

We would like to pay special thanks to the reviewer for valuable comments and constructive suggestions. We took a closer look at all the comments and reviewed the manuscript accordingly. All changes in red fonts have been marked in the revised manuscript. The explicit answers to the comments are given below in blue fonts.

Sporadic sodium layer events are frequently observed by the Na lidars around the world. This study provides another potential underline mechanism that could be helpful for the understanding of this dramatic event in the upper atmosphere. Here, the author utilizes the solitary wave theory to explain several sporadic sodium layer events in the upper mesosphere observed by a Na Doppler lidar at ALO. The waveform of the solitary wave is derived by fitting the residual of the observed Na layer anomalies to the solution of the KdV equation that describes the solitary wave. The author suggests that the observed “solidary wave” is “consistent with the shallow water model”, and presents an example of Na<sub>s</sub> coexisting with a strong wind shear in the lidar data. I know ALO has several nightglow instruments operating as well, so is it possible to check and see if these nightglow instrument captured any of these reported “solidary wave” events in Table 1?

Thank you so much for sharing your opinion. We tried our best to support our results with the observational data. Unfortunately, we were unable to obtain Nightglow Imager Observational data from the ALO station. Additionally, in order to further test our theory, it's required to obtain the data of Na<sub>s</sub> propagation velocity but with the current technical conditions there, acquisition is quite difficult. It would have been interesting to explore this aspect and support our claim through it however, presents situation lags advancement in this regard.

This would strengthen the author's argument with further experimental evidence. In addition, why does not the author fit the 5<sup>th</sup> order KdV solution for all of these event, since it appears the 5<sup>th</sup> order solution generates better fitting?

Thanks for your kind comments and sharing reservations. The grounds for introducing 5<sup>th</sup> order KdV is to explain the tiny wavelets that appear on both wings of the blue dotted line in Figure 2d in the revised manuscript. As stated in the Eq. (46), once the higher order dispersion factor is included into the KdV equation, wavelets begin to appear on both sides along the main peaks of the solitary wave solution and it is more consistent with the fine structure of Na<sub>s</sub> observed. Further, we concentrated on the role of dispersion rather than the sequence of dispersion in defining the fine structure of Na<sub>s</sub>.

On the other hand, my major concern is the author seems to suggest the Na<sub>s</sub> is the product of Na<sup>+</sup> layer in solidary wave form through Na ion-molecular chemistry, if I understand it correctly. A sharp Na<sup>+</sup> layer with high peak ion density near and below 90 km, where neutral-molecular chemistry dominates, is highly unlikely in my

opinion. There is not argument of the chemical reaction time, and how it compares with time of the wave event. In addition, the author completely ignores the possibility that it can also be a dynamic feature.

Technical comments:

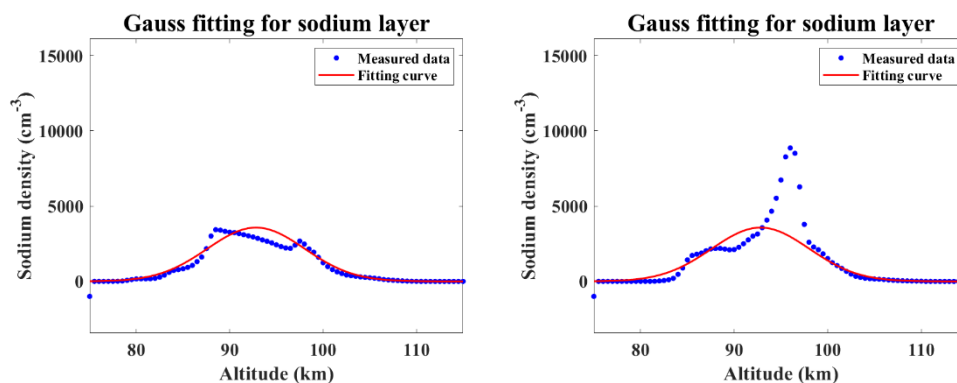
1. The title of the paper does not reflect the key point of the manuscript. Since the whole paper is focusing on solitary wave mechanism for Nas, it would be beneficial to somehow include “solitary wave” in the title.

Thanks for the constructive suggestions. As per your valuable suggestion, we have revised the title of the manuscript as: Solitary wave characteristics on the fine structure of mesospheric sporadic sodium layer.

2. Page 2, Line 2: The author states the Na layer shape is “normally with a Gaussian distribution”. But This is really depending on the temporal resolution of how the lidar data are processed. For short time scale, the layer does not appear to be Gaussian at all. If the “solitary wave” lasted less than one hour in the Na lidar observations, this assumption will not apply. The author should be very careful about this statement. So more clarification would be required.

Thanks for your feedback. We cordially appreciate your concern and totally agree with your opinion that for short time scales, this layer doesn't appear to be Gaussian at all. However, none of the 27 samples we picked had a time range of fewer than seven hours in real data processing. (Table 1\* elaborates the starting and ending time of each case)

In addition, for the case of November 3, 2016, we plotted the density profile at numerous periods other than the peak moment (as shown in Figure 1\*). Their corresponding moments are five, two and one hour before the peak chronologically and one, two and five hours after the peak. The red curve represents a fitted background Gaussian distribution throughout the night.



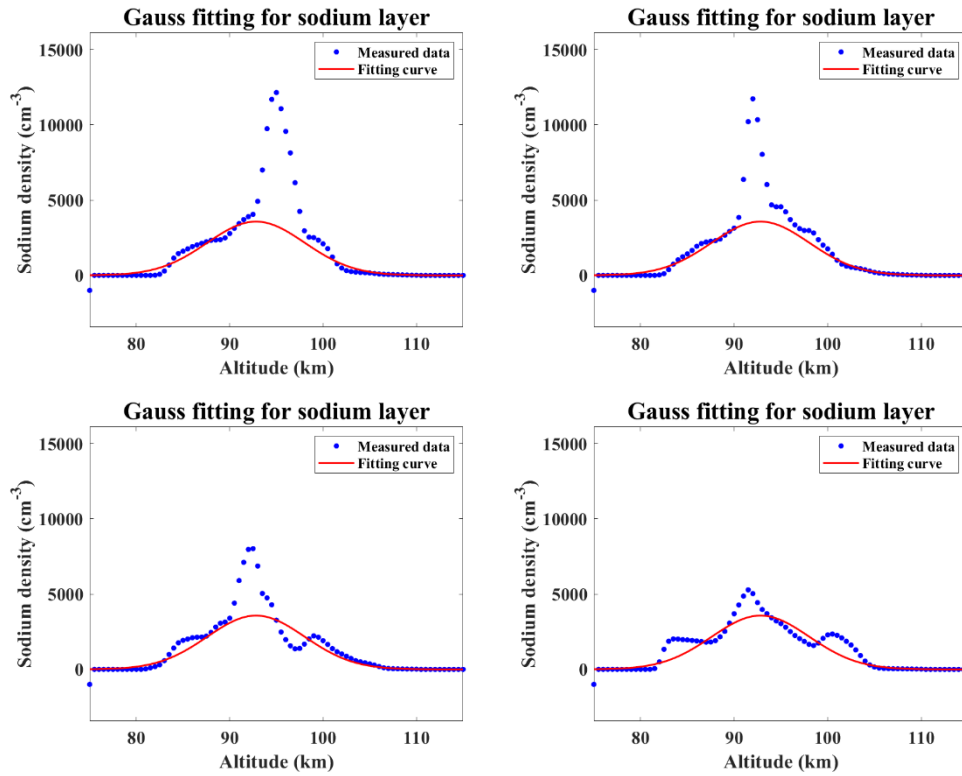


Figure 1\*

Table 1\*

Event	observation start time (UT)	observation end time (UT)	Duration (hour)
2014-08-20	0.5	10.2	9.7
2015-01-30	0.9	9	8.1
2015-02-02	1.6	9	7.4
2015-04-18	-0.5	9.7	10.2
2015-04-19	-0.9	9.5	10.4
2015-04-21	-0.3	9.8	10.1
2015-04-22	-0.3	9.8	10.1
2015-11-06	-0.2	8.8	9
2016-02-25	1.1	9	7.9
2016-03-02	-0.2	8.8	9
2016-03-15	0.1	8.6	8.5
2016-06-06	1.1	10.6	9.5
2016-10-26	-0.5	8.8	9.3
2016-10-28	-0.1	8.3	8.4
2016-11-03	0	8.8	8.8
2016-11-09	0.1	7.8	7.7
2017-04-22	-0.4	9.8	10.2
2017-11-25	0.6	8.5	7.9
2017-11-28	0.1	7.9	7.8
2017-12-16	0.7	8.9	8.2
2017-12-17	0.2	8.9	8.7
2017-12-19	1.9	8.9	7
2017-12-21	0.6	9	8.4

2017-12-22	0.3	8.9	8.6
2019-04-07	-0.4	10.2	10.6
2019-04-09	-0.8	10.1	10.9
2019-07-06	0.9	10.9	10

3. Page 2, Line 10: The author states “the ion-molecular theory is the most possible mechanism for  $\text{Na}_s$ ”. This statement is still debatable. Although there is high correlation between  $\text{Na}_s$  and Es (or  $\text{Na}^+$ ) in the MLT, the dynamic effect cannot be ruled out, since the ion-neutral collision rate is still high, especially near and below  $\sim 100$  km. In fact, some recent simulations indicate the dynamic effect can play an important role in the Na layer structure in the lower thermosphere up to  $\sim 120$  km or even higher.

Thanks for your suggestion. We totally agree with you. Gardner et al. derived the relationship between  $\text{Na}_s$  and gravitational waves in details, which was supported by many subsequent observations. We have corrected the imprecise error statements and added corresponding references (Page 2, line 10).

4. Page 2, Line 24-25: Most of these references were published more than a decade ago, so I would not say they are “recently”.

Thanks for your comment. We have replaced the de facto expression “Recently” with “In the last decade and earlier” (page 2, line 27).

5. Page 3, line 14: should be “in the mesopause region”.

Thanks for the kind suggestion. We cordially apologize for our error and this typo has been corrected (page 3, line 19).

6. Page 3, after equation 2, the author states the variable ‘n’ could be regarded as  $\text{Na}^+$  produced Na. I understand this statement follows the previous one (#3). But, again, I think the author should also consider/include the possibility of dynamic transport of Na atoms in the argument.

Thanks for your valuable comment. We apologize for the manuscript’s fundamental objective being misunderstood. Our research paper has no intention of invoking a new mechanism for the  $\text{Na}_s$  source. It is primarily

concerned about the time series of fine structure. Influenced by your 3rd review opinion above, the expression of the variable 'n' could be regarded as a production from  $\text{Na}^+ \rightarrow \text{Na}$ . According to Xu and Smith, 2003, it appears that the sodium density could also be concentrated through dynamic processes. And the wisdom reviewer 3 proposed the input could be meteor injection, too. Thus, the input 'n' could possibly be generated through molecule reactions, dynamics, or meteor injection. We made some changes to the statements in the revised manuscript (page 3, line 27).

7. Page 5, line 4: It reads somewhat awkward that  $u_2$  is defined before the definition of  $u_1$ . In addition, what is "the limiting wave amplitude"? Please clarify.

Thanks for the valuable comment. There are numerous sorts of solitons within the solitary waves and one common feature of them is that the amplitude of the soliton at infinity is a definite constant (Zabusky and Kruskal, 1965), which is called "the limiting amplitude". In our study, this constant is  $u_2$ .

8. Page 9, line 25-26: This statement need further clarification. The vertical scale of  $N_{as}$  is less than 10 km because of the limitation set up the Na lidar range, but it does not mean the vertical propagation of the "solidary wave" is limited to the same scale. It would be highly possible that the wave propagates beyond the mesospheric Na layer (the Na lidar range) with larger vertical scale.

Thank you for your advice. We have no objection to your opinion and totally agree with you. The solitary wave theory has been utilized to explain well observed phenomena in the lower atmosphere and in rotating and magnetized dusty plasma in the dayside tropical mesosphere. It is therefore reasonable to suspect that solitary waves may be widespread in the earth's atmosphere. We revised the relevant statements and added corresponding references in the manuscript.

9. Page 13 on the author contribution. I do not see any data from the Chinese Meridian Project in this study, but the author Xiankang Dou "provided data from the Chinese Meridian Project"?

Please accept our sincere apologies. Xiankang Dou provided data from the Chinese Meridian Project and conceived this study. However, in

subsequent studies, we found that there was a large error in sodium density data detected by meridian Project (as shown in the Figure 2\*), and this data was not chosen for the final draft.

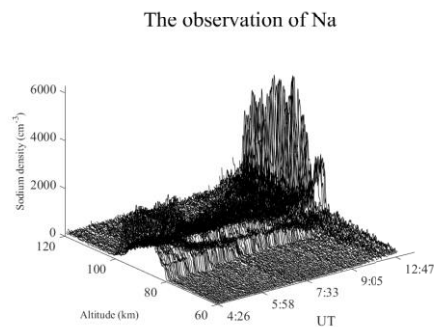


Figure 2\*

10. Page 20 figure 3: The author might need to adjust the color level of the contour plots of horizontal winds, since there are some large chunks of blank area in the two plots, where the lidar data should be still good.

Thanks for your suggestion, we re-examined the raw data from ALO observations. Nothing was discovered to be lacking in the depiction of horizontal wind data.

Table 1, Would it be possible to generate a few figures to make some of the important parameters more visible, in addition to this table? It is difficult to digest the information from this busy table.

Thanks for the commentary. In order to express the information more directly, we showed the parameters  $h_{d/2}$ ,  $d$ ,  $d'$ ,  $u(h_{d/2})$  and  $p_1(n)$  of the case on November 3, 2016 in Figure 2d.

### **Cited References for this Reply:**

- Xu, J., and Smith, A. K.: Perturbations of the sodium layer: controlled by chemistry or dynamics?, *Geophysical Research Letters*, 30, 10.1029/2003gl018040, 2003.
- Zabusky, N. J., and Kruskal, M.: Interaction of "Solitons" in a Collisionless Plasma and the Recurrence of Initial States, *Physical Review Letters*, 15, 240–243, 10.1103/PhysRevLett.15.240, 1965.