We would like to pay special thanks to the reviewer for valuable comments and constructive suggestions. We took a closer look at all the comments and reviewed the manuscript accordingly. All changes in red fonts have been marked in the revised manuscript. The explicit answers to the comments are given below in blue fonts.

The authors apply a solitary wave model to numerous sporadic Na layer (Nas) profiles measured with a lidar at the Andes Lidar Observatory and find that for most events, a solitary wave model provides very good fits to the Nas profiles. The implication is that these Nas are linked to and may be somehow caused by the solitary wave. The paper is adequately referenced but I found the writing quite confusing in places. Section 2, where the fundamental solitary wave theory is discussed needs a major rewrite as do Section 3.1, 3.2 and 3.3 where the fitting of the theory to data is discussed. I understand what they are doing, but it was figure 2, not the text that enabled me to figure things out. Even so I'm puzzled by some of the equations, for example, should equation (11) be written as

$$u(x,t) = u_2 + (u_1 - u_2)sech^2\left[\sqrt{\frac{u_1-u_2}{12\beta}}(x-ct)\right] ?$$

Although the idea that Nas may be related to solitary waves is interesting, my main concern is that the authors have provided no insight into how the Na density could rise to such large values in Nas simply by the passage of a solitary wave through the Na layer. The conventional explanation for Nas is that very high concentrations Na+ are collected in thin layers by the combined effects of the earth's magnetic field and the vertical wind shears caused by large amplitude waves and tides. Chemical reactions then convert the Na ions to neutral Na, thus forming the Nas. While the authors have demonstrated that the solitary wave model provides a good fit to the Nas, this is hardly evidence that solitary waves are involved. Nas are thin, sometimes form rapidly, and often show vertical phase progression that mimic the phase progression of long period waves and tides. The authors do not discuss those issues. At a minimum, they need to show how a solitary wave propagating through the mesopause region would impact the density profile of minor species like Na. Such theoretical work has been done for waves and tides (e.g. Gardner & Shelton, JGR, 90(A2), pp. 1745-1754, 1985), but not for solitary waves, which behave differently.

I recommend that the paper be returned to the authors for major revisions, that address the issues I have raised. I hope they do so because if they can show how a solitary wave produces the thin Nas with vertical phase progression, like that illustrated in Figure 3a, then this would provide convincing evidence that solitary waves are frequent in the mesopause region and deserve more attention from the upper atmosphere research community.

Thanks for the comment. Starting with the Bernoulli's equation for ideal fluid：

$$\begin{aligned} \frac{\partial u}{\partial t} &= -\frac{1}{\rho}\frac{\partial p'}{\partial x} \\ \frac{\partial w}{\partial t} &= -\frac{1}{\rho}\frac{\partial p'}{\partial z} \\ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0 \end{aligned} \tag{1**}$$

Its boundary conditions are:

$$(w|_{z=0} = 0, (\frac{\partial p'}{\partial t} - \rho g w)|_{z=h} = 0. \tag{2**}$$

Where u represents the horizontal velocity and w represents the vertical velocity, p' is pressure.

Let $\quad \boldsymbol{\mathcal{L}} \equiv \frac{\partial}{\partial t}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2})$,

then the formula of Eq. (1**) is written as

$$\mathcal{L}\mathrm{w} = 0 \tag{3**}$$

Let

$$w = W(z)\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{4**}$$

and substitute into Eq. (3**)

$$\frac{d^2W}{dz^2} - k^2 W = 0 \ . \tag{5**}$$

Consequently,

$$W(z) = A\mathrm{e}^{kz} + B\mathrm{e}^{-kz} \ , \tag{6**}$$

where A and B are constants.

Substitute Eq. (6**) into Eq. (3**), we have:

$$w = (A\mathrm{e}^{kz} + B\mathrm{e}^{-kz})\mathrm{e}^{\mathrm{i}(kx-\omega t)} \tag{7**}$$

Then, put Eq. (7**) into Eq. (1**):

$$p' = \frac{\mathrm{i}\omega\rho}{k}(A\mathrm{e}^{kz} - B\mathrm{e}^{-kz})\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{8**}$$

According to the lower boundary conditions：

$$B = -A; \tag{9**}$$

According to the upper boundary conditions：

$$\{(\frac{\omega^2}{k} - g)\mathrm{e}^{kh} - (\frac{\omega^2}{k} + g)\mathrm{e}^{-kh}\}A = 0. \tag{10**}$$

So

$$\omega = \sqrt{gk\tanh(kh)}. \quad (11^{**})$$

In shallow water conditions,

$$\begin{aligned}\omega &= \sqrt{gk[kh - \tfrac{1}{3}(kh)^3]} \approx \sqrt{k^2 c_0^2 (1 - \tfrac{1}{3}k^2 H^2)} \\ &= kc_0(1 - \tfrac{1}{6}k^2 H^2) = kc_0 - \tfrac{1}{6}k^3 c_0 H^2\end{aligned} \quad (12^{**})$$

where ω is a real number. Then phase velocity v_p and group velocity v_p can be obtained respectively:

$$v_p = \frac{\omega}{k} = c_0 - \beta k^2, v_g = \frac{\partial \omega}{\partial k} = c_0 - 3\beta k^2 \quad (13^{**})$$

where

$$\beta = c_0 H^2/6 \ . \quad (14^{**})$$

When β≠0, $v_p \neq v_g$, which fully indicates that the $\beta \frac{\partial^3 u}{\partial x^3}$ term of KdV equation characterizes the dispersion effect.

In addition,

$$\frac{dv_g}{dk} = -6\beta k \quad . \quad (15^{**})$$

Therefore, the effect of $\beta \frac{\partial^3 u}{\partial x^3}$ causes wave dispersion, With the increase of β, the wavelength becomes shorter, and the wave dispersion becomes stronger, such waves are known as dispersion waves. Of course, for long waves (when k is small), it is a weakly dispersive wave, characterized by ω containing only the odd degree term of k.

According to (Gardner and Shelton 1985), since the layer density response is highly dependent on the density gradients occurring in the layer, the steady state layer density profile becomes much important. Large density gradients encourage nonlinearities in the layer response. Therefore, when the nonlinear effect contrived by the gradient of the Na density profile is balanced with the dispersion effect mentioned above, the wave shows neither dispersion nor nonlinear characteristics, but propagates in the form of solitary waves described in the manuscript.

Atmospheric solitary wave is a kind of nonlinear gravity internal wave which is balanced between nonlinear effect and horizontal linear dispersion (Grimshaw, 2002). According to Gardner et al. (Gardner and Shelton 1985), In the quarter period after the maximum vertical wind, the atmospheric density disturbance is the largest, and the layer peak reaches its maximum upward displacement. This has the effect of enhancing the density of secondary components above the steady-state position of the layer peak. We believe that Gardner's view can still explain the relationship between the wind field and the occurrence of the maximum of the minor components. Unfortunately, due to the lack of wind field observation data at the corresponding time, we could not reconfirm Gardner's conclusion when he discussed the linear layer response using gravity wave theory.

Cited References for this Reply:

Gardner, C. S., and Shelton, J. D.: Density response of neutral atmospheric layers to gravity wave perturbations, Journal of Geophysical Research, 90, 10.1029/JA090iA02p01745, 1985.

Grimshaw, R.: Environmental Stratified Flows: TOPICS IN ENVIRONMENTAL FLUID MECHANICS, THE KLUWER INTERNATIONAL SERIES, edited by: Chatwin, D. P., Dagan, D. G., List, D. J., Mei, D. C., and Savage, D. S., Springer, 285 pp., 2002.