Author's response to Referee 1, David Mitchell, 2nd review from 10 Sep 2021
ACPD article title: Mass of different snow crystal shapes derived from fall speed measurements
Author(s): Sandra Vázquez-Martín et al.
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Dear David Mitchell,

We are pleased that you liked our responses and changes. We are thankful for your previous constructive feedback and suggestions, which together with the suggestions by Referee 2 gave us the chance to improve our manuscript considerably. Once again, we appreciate your suggestions to improve clarity and the impact of our study. Please see below our response to your comments. We are repeating your comments in normal font.

Following your comments, we are reporting ***our responses*** *(in blue italics) directly following each point that you have raised.*
Then, we are suggesting **changes to the manuscript** (still in blue).

Kind regards,

The authors

General Comments:

The authors have done an excellent job addressing my review comments and have advanced the science of ice particle fall velocities by adding Appendix C. Most of my comments address Appendix C, followed by a few other specific comments.

Appendix C in this paper demonstrates for the first time (as per my knowledge) that characteristic length L* is superior to the ice particle maximum dimension D for estimating ice particle fall speeds (v). Since Reynold's number Re and Best number X are related through boundary layer theory, this is expected, but this has not been experimentally demonstrated until now. For this reason, I consider Appendix C as seminal work in the field of cloud physics.

Response:
Thank you for underlining the importance of our closure study that we described in Appendix C. Consequently, and to follow the suggestion by the editor, we have added a new point in the Conclusions about this and mention it also in the Abstract.
Changes to the manuscript:
ADDED third bullet point in **Conclusions**:
"On a selection of 75 simple columns from shape group (3), we have done a closure study (see Appendix C) to confirm the Re–X relationship, which is central in our method (see Sect. 3.1) and used by many other studies. For this, the widths and lengths of the columns have been determined

in addition to Dmax. From these geometric dimensions, the masses of the columns have been estimated directly. Then, from each column mass, the Best number X has been determined using Eq. 3. Thus, Re and X have been determined independently and consequently compared to the X–Re relationship given by Eq. 5. This closure showed the superiority of the characteristic length L∗ (Jayaweera, 1971) over Dmax, confirming that Dmax is not suitable to calculate Re and X using Eq. 2 and Eq. 3, respectively, for columns. The closure study also showed that using the modified Best number X∗ (Heymsfield and Westbrook, 2010) instead of the Best number X improved the agreement. The best closure for our subset of simple columns was achieved when using both characteristic length L∗ and modified Best number X∗ together."

Line 361:

ADDED reference to Sect 4.2.1 and Appendix C.

"…width … is more closely related to a suitable characteristic length to determine Re."

CHANGED TO:

"…width … is more closely related to a suitable characteristic length to determine Re (see Sect. 4.2.1 and Appendix C)."

ADDED paragraph to **Abstract** with results of closure study:

"The resulting mass–size relationships indicate that for certain shapes, in particular columns and related shapes, maximum dimension is not suitable to describe the size of snow particles when determining the Reynolds number. Consequently, mass derived from fall speed for these shapes is not reliable. A closure study done on a selection of simple columns, for which mass is determined geometrically, shows that for this shape a characteristic length, similar to the diameter of the basal facet, is superior to the maximum dimension, which is similar to the column length, as size parameter. Using a modified Best number, the Best number reduced as a function of area ratio, resulted in even better agreement in the closure study, confirming that the modified Best number approach adopted in this study represents an improvement for columns."

Other changes to better include Appendix C as closure study:

Lines 227-229:

In Sect 4.2.1, refer more prominently to the Appendix C.

Line 224:

ADDED clarification:

"between Dmax and measured fall speed,…"

CHANGE TO:

"between Dmax and measured fall speed for these shape groups,…"

Lines 450-452:

Improve discussion of size parameter in Eq 2-3:

"Note that Dmax in Eq. 3 comes from Eq. 2, i.e. it represents the size parameter best suited to calculate Re. Thus, also for calculating X one should use the characteristic length L∗ instead of Dmax."

CHANGED TO:

"Note that here, Dmax represents the same size parameter best suited to calculate Re as in Eq. 2. Thus, not only Re should be determined from L∗ (instead of Dmax), but also X."

Title App C:

"Reynolds and Best numbers for simple thick column"

CHANGED TO:

"Closure study---Reynolds and Best numbers for simple thick column"

Lines 436:

New sentences after first sentence to better introduce closure study:
"If Reynolds and Best numbers (Re and X, respectively) can be determined independently, i.e., without using any X-Re relationship, then they can be compared with the X-Re relationship by Bohm (1989) given by Eq**.** **5**. Thus, it represents a closure study that can confirm the X-Re relationship, which this and other studies rely on. The following explains the method and results when applied to simple columns, a small subset of our data." <new paragraph>

As noted in Heymsfield and Westbrook (2010; henceforth HW2010), the Best number X was derived by equating the drag and gravitational forces, where $X = C_d Re^2$, with C_d = drag coefficient. The dimension employed in describing this balance of forces would seem to be related to the particle's boundary layer rather than D. But the method for calculating ice fall speeds in HW2010, as well as other ice fall speed methods, employs D rather than L*. This is natural since L* = A/P where A = total surface area of particle and P = particle perimeter projected to the flow, which are both difficult to measure. Perhaps what is needed in future studies is an expression relating L* to D for each shape category (assuming A and P could be measured somehow).

This is a nice suggestion for future work. The total surface area A is difficult to measure as you have pointed out. However, A could be estimated for the more regular ice crystal shapes. Then, L⋆ to D could be determined for each shape, given sufficiently large datasets of such regular particles.

Appendix C is rich in scientific knowledge that might be missed as it is written. The following clarifications/questions are offered to bring out this knowledge better:

1) The closure experiment consisting of the magenta data points and curve, along with the blue data points, follow a theoretical treatment that demonstrates the superiority of L* over D.

Response:
You are right that we should highlight better, that this represents a closure study that demonstrates the usefulness of L⋆.
Changes to the manuscript:
Line 454-455:
"The points related to Dmax do not match well the empirical relationship X–Re by Böhm (1989) with δ0 = 5.83 and C0 = 0.6. Added period at end of sentence."
CHANGED TO:
"The points related to Dmax (blue triangles) do not match well the empirical relationship X–Re (with δ0 = 5.83 and C0 = 0.6) by Böhm (1989) based on a theoretical treatment by (Abraham, 465 1970)."
Line 456:
"…closer to the empirical relationship."
ADDED sentence:
"…closer to the empirical relationship. Thus, this closure experiment comparing independently determined Re and X to the X-Re relationship demonstrates the superiority of characteristic length L⋆ over Dmax as a particle size parameter when dealing with particle mass m."

2) The closure experiment consisting of the green data points and curve demonstrates better closure is obtained by using X* rather than X, where $X^* = X A_r^{½}$, where X and Re are based on L* and A_r = area ratio. There is no theoretical reason (so far) for multiplying X by $A_r^{½}$.

Response:
Thank you for suggesting a clearer discussion of the improvement when using X⋆ instead of X.
Changes to the manuscript:
Added period at end of sentence.

3) The HW2010 scheme was derived in terms of D; not L*. Moreover, the limiting value of the pressure drag coefficient, C_0, is 0.35, which is not supported by lab experiments. Perhaps a more theoretical (i.e., first principle) treatment of ice fall speeds may be possible using L* (given that correcting potential overestimates in m_{geom} might produce better agreement with the Re-X (Eq. 5) relationships).

Response:
We were also wondering over which value of C0 to use. In combination with the different value in delta0, the different value of C0 produces very similar results. Thus, the changed value of C0 was perhaps not necessary?

4) What happens when the green data points are based on X* calculated from m_{geom}, A, D and A_r and Re is calculated from D and v (as done in HW2010)? Is similar closure obtained? What can this tell us about the viability of the HW2010 scheme?

Response:
Using X⋆ and Dmax is better than X and Dmax.
Using X⋆ and L⋆ is better that X⋆ and Dmax.
From this one can probably not tell much about the HW2010. They focused on open geometries where Dmax may work better than it does for columns (or L⋆ would not be noticeably better than Dmax).
We have now shown that X⋆ works better than X even for columns.
Changes to the manuscript in response to 2)–4):
UPDATED Fig. C2.
L 457:
"…according to X⋆ = X · Ar^1/2."
CHANGED TO:
"…where X⋆ = X · Ar^1/2 is the modified Best number suggested by Heymsfield and Westbrook (2010)."
L 457–458:
"The resulting points (using L∗) are also shown in Fig. C2 and are even closer to the empirical X–Re relationship."
CHANGED TO:
"The resulting points (using L∗) are also shown in Fig. C2 (green 'x') and provide an even better closure to the empirical X–Re relationship."
EXTENDED discussion around X⋆ with new sentences after L 458:
"Heymsfield and Westbrook (2010) used Dmax and not characteristic length L∗ (they focused on shapes with open geometries for which characteristic length is difficult to determine). The closure for our columns using X∗ and Dmax (cyan crosses in Fig. C2) is not as good as using X∗ and L∗,

but still better than using the unmodified Best number X and Dmax. Thus, for columns we can conclude that the modified Best number represents an improvement over the Best number. In addition, the superiority of characteristic length L∗ over Dmax for columns is given also when working with the modified Best number X∗."

L 479:

"Thus, the above discussion remains valid regardless of which relationship is used as comparison."

CHANGED TO:

"Thus, the above conclusions of superiority of characteristic length L⋆ over Dmax and improvement when using modified Best number rather than Best number remain valid regardless of which relationship is used as comparison."

Specific Comments:

Author's response to major comments 3 & 4 (relating to lines 220-229): This may be a minor point but is something I feel the authors should be aware of. Regarding the subset of 75 particles in shape group 3 (hexagonal columns), the authors response states that when width (column diameter) is used instead of column length (i.e., D_{max}), the m-D power law exponent b_D is 2.4. They note this appears consistent with the b_D in Mitchell et al. (1990) for hex-columns, which was 2.6, but also acknowledge the latter b_D depends on D_{max}. However, the columns in M1990 having b_D = 2.6 are short, nearly isometric hex-columns, that are much different than the columns shown in Fig. C1 (i.e., the hex-column subset used in Appendix C). M1990 also features "long columns" (Figs. 1-3) that are comparable with the columns in V-M et al. Fig. C1.

Response:

Thank you for pointing out that most (all but about the smallest ten) of our columns in the closure study are not comparable to C1e Short columns in M1990 but more to N1e Long columns.

Moreover, the mass-width relationship for columns obtained by the authors; $m = a W^{2.4}$, where W = column width, can be converted into a mass-D_{max} relationship by substituting the width-length relationship for columns from Auer & Veal (1970, JAS). From Auer & Veal, $W = 11.3 L^{0.414}$ for L > 200 um, where L = length and units are in microns. Substituting into the V-M et al. relationship gives:

$m = a W^{2.4} = a (11.3 L^{0.414})^{2.4} = a' L^{0.994}$. Thus, a quasi-linear dependence is found between m & L, similar to b_D = 1.1 for columns in Table 1 of V-M et al., but this time derived from column width. This also demonstrates consistency with the Auer and Veal measurements.

Response:

Thank you for this interesting remark. By fitting only columns with L>200um we are left with 66 columns for which $m = a W^{2.2} \Rightarrow a' L^{0.91}$. By fitting directly we get $a' L^{1.3}$, which is still close enough given the uncertainties (we get larger uncertainties when fitting $W=c L^b$).

Line 448: Please define L* as A/P where A = total surface area and P = particle perimeter projected to the flow.

Response:

As you suggest, it is worth here to give the definition rather than only referring to Pruppacher and Klett.

Changes to the manuscript:
Line 447:
"A characteristic length L∗ (see ...)"
REPLACED with:
"A characteristic length $L* = At/P$, where At is the total surface area and P the perimeter of the particle projected to the flow (see …)"
The following two sentences are modified to improve the flow and avoid repetition.
L*, At, and P added to the Nomenclature.
Line 446-447:
In the preceding sentence, we added a reference to Sect. 4.2.1, where we discussed that Dmax is not suitable to calculate Re and mentioned characteristic length:
"… is not a suitable representative size parameter to determine Re."
CHANGE TO:
"… is not a suitable representative size parameter to determine Re,
as we have discussed in Sect. 4.2.1 and Vázquez-Martín et al. (2021)."

Technical Comments:

Line 115: Needs a "period" at the end.

Response:
Thank you.
Changes to the manuscript:
Added period at end of sentence.