ACP-2021-598 - Authors reply to the comments of the 2$^{\mathrm{nd}}$ review

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We thank both reviewers for their constructive comments and suggestions and for their very interesting scientific regarding the work presented in this manuscript. Their comments were helpful both for finding weak points that needed improvement and for improving the communication of the results. We have thanked both reviewers in the *Acknowledgement* section.

Please find below our answers (in blue fonts) to the comments of the reviewers (in black fonts).

1 Reviewer 1

Summary comment

In this revised submission the authors have successfully addressed the majority of my previous concerns. The Methodology section has substantially improved: The experimental setup is well described and the data processing procedure is clear and reproducible. The Results section has also improved, but the manuscript would benefit from a more usable definition of the area of interest in the wake of the tree (mc1) and from a more rigorous mathematical treatment and notation (mc2 and several other mcs). Overall this is an interesting study that makes use of a truly unique experimental setup and data; the modeling community will be able to readily take advantage of its data and findings.

1.1 Minor Comments

1. "We investigate this further by selecting grid points with high u0u0 comparing to the undisturbed flow" -> Why are the author's limiting their analysis to this region of the flow? The way the selection of this region of interest appears as quite convoluted and I would imagine that the modeling community would benefit much more from this analysis if the validity of the eddy viscosity assumption was assessed in the whole wake region, rather than in a thin (albeit dynamically important) layer. This is mostly a recommendation to improve the quality of the manuscript.

   We agree with the reviewer that from a flow modelling perspective ideally the assessment of the validity of the eddy viscosity hypothesis should take place alone the whole cross-section of the wake, but our data is not of sufficiently high quality to do this. The main limitation is the measurement error relative to the magnitude of both the mean wind gradient and the momentum fluxes in locations with a low-turbulence flow. This limitation originates both from the characteristics of the experimental setup (i.e. wind lidar probe lengths) and the random errors in the estimated values of the wind vector that are attributed to the limited length of the acquired data set. The following sentence has been added in the Discussion section after the line 284 of the revised version, to clarify this point:

   The reduction of random errors, in combination with smaller probe lengths of the wind lidar, would enable the study of the relation between the momentum fluxes and the mean gradients even in the center of the wake, where very small gradients are observed.

Furthermore, we changed the following sentence:

Line 221: *We investigate this further by selecting grid points with high*$\langle \hat{u}'\hat{u}' \rangle$ *comparing to the undisturbed flow (for more information regarding the grid selection we refer to the Appendix B).We chose these grid cells both because they represent the area where the mixing takes places.*

as:

Line 221: *We investigate this further by selecting grid points with high* $\langle \hat{u}'\hat{u}' \rangle$ *comparing to the undisturbed flow (for more information regarding the grid selection we refer to the Appendix B). We chose these grid cells because they represent the area where the mixing of momentum between the free and wake flow takes place.*

2. L201. strain -> strain rate (and elsewhere).

   Corrected

3. L215. Equation (3) is saying nothing about the stream-wise gradients of the vertical and cross-stream velocities, so this remark is not correct. In other words, if stream-wise gradients of the vertical and cross-stream velocities were not equal to zero, equation 3 would still be valid since it describes a relation between different quantities. Please rephrase.

   We do not agree with the reviewer in this point. In this study we focus on the transport mechanism of the longitudinal momentum. For this purpose, as we state in the text we construct a momentum vector from the following two components:

$$\langle u_1' u_i' \rangle = \nu_T \frac{\partial u_1}{\partial x_i}, \text{ where } i = 2, 3. \tag{1}$$

   However, this equation originates from the Reynolds stress tensor, according to which:

$$\langle u_i' u_j' \rangle - \frac{1}{3} \langle u_k' u_k' \rangle \delta_{ij} = -\nu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \tag{2}$$

   According to Equation 2 the transverse and vertical momentum fluxes are equal to:

$$\langle u_1' u_2' \rangle = -\nu_T \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) \tag{3}$$

   and

$$\langle u_1' u_3' \rangle = -\nu_T \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right). \tag{4}$$

   With our statement in line 215, we want to argue on where do we base our assumption that the contribution of the terms $\frac{\partial u_2}{\partial x_1}$ and $\frac{\partial u_3}{\partial x_1}$ can be considered negligible in the case of the flow examined in our study. We propose the following rewriting of the text in lines 215 – 219 to clarify this point:

With the above equation we want to express the relation between the momentum flux and mean gradient. This expression originates from Eq. 2, when the along wind gradients of the vertical $\frac{\partial\langle u_2\rangle}{\partial x_1}$ *and transverse components* $\frac{\partial\langle u_3\rangle}{\partial x_1}$ *are considered to be negligible. We base this assumption on the estimated values of the along wind gradients* $\left(\frac{\partial\langle u_2\rangle}{\partial x_1}\right.$ *and* $\left.\frac{\partial\langle u_3\rangle}{\partial x_1}\right)$ *based on the wind lidar and sonic anemometer measurements at the 10 locations, where sonic anemometers were found on the* M_2 *mast.*

4. Caption of Fig. 5. I recommend using index notation for the wind gradient vector as well, since otherwise it is difficult to relate gradients and corresponding momentum fluxes.

   We agree that the current caption can be confusing. The sentence *Direction of (a) the mean gradient* $\left|\left(\frac{\partial\langle u\rangle}{\partial y},\frac{\partial\langle u\rangle}{\partial z}\right)\right|$ *and (b) the covariance* $\langle u_1'u_i'\rangle$ *vectors.* has been rewritten as: *Direction of (a) the mean gradient* $\frac{\partial\langle\hat{u}_1\rangle}{\partial\hat{x}_i}$ *and (b) the covariance* $\langle\hat{u}_1'\hat{u}_i'\rangle$ *vectors..*

5. L229. The simplest parameterization for the eddy viscosity is assuming it is a constant. The mixing length is a level up in terms of complexity. Perhaps it is better to say "a relatively simple. . . "?

   Corrected: *The simplest parameterization of the eddy viscosity* ν_T*...* is now re-written as: *A relatively simple parameterization of the eddy viscosity* ν_T*...*

6. EQ4. Are the authors referring to the L2 norm of the velocity gradient tensor? In this case I would recommend using this symbol: $\|\cdot\|_2$ to avoid confusion.

   The norm in the velocity gradient tensor in Equation 4 refers to the Euclidian norm. However, we would prefer to keep the notation already used. We suggest the following addition in line 233: where $\left|\left(\frac{\partial\langle u\rangle}{\partial y},\frac{\partial\langle u\rangle}{\partial z}\right)\right|$ *represents the Euclidean norm of the transverse gradient of the mean longitudinal wind.*

7. 8. EQ5. Since the authors are only considering i = 1, 2, I recommend writing out the full expression for the momentum flux, i.e. $\sqrt{(u'v')^2+(u'w')^2}$ Same elsewhere.

   We agree with this suggestion. The equation:

$$l_m = \frac{\sqrt{\left|\langle u_1'u_i'\rangle\right|}}{\left|\left(\frac{\partial\langle u\rangle}{\partial y},\frac{\partial\langle u\rangle}{\partial z}\right)\right|}$$

   is rewritten as:

$$l_m = \frac{(\langle u'v'\rangle^2+\langle u'w'\rangle^2)^{1/4}}{\left|\left(\frac{\partial\langle u\rangle}{\partial y},\frac{\partial\langle u\rangle}{\partial z}\right)\right|}.$$

2 Reviewer 2

I am fine with most of the revisions which have been done following my review. I have only some remaining comments on the Eddy-viscosity check which was done in this work. The new equation 3 is a 2D vector alignment, which is derived from the tensorial relation 2. The Boussinesq eddy-viscosity relation is the tensorial one. We have (3) => (2) but the reverse is obviously false. Two minor changes should be done in the manuscript.

1. In the abstract, about the "validity of the eddy-viscosity hypothesis", replace by "the validity of a 2D vectorial relation derived from the eddy-viscosity hypothesis".

   We agree with this suggestion. We changed the sentence accordingly.

2. In the text, Line 237: replace "supports the validity of the eddy-viscosity hypothesis" by "supports a two-dimensional vectorial alignment between vectors, derived from the tensorial eddy-viscosity hypothesis".

   We agree with this suggestion. The sentence:

   Using the same criterion as Schmitt (2007), we find that the observed relative direction of the two vectors supports the validity of the eddy-viscosity hypothesis.

   is now re-written as:

   Using the same criterion as Schmitt (2007), we find that the observed relative direction of the two vectors supports a two-dimensional vectorial alignment, derived from the tensorial eddy-viscosity hypothesis.

References

Schmitt, F. G.: About Boussinesq's turbulent viscosity hypothesis: historical remarks and a direct evaluation of its validity, Comptes Rendus Mécanique, 335, 617 – 627, https://doi.org/https://doi.org/10.1016/j.crme.2007.08.004, 2007.