Constant Flux Layers with Gravitational Settling: with links to aerosols, fog and deposition velocities~~ over water~~.

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Abstract. Turbulent boundary layer concepts of constant flux layers and surface roughness lengths are extended to include aerosols and the effects of gravitational settling. Interactions between aerosols and the Earth's surface are represented via a roughness length for aerosol which will generally be different from the roughness lengths for momentum, heat or water vapor. Gravitational settling will~~These~~ impact vertical profiles and the surface deposition of aerosols, including fog droplets. ~~, especially over water.~~ Simple profile solutions are possible in neutral and stably stratified atmospheric surface boundary layers. These profiles can be used to predict deposition velocities and to illustrate the dependence of deposition velocity on reference height, friction velocity and gravitational settling velocity.

Keywords Constant flux layers • Aerosols • Fog • Gravitational settling • Surface roughness

1. Introduction

Within the turbulent atmospheric "surface layer", typically 0 < z < ~50 m, it is helpful to look at idealized situations where fluxes of momentum, heat or other quantities are considered independent of height, z, above a surface which is a source or sink of the quantity being diffused by the turbulence. Garratt (1992, Chapter 3) or Munn (1966, Chapter 9) discuss this "constant flux layer" concept and, for momentum, the paper by Calder (1939), discussing earlier work by Prandtl, Sutton and Ertel, is an early recognition of the utility of this idealized concept. Monin-Obukhov Similarity Theory (MOST) is based on constant flux layer situations in steady state, horizontally homogeneous, turbulent atmospheric boundary layers and leads to suitably scaled, dimensionless velocity and other profiles being dependent on z/L where z is height above the surface and L is the Obukhov length (defined below). With no sources or sinks of momentum or heat within these constant flux layers one can use dimensional analysis to establish the form of the profiles while observational data or hypotheses are needed to establish the detailed profile forms. Munn (1966, Chapter 9), Garratt (1992, section 3.3) or Kaimal and Finnigan (1994) explain Monin-Obukhov similarity while Monin and Obukhov (1954) is a translation of the original Russian work. The simplest case is with neutral stratification ($1/L = 0$) where dimensional analysis can be used to infer that the velocity shear, dU/dz is simply proportional to u_*/z where the shear stress, assumed constant with height, is ρu_*^2, with ρ as air density.

Integration of this relationship leads to

$$U(z) = (u_*/k)\ln(z/z_{0m}), \qquad (1)$$

with the roughness length for momentum, z_{0m}, being defined as the height at which a measured profile has $U = 0$ when plotted on a U vs ln z graph, and where k is the Karman constant with a generally accepted value of 0.4. Noting that z_{0m} values are generally small compared to measurement heights, and after a z_{0m} value has been established for the underlying surface, it is mathematically convenient to modify the relationship to

$$U = (u_*/k)\ln((z+z_{0m})/z_{0m}), \qquad (2)$$

so that we have $U = 0$ on $z = 0$. In eddy viscosity terms ($u_*^2 = K_m\, dU/dz$) this corresponds to

$$K_m = ku_*(z+z_{0m}). \qquad (3)$$

In situations with constant, or near constant fluxes of heat (H) or water vapour, similar, near logarithmic, MOST profiles and eddy diffusivities can be established, based on measured profiles, involving z/L where the Obukhov length, $L = -\rho c_p \theta u_*^3/(kgH)$ in which c_p is the specific heat of air at constant pressure, g is acceleration due to gravity and θ is the potential temperature. Application of Buckingham's pi theorem, assuming steady state, horizontally homogeneous conditions, with a constant (positive upwards) heat flux, ($H/\rho c_p = -u_*\theta_*$) leads to

$$(kz/\theta_*)\, d\theta/dz = \Phi_H(z/L) \qquad (4)$$

where $\Phi_H(z/L)$, referred to as a dimensionless temperature gradient. This needs to be established experimentally but should approach one when $z/L \rightarrow 0$. In the limit for small z, or large |L|, we again get a logarithmic profile after integration but a complication arises over what we define as surface temperature, or surface water vapour mixing ratio. Integration of Eq (4) and a similar equation for water vapour leads to ~~For~~ potential temperature and water vapour profiles th~~ese~~at can involve additional "scalar" roughness lengths, z_{0h} and z_{0v}. Much has been written about roughness lengths and ratios between z_{0m} and z_{0h}, including Chapter 5 of Brutsaert (1982) and Chapter 4 of Garratt (1992). For momentum transfers, pressure differences and form drag on roughness elements, sand grains, blades of grass, bushes, trees, buildings and water waves can provide most of the drag on the surface. E~~ and, e~~xcept over water, z_{0m} is considered as a Reynolds number independent surface property. Water waves are wind speed dependent and z_{0m} needs to take this into account. For heat and water vapour the final transfers from air to the surface involve molecular diffusion and, as a result, values of z_{0h}, z_{0v} are generally lower than z_{0m}.

For aerosol particles or~~and~~ droplet concentrations we can ~~will~~ introduce an additional roughness length, z_{0c}, on the basis that ~~their~~ interactions with the surface will ~~again~~ be different from momentum and from other ~~quantitie~~scalars. Aerosol type, density and size, as well as u_*, may also cause variability in z_{0c}. As was necessary with the established roughness lengths for momentum and heat, field measurements over a variety of surfaces will be needed to establish appropriate values. As a first approach, for fog droplets and other aerosol particles deposited to water, and other, ~~-~~surfaces we assume $Qc \rightarrow 0$ as $z \rightarrow 0$ and, as a trial value, will generally use $z_{0c} = 0.01$ m for illustration. This is somewhat larger than values typically assumed for water vapour or heat. The main innovation in this short communication will be to combine the effects of turbulent transfer towards an underlying surface with gravitational settling (V_g). This is done in a similar way to that proposed by Venkatram and Pleim (1999) and differs from the additive deposition velocity format used by Zhang et al (2001) and Slinn (1982). The parameter, $S = V_g/ku_*$ plays a key role.~~.~~

2. A simple model with added gravitational settling

We ~~will~~ consider situations where there is aerosol present with a concentration or mass mixing ratio, Qc. For simplicity it is assumed to consist of uniform particles with a constant gravitational settling velocity, V_g, and is at a density low enough to have no impact on the density of the combined air plus~~+~~ aerosol mixture. We assume no mass exchange between the aerosol and the surrounding air, which may be a concern for fog droplets which require an additional assumption that the air is always at 100% relative humidity.

If we have a net upward or downward flux of aerosol we need to discuss the source. If we are considering sand or dust being picked up from the surface by wind then upward diffusion will be countered by downward gravitational settling, while if the source of the aerosol is above our constant flux layer then the turbulent fluxes and gravitational settling combine. This could be the case with long range transport of aerosol in air blowing out over a rural area, a lake or the ocean. An~~Our~~ other example could~~will~~ be fog droplets, formed at the top of a fog layer and being deposited at the underlying surface (Taylor et al, 2021).

In a horizontally homogeneous, steady state situation, and with a simply specified eddy diffusivity (Eq (3) but with z_{0m} replaced by z_{0c}) and neutral stratification we just need to consider vertical turbulent transfers and gravitational settling where V_g represents the gravitational settling velocity. One could then model the constant downward flux of aerosol, F_{Qc}, as

$$V_g Qc + K_{qc} \cancel{ku_*(z+z_{0c})} dQc/dz = F_{Qc} = u_* q_{c*} \quad (5\cancel{4})$$

~~where V_g represents the gravitational settling velocity,~~ Csanady (1973) proposed this approach and Venkatram and Pleim (1999) obtained essentially the same solution as we will find below. They commented, in 1999, "... why not use a formulation that is consistent with the mass conservation equation (Eq. 5)." More recently Giardina and Buffa (2018) raise the same issue. Note that V_g is generally proportional to d^2, where d is the diameter, via Stokes law for small (d <60 μm) spherical particles (Rogers and Yau,1976, p125), and u_* is the friction velocity. We introduce q_{c*} as a mixing ratio scale via this constant flux definition. The eddy diffusivity K_{qc} is assumed to be

$$K_{qc} = ku_*(z + z_{0c}),$$

(6~~5~~)

where z_{0c} is a roughness length for the aerosol with the assumption that $Qc = Qc_{surf}$ at $z = 0$.

The upward flux case with a surface source of aerosol is interesting in the sense that there will only be a steady, horizontally homogeneous, state when the net flux is zero, i.e, upward turbulent transfer is balanced by gravitational settling. Xiao and Taylor (2002), in ~~an aside from~~ relation to a blowing snow study, show, by solving Eq.(5) with $F_{Qc} = 0$, ~~show~~ that this leads to the classic power law solution (e.g, Prandtl, 1952), which in the current context is

$$\ln(Qc(z)/Qc_{surf}) = -S\zeta, \text{ where } \zeta = \ln((z+z_{0c})/z_{0c}) \text{ and } S = V_g/(ku_*)$$

or

$$Qc(z) = Qc_{surf}\ ((z+z_{0c})/z_{0c})^{-S}$$

(7~~6~~)

Profiles of suspended sediment, and velocity, in water currents can be treated in a similar way but there is an interesting twist if the density of the sediment and water mix is sufficient to modify the turbulent mixing through stable stratification. Taylor and Dyer (1977) rediscovered an interesting result due to Barenblatt (1953) showing that a modified solution allowing for stratification effects on the eddy diffusivity could be obtained. Observations were sometimes misinterpreted as power laws with a modified value of k (Graf, 1971, p180).

For the case of downward flux ~~case~~ to the lower boundary in the atmospheric surface layer it is easiest if we assume $Qc_{surf} = 0$, which may be most relevant over water but is also often assumed for dry deposition of particles (Seinfeld and Pandis, 1998, p960). Material starts from a source above the constant flux layer and travels downwards due to both turbulent mixing and gravitational settling. Assuming constant values for z_{0c}, u_* and V_g one can then solve the first order differential equation~~ODE~~, Eq (5~~4~~), by integrating factor techniques. Multiplying Eq. (5~~4~~) by $(z+z_{0c})^{S-1}/(ku_*)$ where $S = V_g/(ku_*)$, gives,

$$(d/dz)[(z+z_{0s})^S Qc] = (q_{c*}/k)(z+z_{0c})^{S-1}$$

(8~~7~~)

and, with $Qc(0) = 0$, the solution is,

$$Qc(z) = (q_{c*}/(kS))\ [1- ((z+z_{0c})/z_{0c})^{-S}].$$

(9~~8~~)

In terms of $\zeta = \ln((z+z_{0c})/z_{0c})$, we can write,

$$Qc(\zeta) = (q_{c*}/(kS))\ [1-e^{-S\zeta}].$$

(10~~9~~)

These can be referred to as Constant Flux Layer with Gravitational Settling, CFLGS, profiles. In the limits as V_g and~~and~~ $S \to 0$, ~~and as $\zeta \to 0$,~~ Eq (10~~9~~) gives $Qc(\zeta) = (q_{c*}/k)\,\zeta$, a standard log profile.

~~3. Dry deposition velocities~~

~~For aerosol dry deposition (i.e. not involving rain or snow – wet deposition) to any surfaces the traditional way to parametrize the process is with a deposition velocity, V_{dep}. Then the flux to the surface is represented as,~~

~~$F_{Qc} = V_{dep}\,Qc\,(z_{ref})$. (10)~~

~~In a numerical model the reference height z_{ref} is often the lowest grid level. If gravitational settling is the main cause of F_{Qc}, we would expect little change in Qc with height, but if turbulent transfer is dominant then the choice of z_{ref} could be important.~~

~~Dry deposition can involve many aspects and is often modelled in terms of a series of resistances. The deposition velocity used generally includes the effects of both gravitational settling and turbulent collisions of particles with vegetation or the ground, or water surface. The expression used for deposition velocity by Zhang et al (2001), and others, is~~

~~$V_{dep} = V_g + 1/(R_a + R_s)$ (11)~~

~~where V_g is the gravitational settling velocity and the resistances to deposition are aerodynamic (R_a) and surface (R_s). The aerodynamic resistance is given as~~

~~$R_a = (\ln(z_{ref}/z_0) - \psi_H)/(ku_*) \approx (\zeta_{ref} - \psi_H)/(ku_*)$~~

~~where z_0 is a roughness length, presumed to be z_{0m} and ψ_H is a stability function from MOST. It is applied with $z_{ref} >> z_0$ and so one can use $\zeta_{ref} = \ln((z_{ref} + z_0)/z_0)$. In neutral stratification $\psi_H = 0$ and for deposition to a water surface it is reasonable to set $R_s = 0$, unless it could be used to differentiate between z_{0m} and z_{0c}. We can then write the relationship as~~

~~$V_{dep} = V_g\,(1 + 1/(S\zeta_{ref}))$ (12)~~

~~From our CFLGS profile (Eq 8) we can derive an alternative expression for deposition velocity,~~

~~$V_{dep} = F_{Qc}/Qc(z_{ref}) = V_g/(1 - \exp(-S\zeta_{ref}))$. (13)~~

~~This has similarities with the Zhang et al (2001) form. First we note that $V_{dep} \geq V_g$. For our over water situation with $R_s = 0$, for large ζ, $V_{dep} \to V_g$ and also in the limit as $V_g \to 0$ both will have $V_{dep} \to ku_*/\zeta_{ref}$ when $R_s = 0$ and $z_{0m} = z_{0c}$, or if we set $R_s = ln(z_{0m}/z_{0c})/(ku_*)$. The Zhang et al (2001) and z_{0c} approaches differ in detail between those limits and an illustration is given in Section 4, Fig 3. The $\zeta_{ref} \to 0$ limit is similar in both approaches with $R_s = 0$ since then $V_{dep} \to \infty$ as $z \to 0$ and the aerodynamic resistance goes to 0.~~

~~There is little discussion of the variation of V_{dep} with z_{ref} in the literature, most of the focus being on variation with particle diameter (d). Farmer et al (2021) comment that "There are serious problems with our current understanding of deposition rates", but provide (Fig 3 in the paper) a summary of observed, and some modelled, values of deposition rate over different types of surface (grassland, forest, water and cryosphere) for a range of particle diameters from 0.01 to 100 µm. Our main concerns are with fog and other aerosol with diameters in the 0.5 to 50 µm range and their deposition to water surfaces. Farmer et al's plot (Fig 3c) shows an approximate $V_{dep} \sim d^2$ relationship, but with $V_{dep} > V_g$. For more general aerosol the particle density and shape will modify V_g and V_{dep} and cause some of the scatter, along with variations in u_* and z_{ref}. Sehmel and Sutter (1974) report on wind tunnel determinations of deposition velocity over water. Their Figure 3 results for uranine particles (density 1500 kg m^{-3}) shows results at low wind speeds with $V_{dep}/V_g \sim 1$, while at higher wind speeds and for diameters in the range 1-30 µm have V_{dep}/V_g increasing from about 3 to about 10.~~

3~~4~~. Some profiles

The expected values of V_g and u_* should be considered. Aerosols come in all shapes and sizes, see for example Farmer et al (2021) who consider diameters from 1nm to 100µm and deposition velocities, resulting from a combination of turbulent mixing and gravitational settling, mostly in the range 0.01 to 100 cm s^{-1}. Farmer et al (2021) also highlight the role of aerosols in climate issues. Fog droplets have a range of sizes but most fall in the diameter range 0-50 µm, often with bimodal distributions and peaks around 6 and 25 µm (see for example Isaac et al, 2020). Applying Stokes law with appropriate values for water droplets (see Rogers and Yau, 1976) for these peak sizes we get V_g values of 0.0011 and 0.0192 m s^{-1}. Aerosol particles of different density and shape may have different V_g values but the focus here will be for situations with $V_g < 2$ cm s^{-1} and diameters in the 1 - 20 µm range. These terminal velocities are clearly small compared to wind speed but for the larger diameter ~~droplets~~fog droplets, ~~where the bulk of the liquid water content, LWC $(=\rho_a Qc)$, is often measured,~~ the terminal velocity can easily reach~~corresponds to~~ 72~~69~~ m per hour and would~~ill~~ represent a considerable removal rate in fog which may last several hours or days. The key parameter in our constant flux with gravitational settling model is

$$S = V_g/ku_* . \tag{11}$$

In moderate winds over the ocean one might expect u_* values in the 0.15-0.6 m s^{-1} range, while in ~~radiation fog in~~ light winds over land it could be lower. The parameter, S will thus generally be in the range 0.0 to 0.3 in marine situations~~over water~~ but could be unlimited in light winds with low u_* over land. With high values of S gravitational settling will be the dominant process except very close to the surface.

At low values of S gravitational settling will have little~~ow~~ impact and the Qc profiles ~~will be~~ are approximately logarithmic.

To illustrate this Fig. 1 shows Qc constant flux profiles with linear and log vertical axes and a range of S values. We have scaled Qc with a value at 50m. The main unknown is the value of z_{0c}. Here we use our first guess value (z_{0c} = 0.01m) indicating relatively efficient capture of water droplets, or other aerosol, by the ~~water~~ surface. These calculations are for uniform sized aerosol particles or droplets. Note that with high S $(=V_g/ku_*)$ values, maybe occurring with low u_* and minimal turbulence, the limiting case would be constant Qc down to z = 0 and a discontinuity to Qc = 0 at the surface. Calculations with S = 1 and 5 (not shown) confirm this. The essential point from Fig. 1 is that, if there is gravitational settling involved then the profiles will depart from the simple logarithmic profiles that one might expect in a neutrally stratified near-surface atmospheric boundary layer. Note that these profiles depend on z_{0c} but not directly on z_{0m}, except via u_*.

For aerosol dry deposition to any surface a traditional way to parametrize the process is with a deposition velocity, V_{dep}, based on a Qc measurement at z_{ref}, and simply defined via,

$$F_{Qc} = V_{dep}(z_{ref})\, Qc(z_{ref}). \qquad (12)$$

In a constant flux layer, $V_{dep}(z_{ref})$, shown in Fig. 2, is simply proportional to the inverse of $Qc(z_{ref})$ provided that F_{Qc} is constant between the surface and z_{ref}. The dependence of V_{dep} on the reference height, z_{ref}, for Qc is seldom acknowledged in papers reporting measured V_{dep} values, or in the review by Farmer et al (2021). The height, z_{ref}, is often not discussed and hard to find, e.g. in Sehmel and Sutter (1974). In addition, there is a strong dependence on u_* and any value of V_{dep} will depend on z_{ref}, u_* and V_g as well as the nature of the underlying surface, which we have characterized through z_{0c}. In a numerical model the reference height z_{ref} is often the lowest grid level. If gravitational settling is the main cause of F_{Qc}, we would expect little change in Qc with height, but if turbulent transfer is dominant then the choice of z_{ref} could be important. Zhang et al (2001) recognize this in their widely used dry deposition scheme, based on Slinn (1982), and z_{ref} (= z_R in their notation) is clearly a factor in their aerodynamic resistance ($R_a = (ku_*)^{-1}\ln(z_{ref}/z_{0m})$, in neutral stratification). Their surface resistance (R_s) could then be interpreted in roughness length terms (as in Garratt, 1992, Section 3.3.3), as $R_s = (ku_*)^{-1}\ln(z_{0m}/z_{0c})$. Note that if $z_{0m} = z_{0c}$ then R_s = 0, and this may be controversial.

Zhang et al (2001), Slinn(1982) and many others (see Saylor et al, 2019, Farmer et al, 2021) combine these resistances with a gravitational settling velocity, through the relationship.

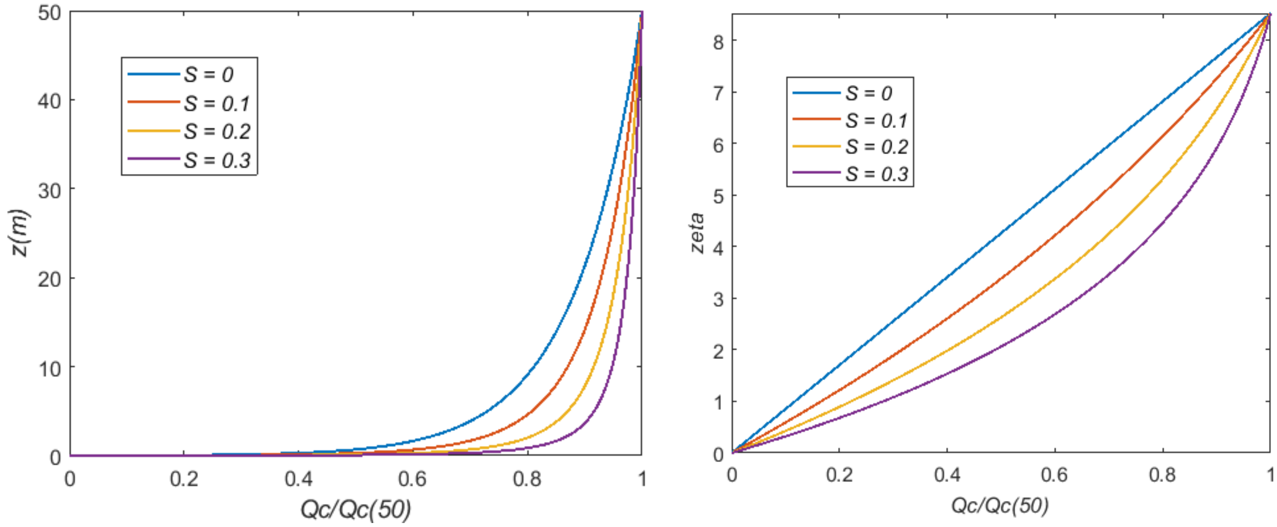
$$V_{dep} = V_g + 1/(R_a + R_s) \quad \text{or} \quad V_{dep}/ku_* = S + 1/[ku_*(R_a + R_s)] \qquad (13)$$

A possible alternative, which takes account of a modified Qc at z_{0m}, is derived by Seinfeld and Pandis (1998, Eq. 19.7), but this is "not consistent with mass conservation" as noted by Venkatram and Pleim (1999).

$$V_{dep} = V_g + 1/(R_a + R_s + R_a R_s V_g). \tag{14}$$

Eq. 14 will give lower V_{dep} values when $R_s > 0$. Neither expression, using the R_a, R_b definitions above, matches our CFLGS model for which, provided $z_{ref} >> z_{0m}, z_{0c}$ we can write, assuming the R_a and R_s relations given above,

$$V_{dep}/ku_* = S/(1 - e^{-S\zeta}) \approx S/(1 - exp(-Sku_*(R_a + R_s))) \tag{15}$$



~~One way to look at the relative importance of gravitational settling for these uniform size droplets is to consider the relative contributions to the total downward flux of water droplets~~ $\cancel{(u_* q_{c*})}$~~. The gravitational contribution is simply~~ $\cancel{V_g Qc}$ ~~while the turbulent diffusion contribution is,~~

$$\cancel{ku_* dQc/d\zeta = u_* q_{c*} e^{-S\zeta}}, \text{ where } \cancel{\zeta = \ln((z+z_{0c})/z_{0c})} \tag{14}$$

~~The ratios of turbulent transfer (TT)/total flux and gravitational settling (GS)/total flux then become~~

$$\cancel{TT = e^{-S\zeta}} \text{ and } \cancel{GS = 1 - e^{-S\zeta}} \tag{15}$$

~~Noting that~~ $\cancel{\zeta = \ln((z+z_{0c})/z_{0c})}$ ~~we can see that these ratios depend on both~~ $\cancel{z_{0c}}$~~, through the~~ $\cancel{z(\zeta)}$ ~~relationship, and~~ $\cancel{S}$ ~~and will vary with~~ $\cancel{z}$~~. Fig. 2 illustrates this. It is important to note that Fig. 2 is based on our relatively low estimate~~

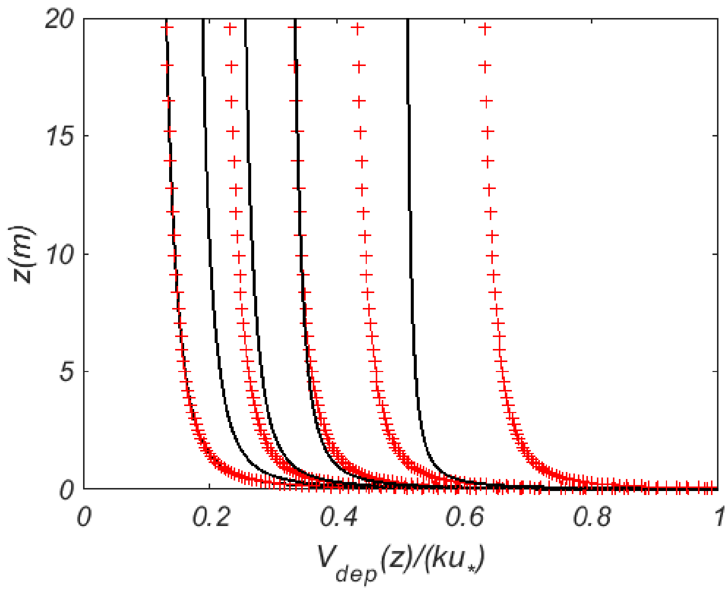
~~for z_{0c}, (0.01 m). If we increase it to z_{0c} = 0.1 m then turbulent fluxes become more important. We can see that the *TT* ratio is formally 1 at the surface, where $Qc = 0$ so there is no gravitational component. For very large ζ the *TT* term would decay to 0 but this would be well above the constant flux layer approximation. At 50 m the value will depend on *S* and z_{0c}.~~

~~a)~~

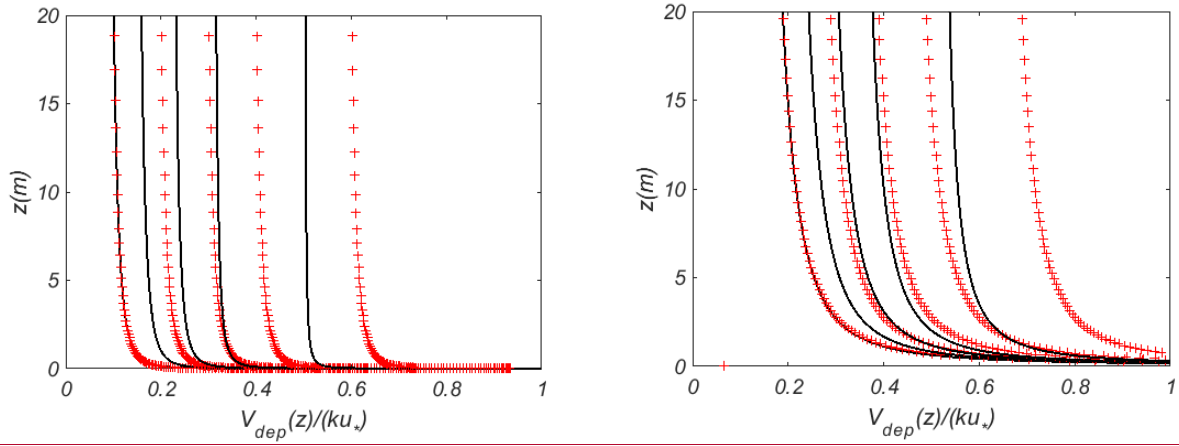
~~b)~~

Fig. 1 *Qc* profiles, scaled by the 50 m value, from the surface to $z = 50$ m in constant flux layers with gravitational settling. The surface roughness length for ~~water droplet~~ aerosol removal, z_{0c} = 0.01 m. Plotted with l~~L~~inear (a) and logarithmic (b) height scales and four S values.

Sample V_{dep} results are shown in Fig 2 when $V_g \geq 0$. In the first case (a) we took $z_{0m} = z_{0c}$ = 0.01 m so that $R_s = 0$. With no gravitational settling both models agree. For $S > 0$, the CFLGS deposition velocities, Eq(15), are lower than those computed from the Zhang/Slinn formulation. Cases b and c keep z_{0m} = 0.01 m but allow z_{0c} to be smaller, $R_s > 0$ in (b) or larger, $R_s < 0$ in (c). The CFLGS relationship, Eq (12c) always shows a modest V_{dep} reduction, relative to the Zhang/Slinn equation, which is typically of order 20%.



a) $z_{0c} = z_{0m}$ = 0.01 m ($R_s = 0$)



b) z_{0c} = 0.001 m; z_{0m} = 0.01 m c) z_{0c} = 0.1 m; z_{0m} = 0.01 m

Fig. 2 V_{dep} profiles, from surface to z = 20 m in constant flux layers with gravitational settling. Solid lines are with the CFLGS model, the + points are from the Zhang/Slinn formulation (ZS). Five cases, left to right are S = 0, 0.1, 0.2, 0.3, 0.5. a) $z_{0m} = z_{0c}$ = 0.01 m, R_s = 0; b) z_{0c} = 0.001 m; z_{0m} = 0.01 m, ku_*R_s = 2.3; c) z_{0c} = 0.1 m; z_{0m} = 0.01 m, ku_*R_s = -2.3.

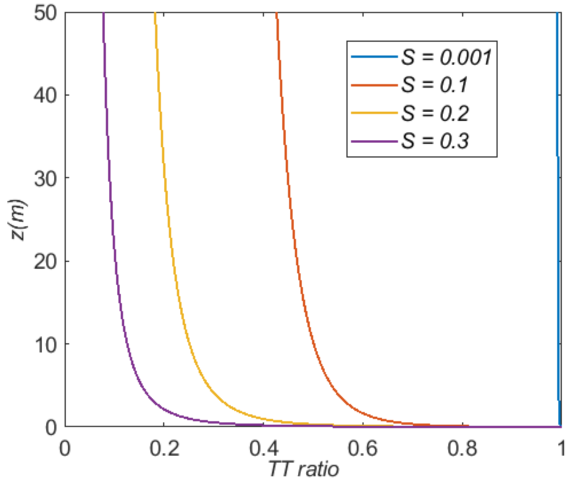
~~On~~Another~~e~~ way to look at the relative importance of gravitational settling for these uniform size droplets is to consider the relative contributions to the total downward flux of ~~water droplets~~ aerosol (u_*q_{c*}). The gravitational contribution is simply V_gQc while the turbulent diffusion contribution is,

$$ku_*dQc/d\zeta = u_*q_{c*}e^{-S\zeta}, \text{ where } \zeta = \ln((z+z_{0c})/z_{0c}) \qquad (16\cancel{14})$$

The ratios of turbulent transfer (TT)/total flux and gravitational settling (GS)/total flux then become

$$TT = e^{-S\zeta} \text{ and } GS = 1 - e^{-S\zeta} \qquad (17\cancel{15})$$

Noting that $\zeta = \ln((z+z_{0c})/z_{0c})$ we can see that these ratios depend on both z_{0c}, through the $z(\zeta)$ relationship, and S and will vary with z. Fig. 3~~2~~ illustrates this. It is important to note that Fig. 3~~2~~ is based on ~~our relatively low estimate for~~ z_{0c}~~, (~~ = 0.01 m~~)~~. If we increase it to z_{0c} = 0.1 m then turbulent fluxes become more important (Fig 2c). We can see that the TT ratio is formally 1 at the surface, where Qc = 0 so there is no gravitational component. For very large ζ the TT term would decay to 0 but this would be well above the constant flux layer approximation. At 50 m the value will depend on S and z_{0c}.


S = 0.001
S = 0.1
S = 0.2
S = 0.3
z(m)
TT ratio
0
10
20
30
40
50
0
0.2
0.4
0.6
0.8
1

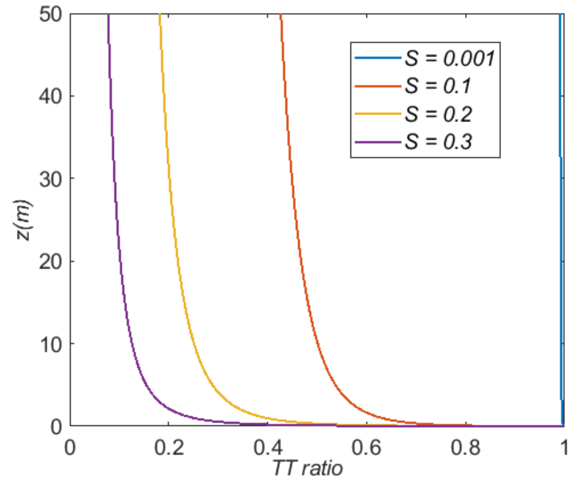
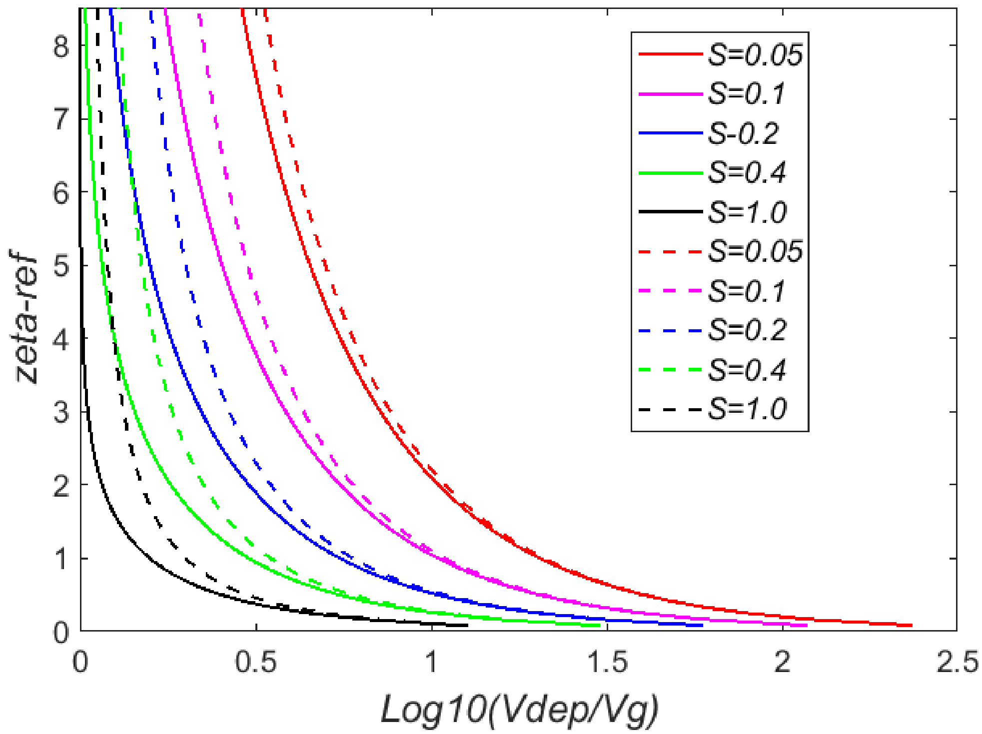


Fig. 3~~2~~ Variation of the Turbulent Transfer fraction of the total *Qc* flux and its variation with *z* and *S*. Note that these *z* values are based on~~ a)~~ z_{0c} = 0.01 m.

~~We can also use Equations (12) and (13) to compute deposition velocities arising from the combination of gravitational settling and, in Zhang et al's (2001) dry deposition terminology, aerodynamic resistance, although we use z_{0c} rather than z_{0m} in the expression for R_a. Results in Fig 3 show similar variations with S, but note we are using log scales for V_{dep}/V_g and for z_{ref}.~~

~~With z_{0c} = 0.01 m and $\zeta = \ln((z+z_{0c})/z_{0c})$ note that z = 50 m corresponds to ζ = 8.517 while ζ = 4 is only z = 0.546 m and ζ = 6 is z = 4.03m. There are differences with the Zhang et al (2001) formulation giving higher V_{dep}/V_g estimates than CFLGS, especially for the higher values of S in the ζ_{ref} > 6 , z_{ref} > 4 m range. Both show dependence on ζ_{ref}, which is rarely commented on when deposition velocity values are reported, the emphasis being placed on aerosol diameter as in Farmer et al's (2021) figures and tables. For aerosols in general we need better determination of deposition velocity, V_{dep}, over all surfaces. Based on the analysis presented here it could be argued that more attention should be paid to the parameter $S = V_g/(ku_*)$ and to the height z_{ref} at which V_{dep} can be applied.~~



~~**Fig3**. Variations of deposition velocity V_{dep}/V_g with ζ_{ref} and S. z_{0c} = 0.01 m. Solid lines are based on CFLGS (Eq 13) and dashed lines are Zhang et al's (2001) model with $R_s = 0$ and R_a (Eq 12) as discussed in the text.~~

4. Stable Stratification Case

For fog applications, ~~O~~over land, radiation fog often occurs at low wind speeds with stable stratification. Advection fog when warm, moist air is advected over a colder surface is another case with stable stratification. For constant flux boundary layers in these circumstances MOST has, for velocity, $K_m = k(z+z_{0m})/\ \Phi_M(z/L)$ and

$$\Phi_M(z/L) = 1 + \beta\,(z+z_{0m})/L : U = (u_*/k)\,(\ln((z + z_{0m})/z_{0m}) + \beta\, z/L). \qquad (18\cancel{6})$$

Observed profiles give $\beta = 5$ (Garratt 1992, p52). In addition $\Phi_H = \Phi_M$ and if we extend this idea to $\Phi_{Qc}(z/L)$ and set $K_{Qc} = k(z+z_{0c})/\ \Phi_{Qc}(z/L)$ ~~with a similar form for Φ_{Qc}~~ we need to solve,

$$V_gQc + [ku_*\,(z + z_{0c})/\ \Phi_{Qc}(z/L)]\ dQc/dz = F_{Qc} = u_*q_{c*}, \qquad (19)$$

or, with $\Phi_{Qc}(z/L) = 1 + \beta\,(z+z_{0c})/L$,

$$dQc/dz + S\{(1+\beta\,(z+z_{0c})/L)/(z+z_{0c})\}Qc=(q_{c*}/k)(1+\beta(z+z_{0c})/L)/(z+z_{0c});\ \text{with}\ \ S=V_g/(ku_*)$$

The Integrating Factor is exp($\int S(1/(z+z_{0c})+\beta/L)dz = (z+z_{0c})^S \exp(S\beta z/L)$ *so that*

$$d\,[(z+z_{0c})^S \exp(S\beta z/L)Qc]\,/dz = (q_{c*}/k)(1+\beta(z+z_{0c})/L)\,(z+z_{0c})^{S-1} \exp(S\beta z/L) \qquad \cancel{(17)}$$

and we need to integrate the RHS. To do this it is convenient to let $\beta(z+z_{0c})/L = x$ and the integral that we need is of

$$(q_{c*}/k)(L/\beta)^{S-1}\exp(-Sx_0)\,\{(1+x)x^{S-1}\exp(Sx)\}, \quad \text{where } x_0 = \beta z_{0c}/L \qquad (20\cancel{18})$$

After some guidance and a few trials one can see that $d/dx\{x^S\exp(Sx)\} = (Sx^{S-1} + Sx^S)\exp(Sx)$ and the integral required is simply $F(x,S) = x^S\exp(Sx)/S$. We then evaluate $F(x,S)$ at $z = 0$, $x = \beta z_{0c}/L$ and any other z to allow us to plot Qc profiles. With stable stratification and light winds the constant flux approximation would only apply to a relatively shallow layer so we normalize with $Qc(ztop)$ and set $ztop$ = 20 m in these cases. If $Qc = 0$ at $z = 0$ we then have,

$$Qc(z) = (q_{c*}/k)(L/\beta)^{-1}\exp(Sx_0)\,[\exp(-Sx)\,x^{-S})]\,[F(x,S) - F(x_0,S)], \qquad (21\cancel{19})$$

and we can then plot the ratio $Qc(z)/Qc(ztop)$ as in Fig. 4. For $S = 0$, with no gravitational settling, the profile will be essentially the same as the velocity profile in Eq. (18)~~(A1)~~ above, i.e.

$$Qc(z) = (q_{c*}/k)\,(\ln((z + z_{0c})/z_{0c}) + \beta z/L). \qquad (22\cancel{20})$$

In addition to z_{0c} and S the key parameter is the Obukhov length, $L = -\rho c_p u_*^3\theta/(kgH)$, (>0). Neutral stratification corresponds to $L \rightarrow \infty$ while stable stratification relationships (H < 0, L > 0) are generally limited to $0 < z/L < 1$. If we are concerned with height ranges up to 10 or 20m then L = 10m would be considered as a very low value maybe with $u_* \approx 0.13\ \mathrm{ms^{-1}}$ and $H \approx -20\ \mathrm{Wm^{-2}}$ as possible values. Figure 4 shows Qc(z)/Qc(20m) profiles in a typical case with our standard value, z_{0c} = 0.01m. We set L = 20m and use a range of S values. For large droplets, S = 0.4, Qc flux is dominated by gravitational settling and reductions in Qc towards 0 only occur in the lowest few m. For smaller particles, S = 0, 0.01, 0.1 turbulent mixing dominates the deposition process. Note that the S = 0 points (log + linear profiles) and the S = 0.01 line, almost overlap as one confirmation of solution form.

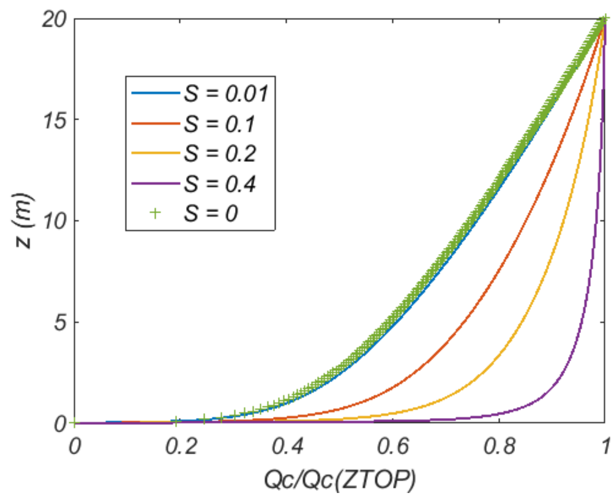


Fig 4. Qc/Qc(ztop) profiles with stable stratification, assuming $\Phi_{Qc}(z/L) = 1 + \beta\,(z+z_{0c})/L$. We set $\beta = 5$, $L =$ 20m and z_{0c} = 0.01m.

~~In addition to z_{0c} and S the key parameter is the Obukhov length, $L = -\rho c_p u_*^3\theta/(kgH)$, (>0). Neutral stratification corresponds to $L \to \infty$ while stable stratification relationships (H < 0, L > 0) are generally limited to $0 < z/L < 1$. If we are concerned with height ranges up to 10 or 20m then L = 10m would be considered as a very low value maybe with $u_* \approx 0.13$ ms^{-1} and H ≈ -20 Wm^{-2} as possible values. Figure 4 shows Qc(z)/Qc(20m) profiles in a typical case with our standard value, z_{0c} = 0.01m. We set L = 20m and use a range of S values. For large droplets, S = 0.4, Qc flux is dominated by gravitational settling and reductions in Qc towards 0 only occur in the lowest few m. For smaller particles, S = 0, 0.01, 0.1 turbulent mixing dominates the deposition process. Note that the S = 0 points (log + linear profiles) and the S = 0.01 line, almost overlap as one confirmation of solution form.~~ In unstable stratification it is generally accepted that $\Phi_H(z/L) \neq \Phi_M(z/L)$ and relatively little is known about stability effects on diffusion of other scalars. For aerosol Jia et al (2021) assume $\Phi_{Qc} = \Phi_H$ in unstable stratification but have proposed a new form, different from Φ_H, for Φ_{Qc} in stably stratified boundary layers. These are all based on Richardson number. In principle one could numerically solve Eq. (19) for any suitable $\Phi_{Qc}(z/L)$ form but our interest is primarily the stable case and it is comvenient that an analytic solution can be found for the generally accepted $\Phi(z/L)$ forms if we assume $\Phi_{Qc} = \Phi_H$. Strictly speaking our $\Phi(z/L)$ functions should be $\Phi((z+z_0)/L)$ functions but we are generally dealing with z >> z_0 and it is customary to ignore that difference.

5. Conclusions ~~and Suggestions~~

The initial~~basic~~ idea behind this analysis was that, in marine fog, cloud droplets can both fall toward the underlying surface through gravitational settling and be diffused towards the surface by turbulence and on contact they can coalesce with an underlying water surface. Taylor et al (2021) apply these ideas to fog modelling with the WRF model. During reviews of that work, and an earlier version of the current paper, it became clear that some reviewers were reluctant to accept that turbulence could cause fog droplets to collide and coalesce with an underlying ~~water~~ surface, and even more reluctant to see this as a constant flux layer situation. Fog droplets are perhaps a special case but the CFLGS~~in that there could be fluctuations in relative humidity allowing transfers between water droplets and water vapour, and variations of droplet size. It can still be argued that our conceptual model of fog droplets and cloud liquid water being generated near the top of a fog layer, perhaps as a result of radiative cooling is useful.~~ concept is equally applicable to aerosol particles or droplets in general, ~~Once created the droplets can travel downward via both gravitational settling and turbulent diffusion towards a sink at the water surface. If the relative humidity is at 100% throughout this descent it seems reasonable assume a constant flux layer.~~

~~The same constant flux layer concept can apply in the case of other aerosols,~~ provided that they are inert and without sources or sinks in the air. Desert dusts, various pollutants or micro-plastic fragments being blown out over lakes or the sea from sources on land may be~~are~~ examples.~~–~~ Here we could anticipate a situation with initial mixing through a relatively deep atmospheric layer over land ~~with minimal deposition~~ being advected over an aerosol capturing water surface so that one could envisage a situation over the water with a constant downward flux of aerosol due to gravitational settling plus turbulent diffusion in a low level constant flux layer.

One implication of the CFLGS model is that simply adding gravitational settling (V_g) to a deposition velocity (V_{dep}) based on aerodynamic and surface resistances may overestimate the combined effects. If we use the CFLGS model it can indicate reductions of order 20%. These are small compared to the uncertainties based on deposition velocity measurements but may well be worth considering.

~~In considering aerosol the recent review of dry deposition by Farmer et al (2021) and the widely used scheme of Zhang et al (2001) clearly show us that deposition velocity frequently exceeds gravitational settling velocity, especially over water. This seems to be readily accepted in the atmospheric chemistry community with models developed such as Eqs (10-12) above, and also for fog deposition to vegetation (Katata, 2014). One can use these ideas in modelling work, adapting the approach of Katata et al (2010, 2011) for radiation fog over forests. This is the approach adopted in Taylor et al (2021) to deal with marine advection fog over the ocean. A critical unknown parameter in this work is the deposition velocity relating~~ ~~Qc~~ ~~at the lowest model level to the downward flux to the surface due to turbulent transfer. As in the analysis above, one can use a roughness length for cloud droplets,~~ ~~z_{0c}~~~~, as a tuning parameter when suitable Qc profile measurements are available.~~

~~The bottom line is that this removal process needs to be taken account of in modelling and forecasting fog occurrence and development and we need to know more about it. Fog is an intermittent phenomenon so setting up 50-m or higher measurement masts in fog-prone locations will be good start. The PARISFOG study (Haeffelin et al,~~

~~2000) included 30-m masts and LANFEX (Price et al, 2018) used 50-m masts but the profile measurements did not include fog water, Qc, or visibility. In-situ vertical profiles of Qc were also missing in field programs like FRAM (Gultepe et al, 2009) and C-Fog (Fernando et al, 2021). C-Fog instrumentation at various sites included 10-m and 15-m masts and also a Radiometrics microwave radiometer for Qc profile measurements. These may well report interesting measurements but better vertical resolution is desirable. There were Qc measurements at two or more levels in earlier field measurements reported by Pinnick et al (1978) and Kunkel (1984) showing increases with height. More such measurements are needed with multiple measurement levels and measuring droplet size distributions, *Qc* or *LWC* values and ideally *Qc* fluxes, along with wind, turbulence, temperature and humidity profiles plus surface pressure and fluxes of momentum, heat and water vapour. Visibility measurements at multiple levels, 4 component radiation and air, aerosol and fog chemistry measurements could also play an important role in fog. From the modelling perspective we need values for~~ z_{0c}~~, which will depend on surface type and, on droplet diameter and on wind speed or friction velocity. Assuming that the lower layers, say 10-30 m of a deep fog layer, are in a relatively steady, constant flux layer situation then the CFLGS profiles developed above could provide a framework for analysis of fogs and the improvement of fog models.~~

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Code/Data Availability

Calculations were made with simple Matlab code, maybe 20 lines for each figure. They can be made available ~~i~~as supplementary material~~f needed.~~.

Author Contribution

This is independent work by the single author.

Competing Interests

None.