

# Constant Flux Layers with Gravitational Settling: with links to aerosols, fog and deposition velocities.

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**Abstract.** Turbulent boundary layer concepts of constant flux layers and surface roughness lengths are extended to include aerosols and the effects of gravitational settling. Interactions between aerosols and the Earth's surface are represented via a roughness length for aerosol which will generally be different from the roughness lengths for 10 momentum, heat or water vapor. Gravitational settling will impact vertical profiles and the surface deposition of aerosols, including fog droplets. Simple profile solutions are possible in neutral and stably stratified atmospheric surface boundary layers. These profiles can be used to predict deposition velocities and to illustrate the dependence of deposition velocity on reference height, friction velocity and gravitational settling velocity.

15     **Keywords** Constant flux layers • Aerosols • Fog • Gravitational settling • Surface roughness

## 1. Introduction

Within the turbulent atmospheric "surface layer", typically  $0 < z < \sim 50$  m, it is helpful to look at idealized situations 20 where fluxes of momentum, heat or other quantities are considered independent of height,  $z$ , above a surface which is a source or sink of the quantity being diffused by the turbulence. Garratt (1992, Chapter 3) or Munn (1966, Chapter 9) discuss this "constant flux layer" concept and, for momentum, the paper by Calder (1939), discussing earlier work by Prandtl, Sutton and Ertel, is an early recognition of the utility of this idealized concept. Monin-Obukhov Similarity Theory (MOST) is based on constant flux layer situations in steady state, horizontally 25 homogeneous, turbulent atmospheric boundary layers and leads to suitably scaled, dimensionless velocity and other profiles being dependent on  $z/L$  where  $z$  is height above the surface and  $L$  is the Obukhov length (defined below). With no sources or sinks of momentum or heat within these constant flux layers one can use dimensional analysis to establish the form of the profiles while observational data or hypotheses are needed to establish the detailed profile forms. Munn (1966, Chapter 9), Garratt (1992, section 3.3) or Kaimal and Finnigan (1994) explain Monin-Obukhov 30 similarity while Monin and Obukhov (1954) is a translation of the original Russian work. The simplest case is with neutral stratification ( $1/L = 0$ ) where dimensional analysis can be used to infer that the velocity shear,  $dU/dz$  is simply proportional to  $u_*/z$  where the shear stress, assumed constant with height, is  $\rho u_*^2$ , with  $\rho$  as air density. Integration of this relationship leads to

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$$U(z) = (u_*/k) \ln(z/z_{0m}), \quad (1)$$

with the roughness length for momentum,  $z_{0m}$ , being defined as the height at which a measured profile has  $U = 0$  when plotted on a  $U$  vs  $\ln z$  graph, and where  $k$  is the Karman constant with a generally accepted value of 0.4.

Noting that  $z_{0m}$  values are generally small compared to measurement heights, and after a  $z_{0m}$  value has been

40 established for the underlying surface, it is mathematically convenient to modify the relationship to

$$U = (u_*/k) \ln((z+z_{0m})/z_{0m}), \quad (2)$$

so that we have  $U = 0$  on  $z = 0$ . In eddy viscosity terms ( $u_*^2 = K_m dU/dz$ ) this corresponds to

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$$K_m = ku_*(z+z_{0m}). \quad (3)$$

In situations with constant, or near constant fluxes of heat ( $H$ ) or water vapour, similar, near logarithmic, MOST profiles and eddy diffusivities can be established, based on measured profiles, involving  $z/L$  where the Obukhov

50 length,  $L = -\rho c_p \theta u_*^3 / (kgH)$  in which  $c_p$  is the specific heat of air at constant pressure,  $g$  is acceleration due to gravity and  $\theta$  is the potential temperature. Application of Buckingham's pi theorem, assuming steady state, horizontally homogeneous conditions, with a constant (positive upwards) heat flux, ( $H/\rho c_p = -u_* \theta_*$ ) leads to

$$(kz/\theta_*) d\theta/dz = \Phi_H(z/L) \quad (4)$$

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where  $\Phi_H(z/L)$ , referred to as a dimensionless temperature gradient. This needs to be established experimentally but should approach one when  $z/L \rightarrow 0$ . In the limit for small  $z$ , or large  $|L|$ , we again get a logarithmic profile after integration but a complication arises over what we define as surface temperature, or surface water vapour mixing ratio. Integration of Eq (4) and a similar equation for water vapour leads to potential temperature and water vapour

60 profiles that can involve additional "scalar" roughness lengths,  $z_{0h}$  and  $z_{0v}$ . Much has been written about roughness lengths and ratios between  $z_{0m}$  and  $z_{0h}$ , including Chapter 5 of Brutsaert (1982) and Chapter 4 of Garratt (1992). For momentum transfers, pressure differences and form drag on roughness elements, sand grains, blades of grass, bushes, trees, buildings and water waves can provide most of the drag on the surface. Except over water,  $z_{0m}$  is considered as a Reynolds number independent surface property. Water waves are wind speed dependent and  $z_{0m}$

65 needs to take this into account. For heat and water vapour the final transfers from air to the surface involve molecular diffusion and, as a result, values of  $z_{0h}$ ,  $z_{0v}$  are generally lower than  $z_{0m}$ .

For aerosol particles or droplet concentrations we can introduce an additional roughness length,  $z_{0c}$ , on the basis that interactions with the surface will be different from momentum and from other scalars. Aerosol type, density and size, as well as  $u_*$ , may also cause variability in  $z_{0c}$ . As was necessary with the established roughness lengths for

70 momentum and heat, field measurements over a variety of surfaces will be needed to establish appropriate values. As a first approach, for fog droplets and other aerosol particles deposited to water, and other, surfaces we assume  $Q_c \rightarrow 0$  as  $z \rightarrow 0$  and, as a trial value, will generally use  $z_{0c} = 0.01$  m for illustration. This is somewhat larger than values typically assumed for water vapour or heat. The main innovation in this short communication will be to

combine the effects of turbulent transfer towards an underlying surface with gravitational settling ( $V_g$ ). This is done  
75 in a similar way to that proposed by Venkatram and Pleim (1999) and differs from the additive deposition velocity  
format used by Zhang et al (2001) and Slinn (1982). The parameter,  $S = V_g/ku_*$  plays a key role.

## 2. A simple model with added gravitational settling

80 We consider situations where there is aerosol present with a concentration or mass mixing ratio,  $Q_c$ . For simplicity it  
is assumed to consist of uniform particles with a constant gravitational settling velocity,  $V_g$ , and is at a density low  
enough to have no impact on the density of the combined air plus aerosol mixture. We assume no mass exchange  
between the aerosol and the surrounding air, which may be a concern for fog droplets which require an additional  
assumption that the air is always at 100% relative humidity.

85 If we have a net upward or downward flux of aerosol we need to discuss the source. If we are considering sand  
or dust being picked up from the surface by wind then upward diffusion will be countered by downward  
gravitational settling, while if the source of the aerosol is above our constant flux layer then the turbulent fluxes and  
gravitational settling combine. This could be the case with long range transport of aerosol in air blowing out over a  
rural area, a lake or the ocean. Another example could be fog droplets, formed at the top of a fog layer and being  
90 deposited at the underlying surface (Taylor et al, 2021).

In a horizontally homogeneous, steady state situation, and with a simply specified eddy diffusivity (Eq (3) but  
with  $z_{0m}$  replaced by  $z_{0c}$ ) and neutral stratification we just need to consider vertical turbulent transfers and  
gravitational settling where  $V_g$  represents the gravitational settling velocity. One could then model the constant  
downward flux of aerosol,  $F_{Qc}$ , as

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$$V_g Q_c + K_{qc} dQ_c/dz = F_{Qc} = u_* q_{c*}. \quad (5)$$

Csanady (1973) proposed this approach and Venkatram and Pleim (1999) obtained essentially the same solution as  
we will find below. They commented, in 1999, "... why not use a formulation that is consistent with the mass  
100 conservation equation (Eq. 5)." More recently Giardina and Buffa (2018) raise the same issue. Note that  $V_g$  is  
generally proportional to  $d^2$ , where  $d$  is the diameter, via Stokes law for small ( $d < 60 \mu\text{m}$ ) spherical particles (Rogers  
and Yau, 1976, p125), and  $u_*$  is the friction velocity. We introduce  $q_{c*}$  as a mixing ratio scale via this constant flux  
definition. The eddy diffusivity  $K_{qc}$  is assumed to be

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$$K_{qc} = k u_*(z + z_{0c}), \quad (6)$$

where  $z_{0c}$  is a roughness length for the aerosol with the assumption that  $Q_c = Q_{c,surf}$  at  $z = 0$ .

The upward flux case with a surface source of aerosol is interesting in the sense that there will only be a steady,  
horizontally homogeneous, state when the net flux is zero, i.e, upward turbulent transfer is balanced by gravitational

110 settling. Xiao and Taylor (2002), in relation to a blowing snow study, show, by solving Eq.(5) with  $F_{Qc} = 0$ , that this leads to the classic power law solution (e.g, Prandtl, 1952), which in the current context is

$$\ln(Qc(z)/Qc_{surf}) = -S\zeta, \text{ where } \zeta = \ln((z+z_{0c})/z_{0c}) \text{ and } S = V_g/(ku_*)$$

or

115  $Qc(z) = Qc_{surf} ((z+z_{0c})/z_{0c})^{-S} \quad (7)$

Profiles of suspended sediment, and velocity, in water currents can be treated in a similar way but there is an interesting twist if the density of the sediment and water mix is sufficient to modify the turbulent mixing through stable stratification. Taylor and Dyer (1977) rediscovered an interesting result due to Barenblatt (1953) showing that 120 a modified solution allowing for stratification effects on the eddy diffusivity could be obtained. Observations were sometimes misinterpreted as power laws with a modified value of  $k$  (Graf, 1971, p180).

For the case of downward flux to the lower boundary in the atmospheric surface layer it is easiest if we assume  $Qc_{surf} = 0$ , which may be most relevant over water but is also often assumed for dry deposition of particles (Seinfeld and Pandis, 1998, p960). Material starts from a source above the constant flux layer and travels downwards due to 125 both turbulent mixing and gravitational settling. Assuming constant values for  $z_{0c}$ ,  $u_*$  and  $V_g$  one can then solve the first order differential equation, Eq (5), by integrating factor techniques. Multiplying Eq. (5) by  $(z+z_{0c})^{S-1}/(ku_*)$  where  $S = V_g/(ku_*)$ , gives,

$$(d/dz)[(z+z_{0c})^S Qc] = (q_{c*}/k)(z+z_{0c})^{S-1} \quad (8)$$

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and, with  $Qc(0) = 0$ , the solution is,

$$Qc(z) = (q_{c*}/(kS)) [1 - ((z+z_{0c})/z_{0c})^S]. \quad (9)$$

135 In terms of  $\zeta = \ln((z+z_{0c})/z_{0c})$ , we can write,

$$Qc(\zeta) = (q_{c*}/(kS)) [1 - e^{-S\zeta}]. \quad (10)$$

These can be referred to as Constant Flux Layer with Gravitational Settling, CFLGS, profiles. In the limits as  $V_g$  and 140  $S \rightarrow 0$ , Eq (10) gives  $Qc(\zeta) = (q_{c*}/k) \zeta$ , a standard log profile.

### 3. Some profiles

The expected values of  $V_g$  and  $u_*$  should be considered. Aerosols come in all shapes and sizes, see for example 145 Farmer et al (2021) who consider diameters from 1nm to 100 $\mu\text{m}$  and deposition velocities, resulting from a combination of turbulent mixing and gravitational settling, mostly in the range 0.01 to 100  $\text{cm s}^{-1}$ . Farmer et al

(2021) also highlight the role of aerosols in climate issues. Fog droplets have a range of sizes but most fall in the diameter range 0-50  $\mu\text{m}$ , often with bimodal distributions and peaks around 6 and 25  $\mu\text{m}$  (see for example Isaac et al, 2020). Applying Stokes law with appropriate values for water droplets (see Rogers and Yau, 1976) for these peak sizes we get  $V_g$  values of 0.0011 and 0.0192  $\text{m s}^{-1}$ . Aerosol particles of different density and shape may have different  $V_g$  values but the focus here will be for situations with  $V_g < 2 \text{ cm s}^{-1}$  and diameters in the 1 - 20  $\mu\text{m}$  range. These terminal velocities are clearly small compared to wind speed but for the larger diameter fog droplets, the terminal velocity can easily reach 72  $\text{m}$  per hour and would represent a considerable removal rate in fog which may last several hours or days. The key parameter in our constant flux with gravitational settling model is

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$$S = V_g/ku_* . \quad (11)$$

In moderate winds over the ocean one might expect  $u_*$  values in the 0.15-0.6  $\text{m s}^{-1}$  range, while in light winds over land it could be lower. The parameter,  $S$  will thus generally be in the range 0.0 to 0.3 in marine situations but could 160 be unlimited in light winds with low  $u_*$  over land. With high values of  $S$  gravitational settling will be the dominant process except very close to the surface. At low values of  $S$  gravitational settling will have little impact and the  $Q_c$  profiles are approximately logarithmic.

To illustrate this Fig. 1 shows  $Q_c$  constant flux profiles with linear and log vertical axes and a range of  $S$  values. We have scaled  $Q_c$  with a value at 50m. The main unknown is the value of  $z_{0c}$ . Here we use our first guess value ( $z_{0c} = 0.01\text{m}$ ) indicating relatively efficient capture of water droplets, or other aerosol, by the surface. These calculations 165 are for uniform sized aerosol particles or droplets. Note that with high  $S$  ( $=V_g/ku_*$ ) values, maybe occurring with low  $u_*$  and minimal turbulence, the limiting case would be constant  $Q_c$  down to  $z = 0$  and a discontinuity to  $Q_c = 0$  at the surface. Calculations with  $S = 1$  and 5 (not shown) confirm this. The essential point from Fig. 1 is that, if there is gravitational settling involved then the profiles will depart from the simple logarithmic profiles that one might 170 expect in a neutrally stratified near-surface atmospheric boundary layer. Note that these profiles depend on  $z_{0c}$  but not directly on  $z_{0m}$ , except via  $u_*$ .

For aerosol dry deposition to any surface a traditional way to parametrize the process is with a deposition velocity,  $V_{dep}$ , based on a  $Q_c$  measurement at  $z_{ref}$ , and simply defined via,

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$$F_{Qc} = V_{dep}(z_{ref}) Qc(z_{ref}) . \quad (12)$$

In a constant flux layer,  $V_{dep}(z_{ref})$ , shown in Fig. 2, is simply proportional to the inverse of  $Qc(z_{ref})$  provided that  $F_{Qc}$  is constant between the surface and  $z_{ref}$ . The dependence of  $V_{dep}$  on the reference height,  $z_{ref}$ , for  $Qc$  is seldom acknowledged in papers reporting measured  $V_{dep}$  values, or in the review by Farmer et al (2021). The height,  $z_{ref}$ , is 180 often not discussed and hard to find, e.g. in Sehmel and Sutter (1974). In addition, there is a strong dependence on  $u_*$  and any value of  $V_{dep}$  will depend on  $z_{ref}$ ,  $u_*$  and  $V_g$  as well as the nature of the underlying surface, which we have characterized through  $z_{0c}$ . In a numerical model the reference height  $z_{ref}$  is often the lowest grid level. If gravitational settling is the main cause of  $F_{Qc}$ , we would expect little change in  $Qc$  with height, but if turbulent transfer is

dominant then the choice of  $z_{ref}$  could be important. Zhang et al (2001) recognize this in their widely used dry  
185 deposition scheme, based on Slinn (1982), and  $z_{ref}$  ( $= z_R$  in their notation) is clearly a factor in their aerodynamic  
resistance ( $R_a = (ku_*)^{-1} \ln(z_{ref}/z_{0m})$ , in neutral stratification). Their surface resistance ( $R_s$ ) could then be interpreted in  
roughness length terms (as in Garratt, 1992, Section 3.3.3), as  $R_s = (ku_*)^{-1} \ln(z_{0m}/z_{0c})$ . Note that if  $z_{0m} = z_{0c}$  then  $R_s = 0$ , and this may be controversial.

Zhang et al (2001), Slinn(1982) and many others (see Saylor et al, 2019, Farmer et al, 2021) combine these  
190 resistances with a gravitational settling velocity, through the relationship.

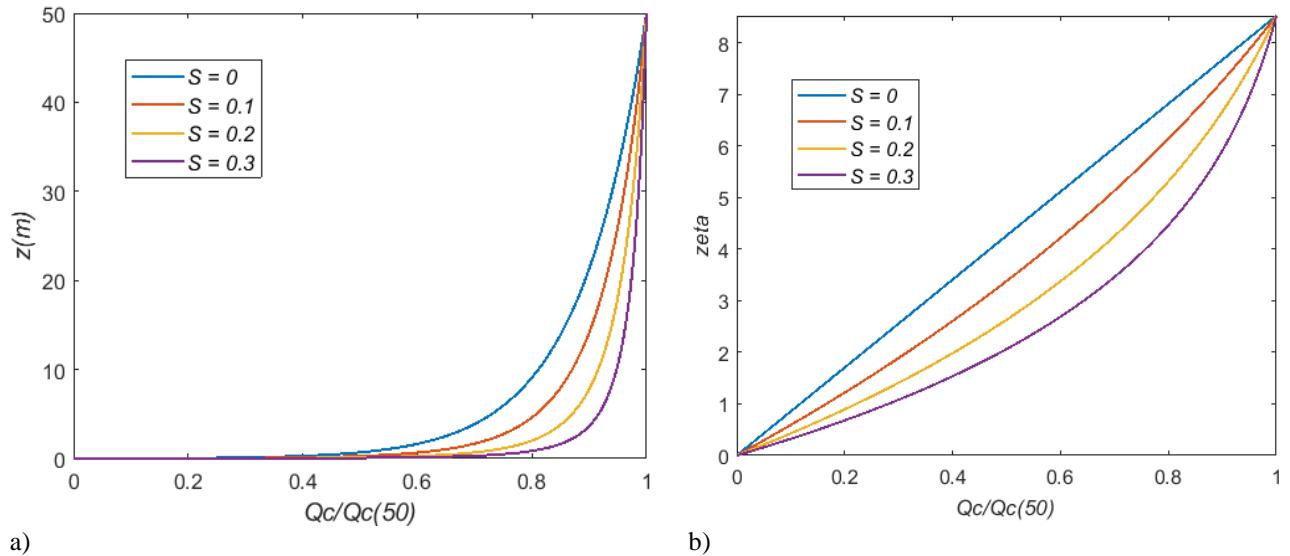
$$V_{dep} = V_g + 1/(R_a + R_s) \quad \text{or} \quad V_{dep}/ku_* = S + 1/[ku_*(R_a + R_s)] \quad (13)$$

A possible alternative, which takes account of a modified  $Qc$  at  $z_{0m}$ , is derived by Seinfeld and Pandis (1998, Eq.  
195 19.7), but this is "not consistent with mass conservation" as noted by Venkatram and Pleim (1999).

$$V_{dep} = V_g + 1/(R_a + R_s + R_a R_s V_g). \quad (14)$$

Eq. 14 will give lower  $V_{dep}$  values when  $R_s > 0$ . Neither expression, using the  $R_a$ ,  $R_b$  definitions above, matches our  
200 CFLGS model for which, provided  $z_{ref} \gg z_{0m}$ ,  $z_{0c}$  we can write, assuming the  $R_a$  and  $R_s$  relations given above,

$$V_{dep}/ku_* = S/(1 - e^{-S\xi}) \approx S/(1 - \exp(-Sku_*(R_a + R_s))) \quad (15)$$

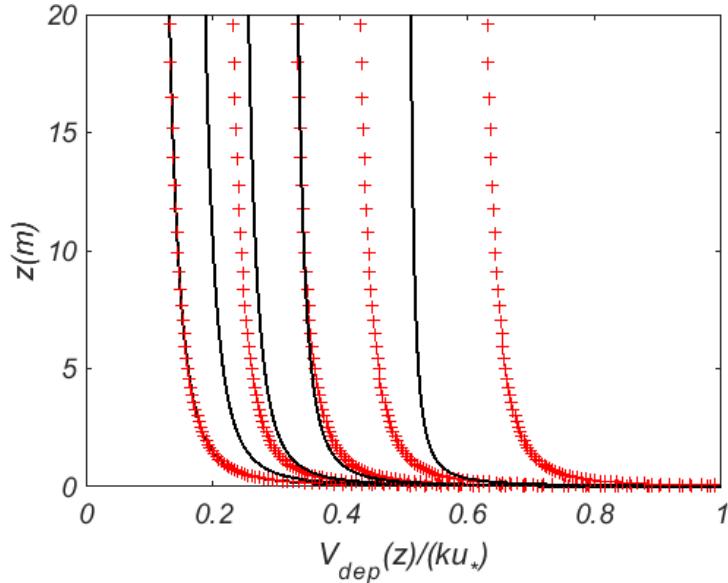


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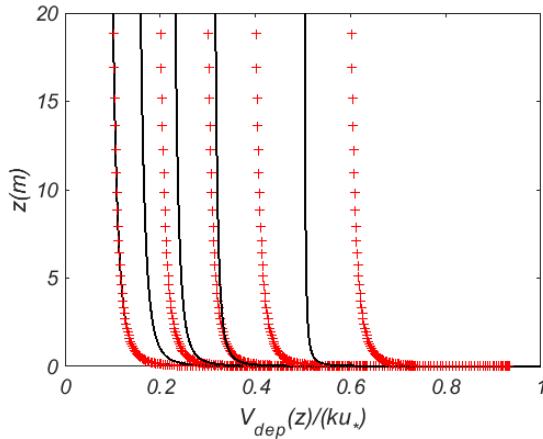
**Fig. 1**  $Qc$  profiles, scaled by the 50 m value, from the surface to  $z = 50$  m in constant flux layers with gravitational settling. The surface roughness length for aerosol removal,  $z_{0c} = 0.01$  m. Plotted with linear (a) and logarithmic (b) height scales and four  $S$  values.

210 Sample  $V_{dep}$  results are shown in Fig 2 when  $V_g \geq 0$ . In the first case (a) we took  $z_{0m} = z_{0c} = 0.01$  m so that  $R_s = 0$ . With no gravitational settling both models agree. For  $S > 0$ , the CFLGS deposition velocities, Eq(15), are lower than those computed from the Zhang/Slinn formulation. Cases b and c keep  $z_{0m} = 0.01$  m but allow  $z_{0c}$  to be smaller,  $R_s > 0$  in (b) or larger,  $R_s < 0$  in (c). The CFLGS relationship, Eq (12c) always shows a modest  $V_{dep}$  reduction, relative to the Zhang/Slinn equation, which is typically of order 20%.

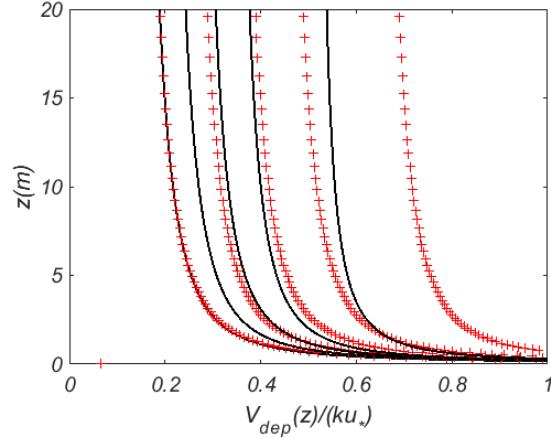
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a)  $z_{0c} = z_{0m} = 0.01$  m ( $R_s = 0$ )



b)  $z_{0c} = 0.001$  m;  $z_{0m} = 0.01$  m



c)  $z_{0c} = 0.1$  m;  $z_{0m} = 0.01$  m

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Fig. 2  $V_{dep}$  profiles, from surface to  $z = 20$  m in constant flux layers with gravitational settling. Solid lines are with the CFLGS model, the + points are from the Zhang/Slinn formulation (ZS). Five cases, left to right are  $S = 0, 0.1, 0.2, 0.3, 0.5$ . a)  $z_{0m} = z_{0c} = 0.01$  m,  $R_s = 0$ ; b)  $z_{0c} = 0.001$  m;  $z_{0m} = 0.01$  m,  $ku_*R_s = 2.3$ ; c)  $z_{0c} = 0.1$  m;  $z_{0m} = 0.01$  m,  $ku_*R_s = -2.3$ .

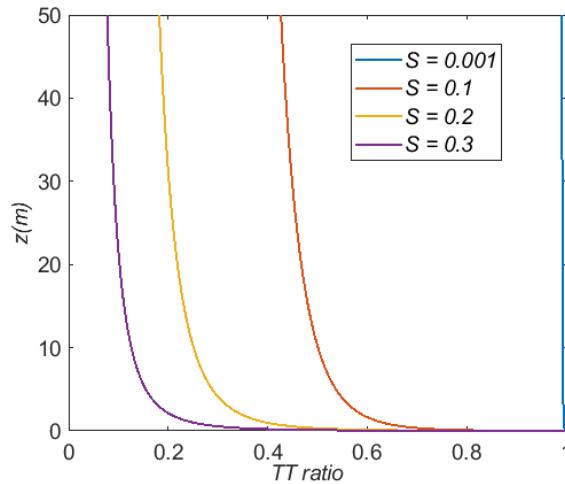
Another way to look at the relative importance of gravitational settling for these uniform size droplets is to consider the relative contributions to the total downward flux of aerosol ( $u_* q_{c*}$ ). The gravitational contribution is simply  $V_g Q_c$  while the turbulent diffusion contribution is,

$$230 \quad k u_* dQ_c/d\zeta = u_* q_{c*} e^{-S\zeta}, \text{ where } \zeta = \ln((z+z_{0c})/z_{0c}) \quad (16)$$

The ratios of turbulent transfer (TT)/total flux and gravitational settling (GS)/total flux then become

$$235 \quad TT = e^{-S\zeta} \text{ and } GS = 1 - e^{-S\zeta} \quad (17)$$

Noting that  $\zeta = \ln((z+z_{0c})/z_{0c})$  we can see that these ratios depend on both  $z_{0c}$ , through the  $z(\zeta)$  relationship, and  $S$  and will vary with  $z$ . Fig. 3 illustrates this. It is important to note that Fig. 3 is based on  $z_{0c} = 0.01$  m. If we increase it to  $z_{0c} = 0.1$  m then turbulent fluxes become more important (Fig 2c). We can see that the TT ratio is formally 1 at the surface, where  $Q_c = 0$  so there is no gravitational component. For very large  $\zeta$  the TT term would decay to 0 but 240 this would be well above the constant flux layer approximation. At 50 m the value will depend on  $S$  and  $z_{0c}$ .



**Fig. 3** Variation of the Turbulent Transfer fraction of the total  $Q_c$  flux and its variation with  $z$  and  $S$ . Note that these  $z$  values are based on  $z_{0c} = 0.01$  m.

#### 4. Stable Stratification Case

For fog applications, over land, radiation fog often occurs at low wind speeds with stable stratification. Advection fog when warm, moist air is advected over a colder surface is another case with stable stratification. For constant flux boundary layers in these circumstances MOST has, for velocity,  $K_m = k(z+z_{0m})/\Phi_M(z/L)$  and

$$\Phi_M(z/L) = 1 + \beta (z + z_{0m})/L : U = (u_*/k) (\ln ((z + z_{0m})/z_{0m}) + \beta z/L). \quad (18)$$

265 Observed profiles give  $\beta = 5$  (Garratt 1992, p52). In addition  $\Phi_H = \Phi_M$  and if we extend this idea to  $\Phi_{Qc}(z/L)$  and set  $K_{Qc} = k(z + z_{0c})/\Phi_{Qc}(z/L)$  we need to solve,

$$V_g Qc + [ku_* (z + z_{0c})/\Phi_{Qc}(z/L)] dQc/dz = F_{Qc} = u_* q_{c*}, \quad (19)$$

270 or, with  $\Phi_{Qc}(z/L) = 1 + \beta (z + z_{0c})/L$ ,

$$dQc/dz + S\{(1+\beta (z+z_{0c})/L)/(z+z_{0c})\}Qc = (q_{c*}/k)(1+\beta(z+z_{0c})/L)/(z+z_{0c}); \text{ with } S=V_g/(ku_*)$$

The Integrating Factor is  $\exp(\int S(1/(z+z_{0c})+\beta/L)dz = (z+z_{0c})^S \exp(S\beta z/L)$  so that

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$$d [(z+z_{0c})^S \exp(S\beta z/L)Qc] /dz = (q_{c*}/k)(1+\beta(z+z_{0c})/L) (z+z_{0c})^{S-1} \exp(S\beta z/L)$$

and we need to integrate the RHS. To do this it is convenient to let  $\beta(z+z_{0c})/L = x$  and the integral that we need is of

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$$(q_{c*}/k)(L/\beta)^{S-1} \exp(-Sx_0) \{(1+x)x^{S-1} \exp(Sx)\}, \quad \text{where } x_0 = \beta z_{0c}/L \quad (20)$$

After some guidance and a few trials one can see that  $d/dx\{x^S \exp(Sx)\} = (Sx^{S-1} + Sx^S) \exp(Sx)$  and the integral required is simply  $F(x, S) = x^S \exp(Sx)/S$ . We then evaluate  $F(x, S)$  at  $z = 0$ ,  $x = \beta z_{0c}/L$  and any other  $z$  to allow us to plot  $Qc$  profiles. With stable stratification and light winds the constant flux approximation would only apply to a relatively

285 shallow layer so we normalize with  $Qc(z_{top})$  and set  $z_{top} = 20$  m in these cases. If  $Qc = 0$  at  $z = 0$  we then have,

$$Qc(z) = (q_{c*}/k)(L/\beta)^{-1} \exp(Sx_0) [\exp(-Sx) x^S] [F(x, S) - F(x_0, S)], \quad (21)$$

and we can then plot the ratio  $Qc(z)/Qc(z_{top})$  as in Fig. 4. For  $S = 0$ , with no gravitational settling, the profile will be

290 essentially the same as the velocity profile in Eq. (18) above, i.e.

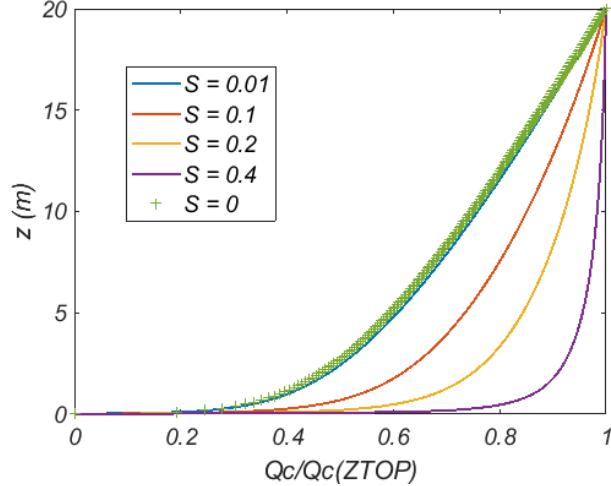
$$Qc(z) = (q_{c*}/k) (\ln ((z + z_{0c})/z_{0c}) + \beta z/L). \quad (22)$$

In addition to  $z_{0c}$  and  $S$  the key parameter is the Obukhov length,  $L = -\rho c_p u_*^3 \theta / (kgH)$ , ( $>0$ ). Neutral stratification corresponds to  $L \rightarrow \infty$  while stable stratification relationships ( $H < 0$ ,  $L > 0$ ) are generally limited to  $0 < z/L < 1$ . If we are concerned with height ranges up to 10 or 20m then  $L = 10$ m would be considered as a very low value maybe with  $u_* \approx 0.13$  ms<sup>-1</sup> and  $H \approx -20$  Wm<sup>-2</sup> as possible values. Figure 4 shows  $Qc(z)/Qc(20m)$  profiles in a typical case with our standard value,  $z_{0c} = 0.01$ m. We set  $L = 20$ m and use a range of  $S$  values. For large droplets,  $S = 0.4$ ,  $Qc$  flux is

dominated by gravitational settling and reductions in  $Q_c$  towards 0 only occur in the lowest few m. For smaller particles,  $S = 0, 0.01, 0.1$  turbulent mixing dominates the deposition process. Note that the  $S = 0$  points (log + linear profiles) and the  $S = 0.01$  line, almost overlap as one confirmation of solution form.

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**Fig 4.**  $Q_c/Q_c(ztop)$  profiles with stable stratification, assuming  $\Phi_{Qc}(z/L) = 1 + \beta(z+z_{0c})/L$ . We set  $\beta = 5$ ,  $L = 20$ m and  $z_{0c} = 0.01$ m.

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In unstable stratification it is generally accepted that  $\Phi_H(z/L) \neq \Phi_M(z/L)$  and relatively little is known about stability effects on diffusion of other scalars. For aerosol Jia et al (2021) assume  $\Phi_{Qc} = \Phi_H$  in unstable stratification but have proposed a new form, different from  $\Phi_H$ , for  $\Phi_{Qc}$  in stably stratified boundary layers. These are all based on Richardson number. In principle one could numerically solve Eq. (19) for any suitable  $\Phi_{Qc}(z/L)$  form but our interest is primarily the stable case and it is convenient that an analytic solution can be found for the generally accepted  $\Phi(z/L)$  forms if we assume  $\Phi_{Qc} = \Phi_H$ . Strictly speaking our  $\Phi(z/L)$  functions should be  $\Phi((z+z_0)/L)$  functions but we are generally dealing with  $z \gg z_0$  and it is customary to ignore that difference.

## 5. Conclusions

The initial idea behind this analysis was that, in marine fog, cloud droplets can both fall toward the underlying surface through gravitational settling and be diffused towards the surface by turbulence and on contact they can coalesce with an underlying water surface. Taylor et al (2021) apply these ideas to fog modelling with the WRF model. During reviews of that work, and an earlier version of the current paper, it became clear that some reviewers were reluctant to accept that turbulence could cause fog droplets to collide and coalesce with an underlying surface, and even more reluctant to see this as a constant flux layer situation. Fog droplets are perhaps a special case but the CFLGS concept is equally applicable to aerosol particles or droplets in general, provided that they are inert and without sources or sinks in the air. Desert dusts, various pollutants or micro-plastic fragments being blown out over lakes or the sea from sources on land may be examples. Here we could anticipate a situation with initial mixing

through a relatively deep atmospheric layer over land being advected over an aerosol capturing water surface so that one could envisage a situation over the water with a constant downward flux of aerosol due to gravitational settling plus turbulent diffusion in a low level constant flux layer.

One implication of the CFLGS model is that simply adding gravitational settling ( $V_g$ ) to a deposition velocity (340  $V_{dep}$ ) based on aerodynamic and surface resistances may overestimate the combined effects. If we use the CFLGS model it can indicate reductions of order 20%. These are small compared to the uncertainties based on deposition velocity measurements but may well be worth considering.

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### **Code/Data Availability**

Calculations were made with simple Matlab code, maybe 20 lines for each figure. Some sample code is made available as SupplementaryMaterial-CFLGS.pdf

395 **Author Contribution**

This is independent work by the single author.

### **Competing Interests**

None.

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