We would like to pay special thanks to the reviewer for valuable comments and constructive suggestions. We took a closer look at all the comments and reviewed the manuscript accordingly. All changes in red fonts have been marked in the revised manuscript. The explicit answers to the comments are given below in blue fonts.

Review "Solitary wave characteristics on the fine structure of mesospheric

sporadic sodium layer" by Qiu et al.

In the current version of the manuscript, Qiu et al. have reported their work on studies of the sporadic sodium layer (Nas), one of the most active research areas in the upper atmosphere subject. They have introduced this subject concisely in the abstract section, for example, the definition of Nas and the possible mechansim (e.g., ion-molecule chemistry mechanism and recombination with electrons) as well as Nas relationship with waves. Then the authors have introduced the solitary wave theory and has applied it to try to explain the fine structure of mesospheric Nas observed by a narrow band lidar at Andes Lidar Observatory.

1. In section 2, the authors have described the solitary wave theory but they have assumed that the particle density changes with time is constant, which is probably wrong in particular for the sporadic Na layer for this study. Acturally I am quite struggled to understand the sections 2 and 3 but it looks that using the the solitary wave fitting method clearly matches the observation data better than after they have selected Nas cases. Based on this and their Figure 4, the authors then conclude that "this solitary wave theory could possibly explain some characteristcs of Nas".

Thanks for the worthy comment. Gardner and Shelton 1985 in previous studies made the following hypothesis about the formation of Nas by gravitational waves: The velocity field of the minor constituent (e.g. sodium) equals to the atmospheric velocity field. Only wave-induced dynamics are considered; no chemical effects associated with atomic sodium are part of these solutions. This has the effect of reducing the source and loss terms, P and Q in Eq. (1***), to zero. Eq. (1***) is written as:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = P - Q = 0 \ , \qquad (1^{***})$$

where

n ~ density of the minor constituent;

V ~ velocity field of the minor constituent;

P ~ source terms;

and

Q ~ loss terms.

In our selected event, solitary wave occurred where both horizontal and vertical winds shear strongly, which means that V in Eq. (1***) is also equal to zero.

Thanks again for the reviewer's brilliant insight. As a result, we can now choose the Nas situations using the solitary wave fitting approach.

2. The authors have also warned that "it is worth noting that the numerical simulation of the higher-order KdV equation is probably only suitable for explaining the events similar to the selected case" and "In contrast, the other events with shorter durations and cloud-like shapes are less consistent with the higher-order simulation results. This discrepancy also implies 30 that the NaS with different characteristics may have different fine structures." This is quite confusing and there is not clear conclusions and convincing results from the current work. It also depends on the method the authors have applied to do the data analysis (for example, they have applied the gaussian fit first for the lidar measurements dataset then subtract it then use the anomlay to do the analysis, see their method in Page 5).

Thanks for the kind and worthy comment. Russell discovered a solitary wave that moves ahead in shallow, narrow channels and maintains its form and speed for an extended period of time. In reality, similar theoretical investigations reveal that solitary wave propagation is relatively stable. Further, theoretical studies show that the propagation of solitary waves is relatively steady. This implies that, as Russell proposes, singular waves tend to travel for a long time. Based on this, we propose the opinion that shorter duration, cloud-like events with poor agreements to higher-order simulations. The results cannot be explained by isolated wave theory, which imply that Nas with different characteristics may have different fine structures.

3. It looks the guassian fit used in the Equation 12 is not suitable for the Nas layer (Shown in the Figure 2c). Is that the reason to apply the solitary fitting for the density aomaly? If so, the caption (measureed data) in the Figure 2d is misleading because it is the Na density difference from Lidar and guassian fit. If you choose to different Guassian fit (for example, super gaussian fit function), will the result be different?

Thanks for the kind comment. During the handling of the raw data, we according to the size of the density value determined the boundary of an exception occurrence and if is not an exception occurs, then select all, there was no abnormal data on highly sequence gauss fitting, fitting results show as Figure 2c red solid line, blue dotted line in Figure 2c said in peak time of the original Na density data. On the other hand, we have modified the legend in Figure 2d according to your suggestion. The mathematical expression of super-Gaussian fitting can be written as:

$$f(x) = \exp(-(\frac{x}{w})^N), N \geq 2.$$

W is a constant. If n is 2, this expression is reduced to a Gaussian distribution, and if n is an even number greater than 2, it is a super-Gaussian distribution. The following figure shows the image when w is 20 and n is 2, 6, and 10 respectively.

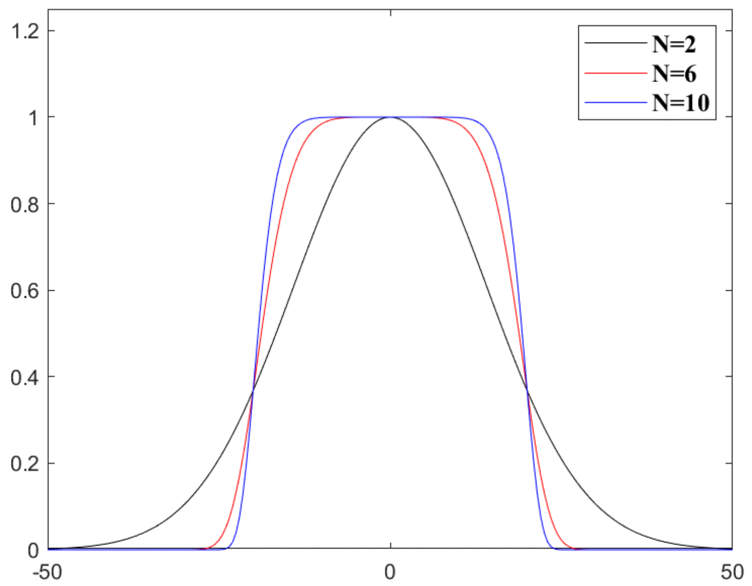

Figure 1***

Apparently, when there is no Nas layer, the super-Gaussian fitting is more inconsistent with the actual observed density distribution of Na layer.

4. Somehow, I am lost in understanding how to obtain the Eq. (21). The parameters used in the fitting expression are different for different cases (which shown in Table 4). So my feeling is that these parameter will give better fit for the data rather than an explaination of Nas layer.

Thanks for your question. Eq. (36) has been obtained from Eq. (35) and ξ_0 (the height of maximum Na density observed by lidar). In other words, by substituting the value of ξ_0 into Eq. (35) and using the least square fitting method to match the data $\vec{p}$, Eq. (36) can be obtained.

5. There are also many assumptions and it is very hard to judge and is unclear if they are reasonable or not. For example, For the equation (2), why Na "could possible be regarded as the input of sodium sources from Na^+ through chemical reactions"? Please keep in mind the source of Na is from the the ablation of incoming meteors.

Thanks for the kind feedback. Propelled by your review opinion above, we think that the expression of the variable 'n' could be regarded as Na^+ produced. And from some previous references, we know the input of sodium could also be the meteor injection (although the amount of the injection could hardly explain the huge increase of sodium density). As the kind reviewer 1 suggests the input could also be controlled by dynamic processes, we have added the three sources for ‘n’. We made some changes to the statements in the revised manuscript (page 3, line 27).

6. Again, why the authors assume the same airmasses (conservation of particle number)

in Equation (2) to let dn/dt equals zero? This mean the production and loss term of Na is always the same. Is this applicable for Nas layer?

Thanks for your question. In previous studies (Gardner and Shelton 1985) on the formation of Nas by gravitational waves, the following hypotheses were made: The velocity field of the minor constituent (e.g. sodium) equals the atmospheric velocity field. Only wave-induced dynamics are considered; no chemical effects associated with atomic sodium are included in these solutions. This has the effect of reducing the Source and loss terms, P and Q in Eq. (2***), to zero. Eq. (2***) is written as:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = P - Q = 0 \quad , \qquad (2^{***})$$

where

n ~ density of the minor constituent;
V ~ velocity field of the minor constituent;
P ~ source terms;

and

Q ~ loss terms.

In our selected event, solitary wave occurs where both horizontal and vertical winds shear strongly, which means that V in Eq. (2***) is also equal to zero. That is the Eq. (2***) in our manuscript.

7. Why the authors only consider "the dispersion term of a surface wave in incompressible shallow fluid" In equation (8) and how this equation 8) is derived? Does this mean that only one single Nas layer can be done in the current method? However, from the Lidar observations, it looks that the peak Nas layer occurrs at different altitude (here just shows one case 2015-02-02 used in the Table 1, see the figure at

http://lidar.erau.edu/data/nalidar/plots/2015/20150202_Dmerge_15min_0.5km_90s_20_p.jpg)

Thanks for your question. With Bernoulli's equation for ideal fluid：

$$\begin{aligned} \frac{\partial u}{\partial t} &= -\frac{1}{\rho}\frac{\partial p'}{\partial x} \\ \frac{\partial w}{\partial t} &= -\frac{1}{\rho}\frac{\partial p'}{\partial z} \\ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0 \end{aligned} \qquad (3^{***})$$

Its boundary conditions are:

$$(w|_{z=0} = 0, (\frac{\partial p'}{\partial t} - \rho g w)|_{z=h} = 0. \qquad (4^{***})$$

Where u represents the horizontal velocity and w represents the vertical velocity, p' is pressure.

Let $\quad \boldsymbol{\mathcal{L}} \equiv \frac{\partial}{\partial t}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2})$

Eq. (3***) is written as

$$\mathcal{L}w = 0 \tag{5***}$$

Let

$$w = W(z)e^{i(kx-\omega t)}, \tag{6***}$$

Substitute into equation (5***)

$$\frac{d^2W}{dz^2} - k^2W = 0 \tag{7***}$$

consequently,

$$W(z) = Ae^{kz} + Be^{-kz} \tag{8***}$$

where A and B are constants.

Eq. (8***) Substitute into Eq. (5***), get:

$$w = (Ae^{kz} + Be^{-kz})e^{i(kx-\omega t)} \tag{9***}$$

Then, put Eq. (9***) into Eq. (3***)

$$p' = \frac{i\omega\rho}{k}(Ae^{kz} - Be^{-kz})e^{i(kx-\omega t)}, \tag{10***}$$

According to the lower boundary conditions：

$$B = -A; \tag{11***}$$

According to the upper boundary conditions：

$$\{(\frac{\omega^2}{k} - g)e^{kh} - (\frac{\omega^2}{k} + g)e^{-kh}\}A = 0. \tag{12***}$$

So

$$\omega = \sqrt{gk\tanh(kh)}. \tag{13***}$$

In shallow water conditions,

$$\begin{aligned} \omega &= \sqrt{gk[kh - \tfrac{1}{3}(kh)^3]} \approx \sqrt{k^2c_0^2(1 - \tfrac{1}{3}k^2H^2)} \\ &= kc_0(1 - \tfrac{1}{6}k^2H^2) = kc_0 - \tfrac{1}{6}k^3c_0H^2. \end{aligned} \tag{14***}$$

Eq. (14***) is the dispersion relation.

H. Ikezi et al. 's research (Ikezi et al., 1970) on solitons shows that they have properties similar to particle collisions, an apparent linear interaction (two peaks temporarily merging into one larger peak) is observed when the two solitons pass through each other from opposite directions. Observations of Na atoms in the mesosphere also frequently show this characteristic, and our hypothesis is that the Nas duration in a relatively short period of time may be the result of the interaction of two or more solitons. Since lidar observations are made only at a certain location, it is unrealistic to describe the possible collision process from the available data. The following figure shows several consecutive Nas events that may support our view.

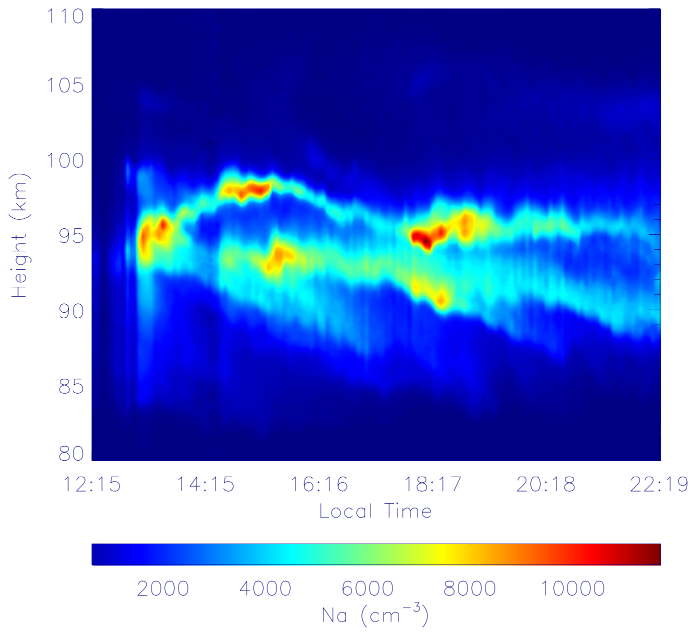

Figure 2***

8. What is the value of "the fluid depth h" used for different cases because this is required to calculate the depth of wave d?

Thanks for the valuable comment. Grimshaw et al. pointed out that different nonlinear evolution equations derived by applying different boundary conditions in atmospheric media, and KdV equation can be derived when the boundary conditions are set to rigid cover (Grimshaw, 2002). Crook believed that for any real atmospheric region with long wave and enough drastic stability changes, it could be regarded as the rigid cap hypothesis, that is, the KdV equation was derived (crook, 1988; crook, 1986). However, due to the limitation of observation conditions, we cannot give the numerical value of the fluid layer depth, but it is natural to infer that the isolated waves observed should be related to the boundary between the stable and unstable regions of the atmosphere. Fortunately, the stability of atmospheric regions can be reflected by Richardson Numbers, which may help us to derive some information about the depth of the fluid layer.

9. My other major concern is the lack of the explaination for sporadic Na Layer formation in the current version of the manuscript, which seems to me it still unclear why solitary wave causes the sporadic Na layer. If we look at one case used in the current manuscript, for example their Figure 3, there is strong correlation of Nas layer (Figure 3a) with zonal mean wind shear in Figure3f where Richardson number is calculated from the Eqauation 36, which has the zonal mean wind changes with altitude from the lidar data. Of course, this suggests that atmospheric waves are related to the Nas formation, but how the authors can attribute it to solitary waves, instead of gravity waves or tidal waves etc. This may be not true because the authors have not applied a

similar analysis for the the neutral Na data without Nas layer. If the result using the neutral Na data by ignoring Nas is similar as presented in the current manuscript, then that would indicate the current conclusion is wrong. To be specified, what the results will look like if the authors apply the same method to the neutral Na data excluding Nas layer? I understand that may be tough since there are some creteria to be met (for example, set the Na concentration and layer depth).

Thanks for the comment. Absolutely this paper does not hope to invoke a new mechanism for the source of the Na_S, it just focuses on the time series of fine structure. The authors always believe that the source of sodium atoms comes from the ion-molecule reactions or dynamic effects. The results from our data processes indicate Na_S could be a possible tracer for nonlinear wave studies through its time series. The fine structure of Na_S exhibits distinct wave fluctuations (shown as Figure 4 in the revised manuscript) and wing-like features (shown as Figure 2d). And as we know the C structure billow-like feature of Na_S (shown as the following image) could be hardly explained by only the source of sodium atoms. As a result, the fine structure of the Na_S could be a possible tracer to study the wave fluctuation and atmospheric instability.

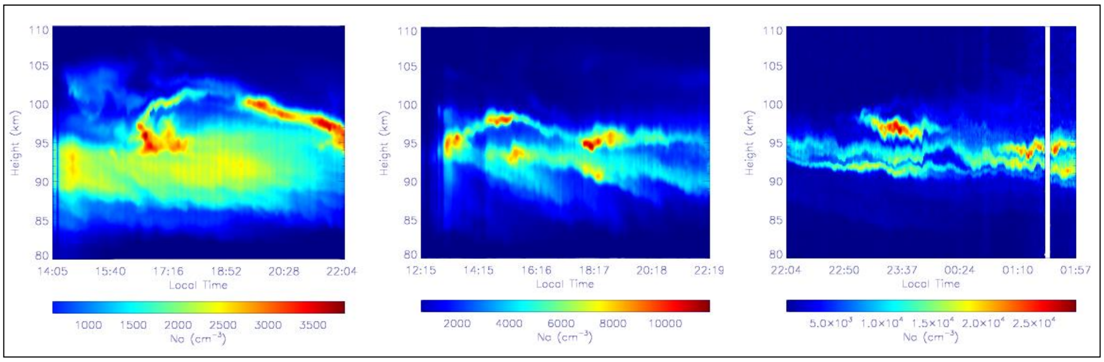

Figure 3***

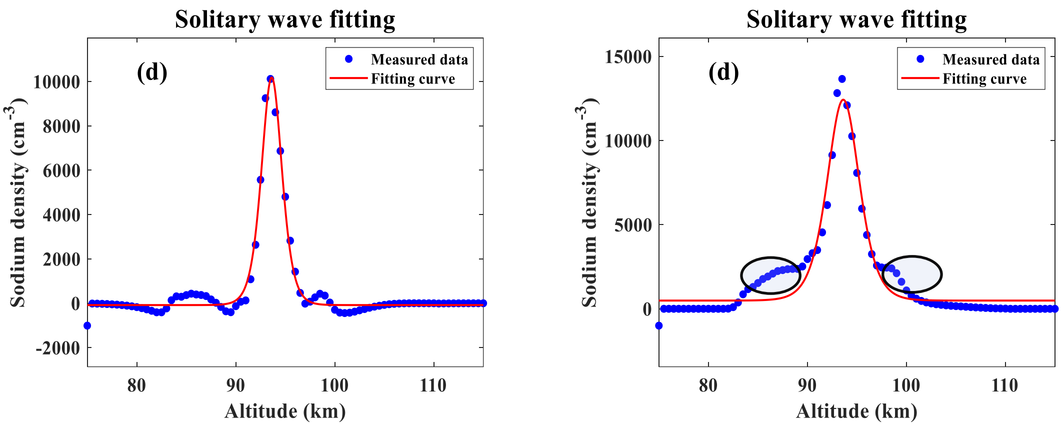

Figure 4***

The left of Figure 4*** is the fitted solitary wave image obtained according to our data processing method, and the right of Figure 4*** is the image obtained by applying the same method to the neutral Na data excluding Na_S layer. According to the mode you describe, we cannot account for the anomalies circled in the picture on the right. Therefore, our view is that the method of acquiring background Na density can be improved, but this process is indispensable in our data processing process.

11. It has been also widely accepted ion–molecule chemistry in plasma layers is the major mechanism for producing Nas layers at different latitudes, which based on the work from Cox and Plane (1998) (i.e., downward motion of sporadic plasma layer Es as a source of Nas formation by neutralized Na+ via an ion–molecule mechanism). This has been tested and supported from the observations including Na lidar measurements from different locations. Can you do a similar to see if this mechanism won't explain the Nas layer occurrs over Ande station?

The authors have never rejected the idea that ionic molecular chemistry near the mesosphere is one of the causes of Na_S. The main purpose of our work is to explore the evolution of a class of Na_S with large intensity and long duration and the relationship between these Na_S and atmospheric fluctuations and instability.

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