Response to reviewer #1

Overall, this is an excellent manuscript that is generally well-written and elegantly presents an important overlooked aspect of ice nucleation that is valuable to the modelling community. Some aspects of the manuscript could be elaborated and clarified as outlined below. I would recommend publication of the manuscript once these few minor points have been taken into consideration.

We thank the reviewer for their positive assessment of our study, pointing out its high relevance for many in the modeling community, as it addresses an important issue in simulating aerosol-cloud interactions which can easily be overlooked.

The review mentions two points to consider for revisions, which we address as follows.

A more specific discussion on how AFs are relevant to models on various spatial scales on lines 156-160 would be helpful for the reader to understand the feasibility of the approach.

We acknowledge the wish for a more detailed discussion of the relevance of AFs for cloud models. Yet, how AFs should be included in models needs to take details of numerical implementation into account, depending on cloud models/schemes. Especially in the case of global models with long time steps such a discussion can be tricky if details of implementation are not fully disclosed, and presumably differs from model to model. Conversely, the use of differential AFs in detailed cloud models is straightforward and should be clear from the discussion of particle budgets (Section 3.2).

Instead of expanding on the subtleties of how AFs should be used in models on various spatial scales, we mention in the revised manuscript the one situation where differential AFs are not needed. Namely, in studies of a single ice formation event which do not require removing INP after nucleation, e.g., in parameterizations and underlying parcel simulations. We now also mention that the effect of wrongly using cumulative AFs as illustrated in section 3.2 depends on the rate of cooling (as the change in predicted ice crystal numbers is likely small in situations with efficient INPs and high cooling rates) and will depend on whether the INP budgets will also be affected by spatial transport.

We add two paragraphs to accommodate these revisions:

After L154 in section 3.2: "The impact on cloud properties of wrongly using cumulative AFs in specific simulations cannot be judged based on the results shown in Fig. 3 alone. For example, in cirrus simulations, the change in total nucleated ice crystal numbers is likely small in situations with efficient INPs (with large $d\varphi/ds$ *near* s_**) and high cooling rates, as most INPs will activate straightaway and the time needed for s to increase above the 50% activation level is short."*

After L174 in section 4: "A general recommendation on how to include differential AFs in models cannot be given, as this depends on by details of the numerical implementation of aerosol-cloud interactions, especially in global models where INP budgets are affected by both microphysics and transport. However, differential AFs are straightforward to implement in cloud models when making use of the budget equations (12) and (13) in combination with equation (6). Cumulative AFs may be used only in studies of single ice formation events, which do not require removing INP after nucleation, e.g., in parameterizations and underlying parcel simulations."

Why was soot used as the example in Figure 4? Given its relative low ice-nucleating ability, perhaps dust aerosol particles could be used to better illustrate the effectiveness of differential AFs? I would also suggest including other examples of INPs and particle size distributions to determine the relative impact of differential AFs compared to cumulative AFs.

We agree that many dusts are better INPs than aviation soot. We have chosen aviation soot because firstly, a physical model providing AFs as a function of both ice supersaturation and particle mobility diameter is available in that case. Secondly, aviation-soot particle size distributions are well constrained allowing us to predict size-integrated AFs with confidence.

That said, using the same dust particles would not allow us to compute size integrated AFs (Fig.4) with the same level of confidence as in the case of contrail-processed aviation soot. In the latter case, size distributions as a function of the particle mobility diameter are available that can directly be integrated over the mobility diameter to obtain ice supersaturation-dependent AFs applying a physically-based parameterization. In the case of dust, the empirical parameterization provided by Ullrich et al. (2017) requires knowledge of representative particle surface size distributions, which are not readily available and can be inferred accurately only using assumptions (e.g. regarding particle shape determining number-surface relationships).

The bias introduced by using cumulative instead of differential AF becomes relevant when INPs are activated stepwise as supersaturation increases: the smaller the step-size the larger the bias. As such, it does not depend on the choice of a specific type of INP. Moreover, a correct implementation of AF in models should be the aim, independent of the effect a faulty one has. Therefore, we opt to keep the analysis presented in the manuscript as concise and generic as possible.