Interactive comment on “Mass of different snow crystal shapes derived from fall speed measurements” by Sandra Vázquez-Martín et al.

Referee #2

Received and published: 5 May 2021

Dear referee,

We thank you for your constructive feedback and appreciate the time you spent to read and evaluate our work. Please see below our response to your comments.

We are reporting our response (in blue italics) directly following each point that you have raised. Then, we are suggesting changes to the manuscript (still in blue).

- **General comments:**

I have read the manuscript “Mass of different snow crystal shapes derived from fall speed measurements” by Vázquez-Martín, Kuhn, and Eliasson and I find it an interesting manuscript. The authors use a dataset of measurements of 2461 ice crystals including fall speed, area, and maximum dimension. From these data they derive particle mass using standard Reynolds – Best number approaches. The dataset is then broken down into 15 different particle habit classifications and relationships between maximum dimension and the other parameters are presented and compared to previous literature.

While it is a nice study that merits being published, there are a few shortcomings that I feel should be addressed before full acceptance.

1) The project is highly dependent on the Re – X number relationship for relating mass and terminal velocity. Traditionally, this is used in the other direction where Vt is being calculated as opposed to using Vt to pull out mass. I would like to suggest that the authors should consider the Heymsfield and Westbrook (2010) relationship. In that publication, they showed that a small modification to the Re-X relationship led to much better Vt calculations. The adjustment was a factor of the square root of the area ratio. It should be simple to determine the area ratio for the particles in the dataset and test this. Would the use of this modification improve the results (tighten the scatter)?

Response:
Thank you for suggesting the modified Re-X relationship by Heymsfield and Westbrook 2010 (H&W2010). It is indeed easy to determine the area ratio in our dataset, and it is already included as a parameter. We have therefore added the modified Best number X as suggested by H&W2010 to our*

methodology and recalculated mass for all particles. This results effectively in scaling up mass by a factor of 1/Ar$^{0.5}$*. This correction factor ranges between 1.0 and 3.8 (median 1.5). It is largest for the particles with the lowest area ratios Ar. The results improve in terms of increased correlation (higher R2) and slightly smaller uncertainties in the fit parameters aD, bD, aA, bA, am, and bm. The slopes bD are higher for most shape groups due to the general size dependence of Ar resulting in an increasing correction factor with increasing size. The changes are most noticeable in shape groups (1)-(3), which have the lowest area ratios. Both values for bD and R2 increase considerably, for example for shape group (3) from bD= 0.81 ± 0.33 (R2=0.51) to bD= 1.13 ± 0.28 (R2=0.73).*

Changes to the manuscript:

When revising the manuscript, the following changes will be done:

Method section: introduce/explain the modified X approach by H&W2010.

Tables: update values.

Figures: update.

Discussion of R^2**s and slopes:** adjust.

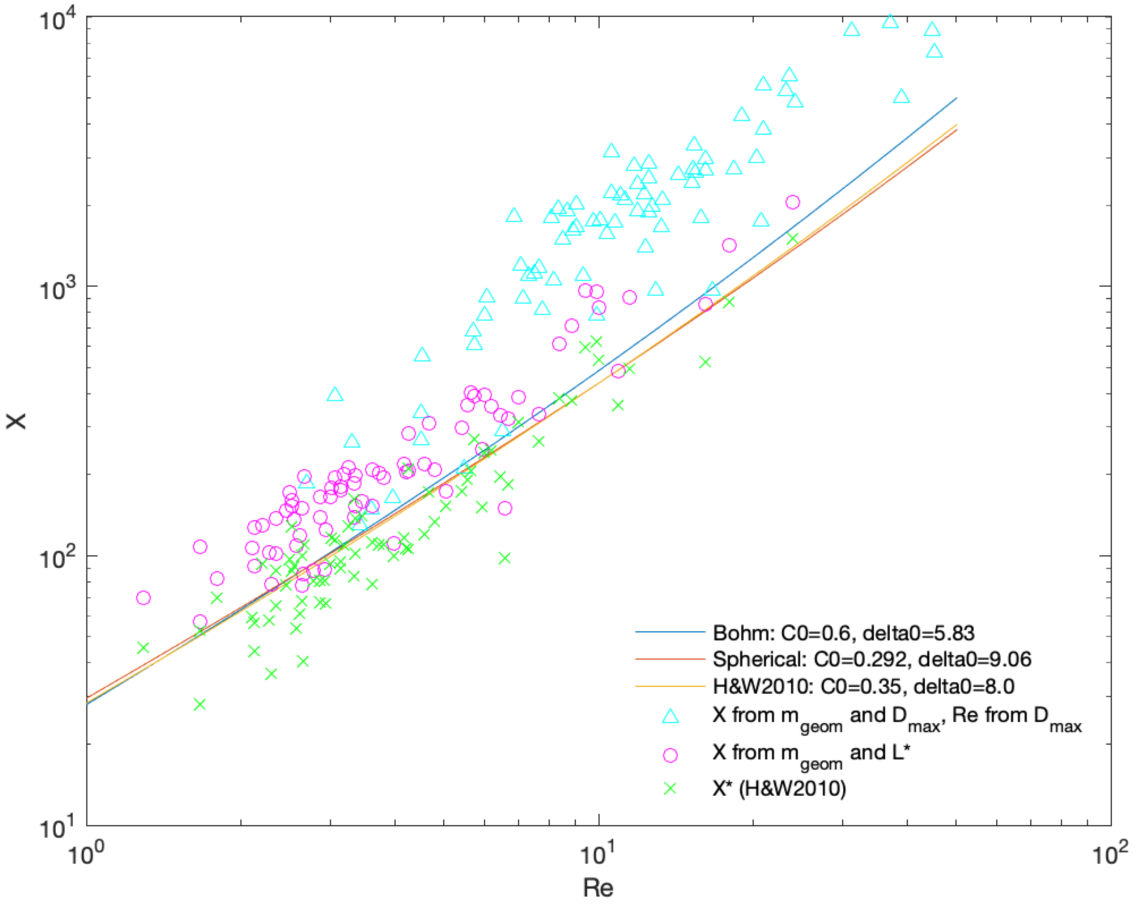
2) The dataset also presents an interesting opportunity for manual comparisons, which would also assist in the selection of the traditional Re-X relationship or the Heymsfield and Westbrook modification. The dataset is said to contain 317 needles and 103 "thick columns". I expect that for at least a portion of these, it should be possible to geometrically estimate the mass. These particles will have the lowest area ratio values and thus would be impacted the most by the Heymsfield and Westbrook modification. It would be a great addition to the manuscript to include a small closure study between Vt, mass, and area with the Re-X relationship, either the traditional relationship or the modified relationship.

Response:

Thank you for suggestion, it is an interesting case study indeed. Our shape group (1) Needles contains many bundles of needles and only few pristine needles yet. Shape group (3) Thick columns, on the other hand, contains many simple columns. With these we have tested the closure you are suggesting. We have determined manually the width and the length of 75 columns out of the 103 particles in shape group (3). Most columns fall horizontally so that width and length can be easily determined from the top-view images. We estimate that the length may be underestimated on the order of up 15% due to deviation from alignment of the column axis in the image plane. On the other hand, the geometrically determined mass, m_{geom}*, may be overestimated for part of the columns that show signs of cavities or hollowing of faces.*

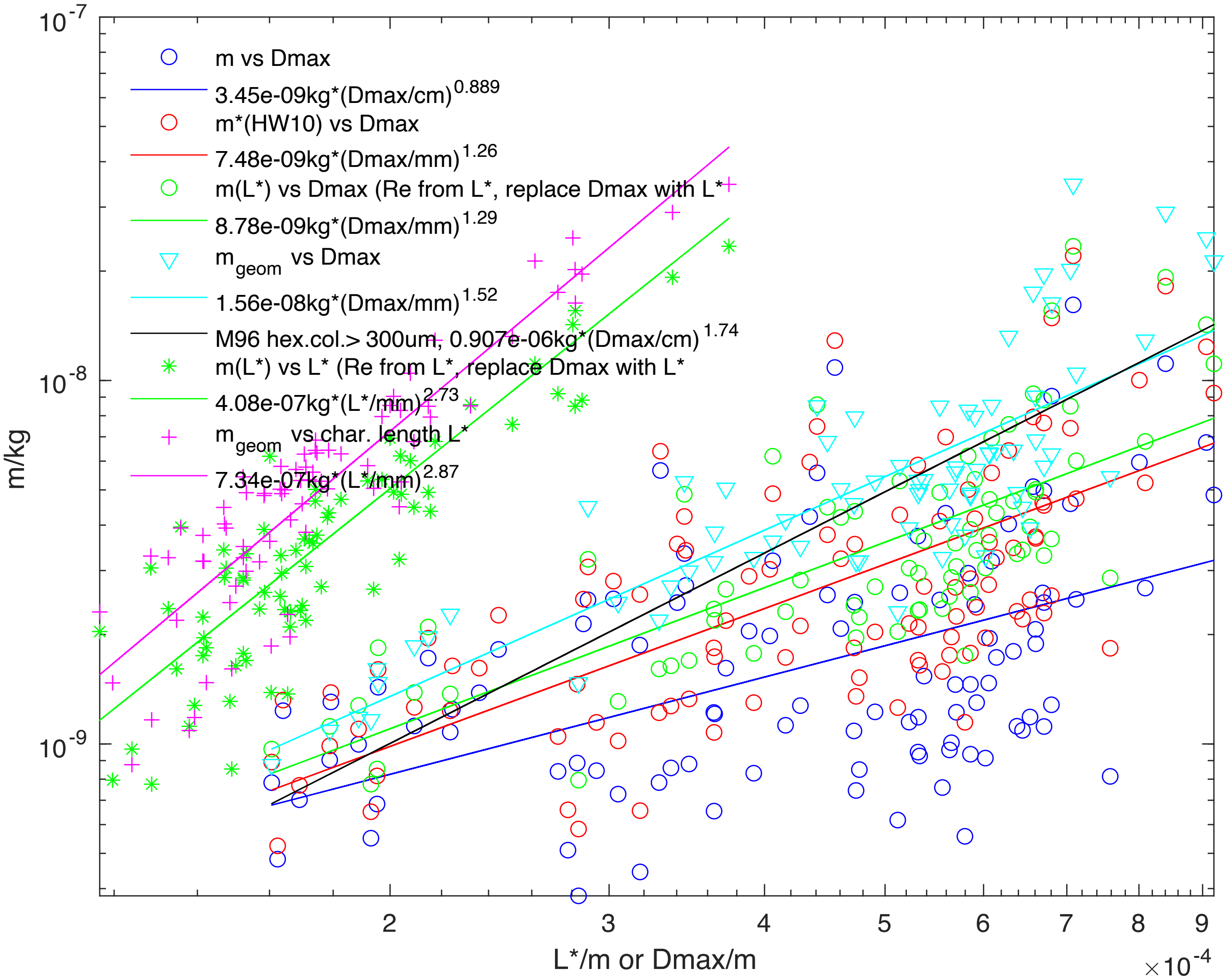
From width and length we have determined a characteristic length L as defined by Jayaweera 1971, JAS (and suggested by referee 1, David Mitchell.*

Now, Re can be determined from measured fall speed and L. The Best number can be determined from measured cross-sectional area A and the characteristic length L*. Thus, X-Re can be plotted and compared to the X-Re relationship (Eq. 5). The following figure shows the results:*



Points are shown for both X-Re and modified X-Re where* $X^* = X \cdot Ar^{0.5}$ *and X calculated from Eq 3. As can be seen, the modified X*-Re agrees better with the X-Re relationship (Eq 5). The figure also shows X-Re where Dmax was used instead of the characteristic length L*. It shows that L* is indeed more suitable to determine Re and X as the points determined using Dmax are further away from the X-Re relationship (Eq 5). As we now have three different sets of parameters used in Eq5, all three have been used to plot the X-Re relationship. They differ by at most about a factor 1.5 at Re = 30 and by less for lower Re. Thus, the same conclusions can be drawn regardless of the chosen parameters.*

We can also examine mass vs size. The following figure compares mass as determined in our manuscript and geometrically calculated mass versus Dmax or characteristic length L:*



Circles and triangles show mass of the 75 columns from shape group (3) against Dmax. The lowest mass is that determined as described in Sect 3.1 and Sect. 3.2 in the manuscript. The mass using modified X is larger by the factor 1/$Ar^{0.5}$ (as explained in our response to your general comment 1) but has apparently a similar spread. Mass determined using characteristic length L* instead of Dmax in Eq.s 2 and 6 is yet larger and it is somewhat better correlated (R^2=0.57 vs 0.44 an 0.21 for mass from modified X* and mass as in the manuscript, respectively). The mass geometrically calculated from width and length is again larger. It is also better correlated to a power law (R^2=0.69) and has a steeper slope (bD=1.52, as reported in the legend). It is also most similar to the relationship by M96 for hexagonal columns (also shown in the figure). If mass cannot be determined geometrically, then either mass using L* or modified X* is a better estimate of mass than mass using the unmodified X as described so far in our manuscript.*

Mass vs characteristic length is also shown in the figure. Both mass determined by the method of our manuscript, but using characteristic length L instead of Dmax in Eq.s 2 and 6, and mass calculated geometrically have slopes of the fitted power laws of close to 3. Mass using L* is lower but comes within a factor of 1.5 to the geometrically calculated mass.*

Changes to the manuscript:

We are considering adding these results and the discussion to the Appendix.

3) The authors go to great efforts to present results by different shape categories. As they point out though, remote sensing cannot identify the microphysical characteristics of particles, so shouldn't we be emphasizing overall characteristics of the particle populations? Rather than emphasizing the differences in characteristics between different particle categories, I'd suggest showing more "average values" and characterize the uncertainty that could exist. I seriously doubt that weather modeling will get to the point where we can predict 15 different particle categories, so understanding the intimate details of each isn't going to be too critical too soon.

Response:
We understand your concerns considering that models unlikely will incorporate 15 different shape categories any time soon. Sorting our data into 15 shape groups is an attempt to use the shape information that we have to describe how particle properties depend on shape. While we do not think that these 15 groups are ultimately best suited to do that, it has been a starting point for our studies Vázquez-Martín et al. 2020 Appl. Sci. and Vázquez-Martín et al. 2021 ACP. Analysing differences between different shapes will help to characterize ranges and uncertainties related to greatly varying snow particle properties. Thus, it may also be useful in case models want to include shape dependencies starting with only two or three shape categories.

- **Other comments:**

1) The "Dataset" section could have an additional paragraph with a brief description / summarization of how the dataset was created. While the reader can obviously go to the two Vázquez-Martín et al publications for more information, a one paragraph summary could potentially save the reader some time.

Response:
Thank you for pointing this out. We will add your suggestions to the "Dataset" section.
Changes to the manuscript:

- Lines 59-60, between "The dataset consists of 2461 … and other hydrometeors" and "The data have been collected using D-ICI …" ADDED a new sentence:

"The dataset consists of 2461 … and other hydrometeors. Same dataset have been used in Vázquez-Martín et al. (2020b). The data have been collected using D-ICI …"

- Lines 61-62, between sentences "… the winters of 2014/2015 to 2018/2019" and "Snow particles are imaged simultaneously from …" ADDED a new sentence:

"… the winters of 2014/2015 to 2018/2019. The images are determined when the snow particles fall into the inlet and consequently fall down the sampling tube and traverse the optical cell. In the centre of the optical cell is the sensing volume. If particles are falling through the sensing volume they are detected by the detecting optics (detailed description in Kuhn and Vázquez-Martín (2020). Upon detection, the particles are optically ~~Snow particles are~~ imaged simultaneously from two different viewing directions."

2) Fitting the dataset: I applaud the appropriate use of data. Reasonable binning and using the median values is something that eludes many. Good job.

Response:
Thank you for your positive feedback.

3) In general, it might be nice to have a table with the symbols / variables used. You use a lot of sub scripts and super scripts and tildes etc. While it is easy to follow what each means after reading it a few times, it took me a few times reading the equations to completely follow the variables.

Response:
Thank you for your suggestion, we are considering to add a list of symbols/variables and acronyms used as a new appendix.
Changes to the manuscript:
Appendix C: List of Symbols, Variables, and Acronyms.

4) Table 1: Some of your shape categories had as few as 37 particles. When you separated into size bins, did you have a minimum number of particles per bin? (Did you do six bins of 6-7 particles or 10 bins of 3-4 particles?) You might consider noting the temperature range that particles in each type were observed if that were possible, and, if possible, the number of days when observations of each particle type were observed. 5 winters of data may not seem biased, but if all 37 graupel particles were observed on one day, then there could be a bias.

Response:
As we have explained in Vázquez-Martín et al. (2020b ACPD, now 2021 ACP), the data are first binned into 10 particle size or cross-sectional area bins before fitting to Eqs. (11) and (12), respectively, where the binning uses flexible bin widths so that each bin contains as close to the same number of particles as possible. Therefore, the bin widths are different for each shape group and thereby avoid the problem of individual bins having a disproportional effect on the fit. The number of bins (10) is a compromise to a small enough number of bins to contain enough particles per bin and a large enough number of bins to allow for a good fit to the measurements. So, as you have pointed out, for a shape group with only few particles, such as (12) graupel with 37, only few particles are included in each bin, such as 3-4 for graupel. We found about 40 particles to be a limit where the binning can still be used. However, at such low particle numbers, the binning method is not better than fitting to individual data.
In most shape groups, particles come from a number of different days and winters. In Graupel for example, particles had been collected on 12 days in 3 winters. Therefore, the risk for potential biases due to sampling under very few special conditions is reduced.
We could not find clear relations between our measured ground temperature and shape groups. Temperature ranges were in general broad. While this at first may seem unexpected, it is understandable considering that the temperature on the ground does not have a unique relation to the conditions at formation and thus is not directly related to snow habit. We decide therefore to not report these ground temperature ranges since they alone did not convey any useful information.

Changes to the manuscript:
Add to the end of Sect. 3.2, Line 125:
Vázquez-Martín et al. (2021 ACP) found that about 40 particles in a shape group (currently the lowest number in our dataset is 37) is the limit where binning can still be used. The advantages of binning, however, become prominent only at larger numbers of particles.

5) Figure 1: The "fit to all data" appears to have a tighter uncertainty range. Also, the size range doesn't go as far as one might expect. Again, statistically, how far off would a user be if they used an average value and the particles were mostly one of the categories?

Response:
The uncertainty range of the fitted power law, i.e. the 68% confidence region of the fit, is indeed more narrow than for most of the shape groups. The "fit to all data" is the fitted power law to all 2461 particles. This large number is likely the main reason for the relatively narrow confidence region. Some shape groups, however, do have a narrower range, such as for example for m-Dmax of groups (8) Branches and (9) side planes, which have the highest number of particles among the shape groups. Thus, "all data", being a mix of different shapes, has a wider confidence region as these shapes with "good statistics". One should also look at the prediction band corresponding to the fits without using data binning, i.e. the range given by the expected 16th and 84th percentiles of mass (where the majority of new measurements would be expected). To avoid confusion with too many lines, we have decided to not show these regions in Fig. 1. In Fig 1a, this prediction band for "all data" would span over about a factor of 4 in mass.
The length of any of the fit lines, including the line for all data, is defined by the percentiles 16th and 84th of all values of Dmax (m vs Dmax), of A (m vs A), and of m (v vs m). Thus, the length indicates the Dmax, A, or m regions where most data are found. Had we shown the percentiles 0 and 100 then the regions for all data would encompass all regions of all shape groups.

6) Figure 2: I'd suggest removing this figure as it doesn't seem important.

Response
This figure is not essential. The related discussion can be shortened without referring to it while still remaining clear. We will remove the figure and make the necessary modifications to the text.
Changes to the manuscript:
Remove Fig. 2
Lines 165-169:
"Figure 2 compares the coefficients RD2 and RA2 of all the shape groups, and for most shape groups, the two are similar. Only the four groups (1), (2), (3), and (10), mentioned above with lower correlation in one of the relationships, have a distinct difference between RD2 and RA2 . These are clearly above and below the line representing RA2 = RD2 in Fig. 2. Of these, the three shape groups (1)–(3) above the line have a better correlation for m vs A than for m vs Dmax, which is consistent…"
CHANGE TO:
"In most shape groups, the coefficients RD2 and RA2 are similar. Only the four groups (1), (2), (3), and (10), mentioned above with lower correlation in one of the relationships, have a distinct difference

between RD2 and RA2 . Of these, the three shape groups (1)–(3) have a better correlation for m vs A than for m vs Dmax, which is consistent…”

Line 309:

“…similar to the corresponding values of RA2 (see Fig. 2).”

CHANGE TO:

“…similar to the corresponding values of RA2.”

7) Figure 3: It would be interesting to look at the ratio of the powers in the mass to D relationship versus the Area to D relationship. Mitchell 1996 has many comparisons and Schmitt and Heymsfield 2010 show that this ratio would be ~1.3 for fractal particles and of course, it would be 1.5 for spheres.

Response

Thank you for this suggestion. Looking at the ratios of the powers in the m-Dmax relationship versus the A-Dmax relationship reveals some problematic shape groups where the power exponent in the m-Dmax relationship, bD, is somewhat smaller than the power exponent in A-Dmax, b. It was not surprising to find groups (1)-(3) among the offending shape groups. For example, in group (3) bD=1.1 and b=1.2. The only other two groups with this problem are (9) Side planes and (10) Spatial plates. Group (10) has few particles and was noticed before for bad correlations between fall speed and any of Dmax or A. For group (9) bD=1.4 and b=1.8. This problem likely indicates for which shape groups Dmax is not suitable as size parameter to calculate Re. Then, for these shapes or shape groups one should use a suitably defined characteristic length in Eqs. 2 and 6. As mentioned earlier, currently we cannot easily determine characteristic length, as we have done for some regular columns, for all particles. Thus, at the moment the modified X approach according to H&W2010 remains the best for our study, it lessens the problem considerably for groups (1)-(3).*

Changes to the manuscript:

Lines 206-207:

“Instead, the X–Re relationship given by Eq. 5 not describing well this shape group may be responsible again.

CHANGE TO:

“<new paragraph>

Shape group (9) is also noticed when comparing bd, the exponent of the m-Dmax relationship, to b, the exponent of A-Dmax. Intuitively, bD should be larger than as also confirmed by literature, such as by M96. For some shape groups, however, b is larger than bD. Not surprisingly, groups (1)-(3) that were noticed earlier by having the lowest bD values are among these groups, as well as groups (9) and (10). The latter was noticed in Vazquez-Martin (2021 ACP) with very poor correlations between Dmax or A and fall speed. This problem likely indicates that for these shape groups Dmax is not suitable as size parameter to calculate Re. While suitable substitutes exist for regular shapes, such as the characteristic length suggested by Jayaweera (1971, JAS), for an arbitrary shape our current image analysis methods cannot determine a similar quantity. Thus, the modified X* approach according to H&W2010 remains the best alternative for our study, it lessens the problem considerably for groups (1)-(3).

Summary and conclusions:

Add another bullet point about this issue of Dmax not being well suited to calculate Re for some shapes.