud water mixing

822 ratio (q_c) (g/kg), (b) cloud droplet number concentration (N_c) (/cm^3), (c) cloud droplet

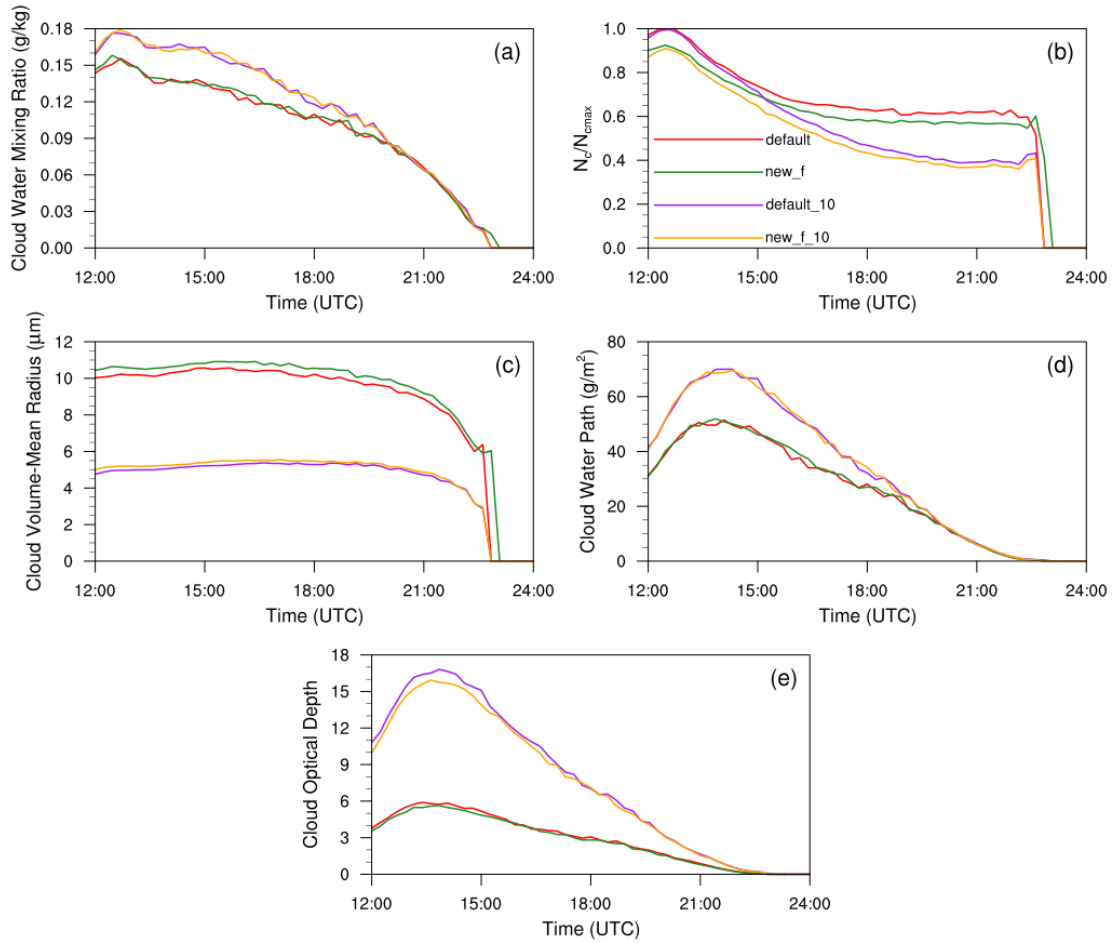
823 volume-mean radius (r_v) (μm), (d) cloud water path (CWP) (g/m^2), and (e) cloud optical

824 depth (τ). In (b), N_c in the experiments *default* and *new_f* are normalized by the

825 maximum cloud droplet concentration (N_{cmax}) from *default*, respectively; N_c in the

826 experiments *default_10* and *new__f_10* are normalized by N_{cmax} from *default_10*,

827 respectively. *new_f* and *new_f_10* are the experiments using entrained air relative

828 humidity.

829

830   Figure 10. The temporal evolutions of main cloud microphysical and optical properties

831   in all simulation experiments for the stratocumulus case, including (a) cloud water

832   mixing ratio (q_c) (g/kg), (b) cloud droplet number concentration (N_c) (/cm^3), (c) cloud

833   droplet volume-mean radius (r_v) (μm), (d) cloud water path (CWP) (g/m^2), and (e) cloud

834   optical depth (τ). In (b), N_c in the experiments *default* and *new* are normalized by the

835   maximum cloud droplet number concentration (N_{cmax}) from *default*, respectively; N_c in

836   the experiments *default_10* and *new_10* are normalized by N_{cmax} from *default_10*,

837   respectively. *new_f* and *new_f_10* are the experiments using entrained air relative

838   humidity.

839