Supplement

(a) Exponential

Aerosol profile

\[ h_E = 0.2\text{km} \]

\[ h_E = 0.5\text{km} \]

\[ h_E = 1.0\text{km} \]

NO\textsubscript{2} profile

\[ h_E = 0.2\text{km} \]

\[ h_E = 0.5\text{km} \]

\[ h_E = 1.0\text{km} \]

(b) Boltzmann

Aerosol profile

\[ h_B = 1.0\text{km} \]

\[ h_B = 1.5\text{km} \]

\[ h_B = 2.0\text{km} \]

NO\textsubscript{2} profile

\[ h_B = 1.0\text{km} \]

\[ h_B = 1.5\text{km} \]

\[ h_B = 2.0\text{km} \]
Figure S1. Overview on the tested profile shapes using the 3 different parameterisations with different height parameters: (a) exponential shape, (b) Boltzmann shape, and (c) Gaussian shape. The colors refer to the AOD/VCD values shown at the top right. The VCD in the figure legend is given in $10^{16}$ molec. /cm$^2$. 

(c) Gaussian

Aerosol profile

<table>
<thead>
<tr>
<th>Height, σ</th>
<th>Profile Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>h$_G$=0.5km, σ=0.2km</td>
<td><img src="image1" alt="Profile 1" /></td>
</tr>
<tr>
<td>h$_G$=0.5km, σ=0.5km</td>
<td><img src="image2" alt="Profile 2" /></td>
</tr>
<tr>
<td>h$_G$=0.5km, σ=1.0km</td>
<td><img src="image3" alt="Profile 3" /></td>
</tr>
<tr>
<td>h$_G$=0.5km, σ=1.5km</td>
<td><img src="image4" alt="Profile 4" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=0.2km</td>
<td><img src="image5" alt="Profile 5" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=0.5km</td>
<td><img src="image6" alt="Profile 6" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=1.0km</td>
<td><img src="image7" alt="Profile 7" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=1.5km</td>
<td><img src="image8" alt="Profile 8" /></td>
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</table>

NO$_2$ profile

<table>
<thead>
<tr>
<th>Height, σ</th>
<th>Profile Shape</th>
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</thead>
<tbody>
<tr>
<td>h$_G$=0.5km, σ=0.2km</td>
<td><img src="image1" alt="Profile 1" /></td>
</tr>
<tr>
<td>h$_G$=0.5km, σ=0.5km</td>
<td><img src="image2" alt="Profile 2" /></td>
</tr>
<tr>
<td>h$_G$=0.5km, σ=1.0km</td>
<td><img src="image3" alt="Profile 3" /></td>
</tr>
<tr>
<td>h$_G$=0.5km, σ=1.5km</td>
<td><img src="image4" alt="Profile 4" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=0.2km</td>
<td><img src="image5" alt="Profile 5" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=0.5km</td>
<td><img src="image6" alt="Profile 6" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=1.0km</td>
<td><img src="image7" alt="Profile 7" /></td>
</tr>
<tr>
<td>h$_G$=1.0km, σ=1.5km</td>
<td><img src="image8" alt="Profile 8" /></td>
</tr>
</tbody>
</table>
Figure S2. Comparison of aerosol profiles retrieved by PriAM for 360nm (first line) and 477nm (second line) with the corresponding input aerosol profiles for exponential shape with scale heights of (a) $h_E=0.2\text{km}$, (b) $h_E=0.5\text{km}$, and (c) $h_E=1.0\text{km}$.

The black and red curves indicate the input and retrieved aerosol profiles by PriAM, respectively.
Figure S3. Comparison of aerosol profiles retrieved by PriAM for 360nm (first line) and 477nm (Second line) with the corresponding input aerosol profiles for Boltzmann shape with height parameter (a) $h_B=1.0\text{km}$, (b) $h_B=1.5\text{km}$, and (c) $h_B=2.0\text{km}$

The black and red curves indicate the input and retrieved aerosol profiles by PriAM, respectively.
(I) Gaussian @ $h_0 = 0.5 \text{ km}$

<table>
<thead>
<tr>
<th>360mm</th>
<th>477nm</th>
<th>360mm</th>
<th>477nm</th>
<th>360mm</th>
<th>477nm</th>
<th>360mm</th>
<th>477nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.2 \text{ km}$</td>
<td></td>
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<tr>
<td>$\sigma = 0.5 \text{ km}$</td>
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<td></td>
</tr>
<tr>
<td>$\sigma = 1.0 \text{ km}$</td>
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<td></td>
<td></td>
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<tr>
<td>$\sigma = 1.5 \text{ km}$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

The aerosol extinction coefficient [km$^{-1}$]
Figure S4. Comparison of aerosol profiles retrieved by PriAM for 360nm (first line) and 477nm (Second line) with the corresponding input aerosol profiles for Gaussian shape with different peak heights \( z_{peak} \) (I) \( h_0=0.5\text{km} \) and (II) \( h_0=1.0\text{km} \) of full width at half maximum (FWHM) \( \sigma \) (a) \( \sigma=0.2\text{km} \), (b) \( \sigma=0.5\text{km} \), (c) \( \sigma=1.0\text{km} \), and (d) \( \sigma=1.5\text{km} \).

The black and red curves indicate the input and retrieved aerosol profiles by PriAM, respectively.
Figure S5. Comparison of NO$_2$ profiles retrieved by PriAM for 360nm (first line) and 477nm (Second line) with the corresponding input NO$_2$ profiles for exponential shapes with scale heights of (a) $h_r=0.2$km, (b) $h_r=0.5$km, and (c) $h_r=1.0$km. The AOD was set to 1.0. The black and red curves indicate the input and retrieved NO$_2$ profiles by PriAM, respectively.
Figure S6. Comparison of NO₂ profiles retrieved by PriAM for 360nm (first line) and 477nm (Second line) with the corresponding input NO₂ profiles for Boltzmann profile shapes with height parameter (a) $z=1.0\text{km}$, (b) $z=1.5\text{km}$, and (c) $z=2.0\text{km}$. The AOD was set to 1.0.

The black and red curves indicate the input and retrieved NO₂ profiles by PriAM, respectively.
(l) Gaussian \( h_0 = 0.5 \text{ km} \)

<table>
<thead>
<tr>
<th>( h_0 )</th>
<th>( \sigma )</th>
<th>( NO_2 ) number density ( \times 10^{12} \text{ molecules cm}^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 km</td>
<td>0.1 km</td>
<td>![Graphs for different ( \sigma ) values]</td>
</tr>
</tbody>
</table>

**Graphs:**
- **Graph 1:** Shows the input \( NO_2 \) profile and the \( NO_2 \) profile retrieved by PhAM for different \( \sigma \) values and heights.
- **Graph 2:** Illustrates the variation of \( NO_2 \) number density with height for different \( \sigma \) values.
- **Graph 3:** Displays the relationship between \( NO_2 \) number density and height for various \( h_0 \) and \( \sigma \) combinations.

- **Legend:**
  - **Input \( NO_2 \) profile**
  - **\( NO_2 \) profile retrieved by PhAM**
Figure S7. Comparison of NO2 profiles retrieved by PriAM for 360nm (first line) and 477nm (Second line) with the corresponding input NO2 profiles for Gaussian shape with different peak heights $z_{\text{peak}}$ (I) $h_{\text{G}}=0.5\text{km}$ and (II) $h_{\text{G}}=1.0\text{km}$ of full width at half maximum (FWHM) $\sigma$ (a) $\sigma=0.2\text{km}$, (b) $\sigma=0.5\text{km}$, (c) $\sigma=1.0\text{km}$, and (d) $\sigma=1.5\text{km}$. The AOD was set to $1.0$. The black and red curves indicate the input and retrieved NO2 profiles by PriAM, respectively.
Figure S8. Comparison of the aerosol profiles retrieved by PriAM and MAPA for 360 nm (first line) and 477 nm (second line) and the corresponding input aerosol profiles for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape.

The red and blue curves indicate the results from PriAM and MAPA, respectively.
Figure S9. Absolute deviations between the profiles retrieved by PriAM and MAPA for 360 nm (first line) and 477 nm (second line) and the corresponding input aerosol profiles for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape. The red and blue curves indicate the results from PriAM and MAPA, respectively.
Figure S10. Correlation plots between $O_3$ DSCDs simulated by MCARTIM for different SSA and AP and SCIATRAN for SSA of 0.90 and AP of 0.72.

The red and black circles indicate the MCARTIM results for SSA=0.95, AP=0.68 and SSA=0.9, AP=0.72, respectively.
Figure S11. Comparison between the AODs retrieved from MAPA and the input AODs for the 4 aerosol profile shapes listed in table 1. The MAPA AODs were retrieved from the O4 DSCD simulated by MCARTIM for different SSA and AP and SCIATRAN for SSA of 0.90 and AP of 0.72.

The set of SSA and AP is shown in brackets on top.
Figure S12. Comparison of the PriAM inversion results with different alternative a priori profiles and the results for the universal a priori (exponential shape with AOD 0.2). The first line in every panel denotes the results for 360 nm, and the second line denotes the results for 477 nm. Colors indicate the shapes and AODs shown at the top. ‘Corresponding a priori profile’ means that the same profile type as the simulated profiles is also used as a priori profile.
Figure S13. Absolute deviations of the PriAM aerosol inversion results with different alternative a priori profiles and the results for the universal a priori (exponential shape with AOD 0.2). The first line in every panel denotes the results for 360 nm, and the second line denotes the results for 477 nm. Colors indicate the shapes and AODs shown at the top. ‘Corresponding a priori profile’ means that the same profile type as the simulated profiles is also used as a priori profile.
Figure S14. Retrieved aerosol profiles by PriAM using different SSA and AP (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape. For these inversions, the same SSA and AP were used for the simulations of the $O_3$ DSCDs and for the PriAM inversions. The first line in every panel denotes the results for 360 nm, and the second line denotes the results for 477 nm.

The colors refer to the corresponding SSA and AP values shown at the top.
Figure S15. Absolute deviations between the PriAM results using different SSA and AP and the input aerosol profile results of (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape. For these inversions, the same SSA and AP values were used for the simulations of the $O_3$ DSCDs and for the PriAM inversions. The first line in every panel denotes the results for 360 nm, and the second line denotes the results for 477 nm.

The colors refer to the corresponding SSA and AP shown at the top.
Figure S16. The retrieved profiles using incorrect SSA and AP values from the retrieved profiles with the correct SSA and AP values for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape.

The colors refer to the SSA and AP values shown at the top.
Figure S17. Absolute deviations of the retrieved aerosol profiles using incorrect SSA and AP values from the profiles retrieved with the correct SSA and AP values for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape.

The colors refer to the SSA and AP values shown at the top.
Figure S18. Retrieved NO$_2$ profiles by PriAM and MAPA for scenario S1 (see text) for aerosol profiles with 3 selected AODs (0.3, 1.0, and 3.0) and 5 NO$_2$ VCDs for of (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape.

The solid and dotted colored lines refer to the AODs and algorithms shown at the top.
Figure S19. Absolute deviation of the retrieved NO₂ profiles by PriAM and MAPA from the input NO₂ profiles for scenario S1 (see text) for aerosol profiles with 3 selected AODs (0.3, 1.0, and 3.0) and 5 NO₂ VCDs for of (a) exponential shape with h = 0.5 km, (b) exponential shape with h = 1.0 km, (c) Boltzmann shape, and (d) Gaussian shape. The solid and dotted colored lines refer to the AODs and algorithms shown at the top.
Figure S20. Retrieved NO$_2$ profiles by PriAM and MAPA for Boltzman NO$_2$ input profiles for scenario S1 (see text) and for 3 aerosol profile shapes ((a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape) with 3 selected AODs (0.3, 1.0, and 3.0) and 5 NO$_2$ VCDs. The solid and dotted colored lines refer to the AODs and algorithms shown at the top.
Figure S21. Absolute deviations of the retrieved NO$_2$ profiles for Boltzmann NO$_2$ input profiles by PriAM and MAPA from the input NO$_2$ profiles for scenario S1 (see text) and for 3 aerosol profile shapes (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape) with 3 selected AODs (0.3, 1.0, and 3.0) and 5 NO$_2$ VCDs. The solid and dotted colored lines refer to the AODs and algorithms shown at the top.
Figure S22. Retrieved NO$_2$ profiles retrieved by PriAM for scenarios S1 and S2 and input NO$_2$ profiles for 4 AODs (0.3, 1.0, 3.0, and 5.0) and 5 VCDs for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape.

The first line in each panel denotes the results for 360 nm, and the second line denotes the results for 477 nm. The solid and dotted colored lines refer to the AODs and strategies shown at the top.
Figure S2.3. Absolute deviations between the NO$_2$ profiles retrieved by PriAM for scenarios S1 and S2 and the input NO$_2$ profiles for 4 AODs (0.3, 1.0, 3.0, and 5.0) and 5 VCDs and for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape. The first line in each panel denotes the results for 360 nm, and the second line denotes the results for 477 nm. The solid and dotted colored lines refer to the AODs and strategies shown at the bottom right.
Figure S24. Retrieved NO$_2$ profiles by PriAM for Sa of 0.1 and 0.5 and AOD of 0.3 and 5.0, along with the input NO$_2$ profiles for the (a) exponential shape with h = 0.5 km, (b) exponential shape with h = 1.0 km, (c) Boltzmann shape, and (d) Gaussian shape.

The first line in each panel denotes the results for 360 nm, and the second line denotes the results for 477 nm. The solid and dotted colored lines refer to the AODs and Sa shown at the top.
Figure S25. Absolute deviations between NO$_2$ profiles retrieved by PriAM and the input NO$_2$ profiles for Sa of 0.1 and 0.5 and AOD of 0.3 and 5.0 for (a) exponential shape with $h = 0.5$ km, (b) exponential shape with $h = 1.0$ km, (c) Boltzmann shape, and (d) Gaussian shape. The first line in each panel denotes the results for 360 nm, and the second line denotes the results for 477 nm. The solid and dotted colored lines refer to the AODs and Sa shown at the top.