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Technical note: On comparing greenhouse gas emission metrics

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Abstract. Many metrics for comparing greenhouse gas emissions can be expressed as an instantaneous Global Warming Potential multiplied by the ratio of airborne fractions calculated in various ways. The Forcing Equivalent Index (FEI) provides a specification for equal radiative forcing at all times at the expense of generally precluding point by point equivalence over time. The FEI can be expressed in terms of asymptotic airborne fractions for exponentially growing emissions. This provides a reference against which other metrics can be compared.

Four other equivalence metrics are evaluated in terms of how closely they match the timescale dependence of FEI, with methane, referenced to carbon dioxide, used as an example. The 100-year Global Warming Potential over-estimates the long-term role of methane while metrics based on rates of change over-estimate the short-term contribution. A recently-proposed metric, based on differences between methane emissions 20 years apart, provides a good compromise. Analysis of the timescale dependence of metrics, expressed as Laplace transforms, leads to an alternative metric that gives closer agreement with FEI at the expense of considering methane over longer time periods.

The short-term behaviour, which is important when metrics are used for emissions trading, is illustrated with simple examples for the four metrics.

1 Introduction

Anthropogenic contributions to global climate change come from a range of so-called greenhouse gases. Comparisons between them have been facilitated by defining emission equivalence relations (which we denote by $\equiv$), usually using CO_2 as a reference.

The climatic influence of greenhouse gases is commonly represented in terms of radiative forcing, F, expressed in terms of M_{X}, the atmospheric content of gas X, with the perturbations linearised as

$$\Delta F = a_{\mathrm{X}} \Delta M_{\mathrm{X}} \tag{1}$$

Equivalence relations between sources of greenhouse gases are complicated because various gases are lost from the atmosphere on a range of different timescales. This behaviour is often represented using linear response functions, where the

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response function, $R_\mathrm{X}(t)$, represents the proportion of ΔS_X, the perturbation in emissions of constituent X, that remains in the atmosphere after time t. Thus the mass perturbation, ΔM_X, is given as a convolution integral:

$$\Delta M_\mathrm{X}(t) = \int_0^t R_\mathrm{X}(t-t')\,\Delta S_\mathrm{X}(t')\,dt' \tag{2}$$

The outline of this note is as follows. In Section 2 we show how the prescription by Wigley (1998), which gives exact equivalence in radiative forcing between different time histories of emissions, may be elegantly expressed in terms of Laplace transforms. In Section 3, we adapt this representation to other metrics of emission equivalence, and use it as inspiration for a new metric with a single adjustable parameter which accurately approximates equivalence in radiative forcing over timescales from decades to multiple centuries. In Section 4, we compare the different metrics in the time domain, and we conclude in Section 5. An appendix lists the notation.

2 Metrics: FEI

Wigley (1998) defined an equivalence between emission histories, termed the Forcing-Equivalent Index (FEI). Two emission histories are FEI-equivalent if they lead to equivalent forcing at all times. In most cases, this requirement precludes point-by-point emission equivalence at all times.

Equivalent radiative forcing over all time from perturbations ΔS_X and ΔS_Y in the emissions of gases X and Y requires:

$$a_\mathrm{Y}\int_0^t R_\mathrm{Y}(t-t')\,\Delta S_\mathrm{Y}(t')\,dt' = a_\mathrm{X}\int_0^t R_\mathrm{X}(t-t')\,\Delta S_\mathrm{X}(t')\,dt' \qquad \text{for all } t \tag{3}$$

as the condition for

$$\Delta S_\mathrm{Y}(t) \underset{\mathrm{FEI}}{\equiv} \Delta S_\mathrm{X}(t) \tag{4}$$

Subject to the conditions of linearity, this equivalence defines exact equality of radiative forcing. However it is an equivalence for emission profiles and not for instantaneous values.

A special case of FEI-equivalence (e.g. Enting, 2018) is when ΔS_X and ΔS_Y both grow exponentially, with growth rate α and amplitudes c_X and c_Y at $t = 0$. Exponential growth has

$$\Delta M_\mathrm{X}(t) = \int_{-\infty}^t R_\mathrm{X}(t-t')\,c_\mathrm{X}\exp(\alpha t')\,dt' = c_\mathrm{X}\exp(\alpha t)\int_0^\infty R_\mathrm{X}(t'')\exp(-\alpha t'')\,dt'' \tag{5}$$

The integral on the right is $\tilde{R}_\mathrm{X}(p)$, the Laplace transform of $R_\mathrm{X}(t)$, evaluated at $p = \alpha$. Interpreting these relations in terms of Laplace transforms can help clarify the different forms of equivalence metrics in the general case.

As a Laplace transform, the condition for FEI-equivalence is defined by the transform of (3):

$$a_\mathrm{Y}\Delta\tilde{S}_\mathrm{Y}(p)\tilde{R}_\mathrm{Y}(p) = a_\mathrm{X}\Delta\tilde{S}_\mathrm{X}(p)\tilde{R}_\mathrm{X}(p) \tag{6}$$

giving

$$\frac{a_{\rm Y}}{a_{\rm X}}\frac{\tilde{R}_{\rm Y}(p)}{\tilde{R}_{\rm X}(p)}\Delta\tilde{S}_{\rm Y}(p) \underset{\rm FEI}{\equiv} \Delta\tilde{S}_{\rm X}(p) \tag{7}$$

In this expression $\tilde{R}_{\rm Y}(p)/\tilde{R}_{\rm X}(p)$ is the Laplace transform of an integro-differential operator that, in the time domain, acts on $\Delta S_{\rm Y}(t)$. Differentiation of (5) shows that, for exponentially growing emissions, the asymptotic airborne fraction of a gas X is $\alpha\tilde{R}_{\rm X}(\alpha)$ (e.g. Enting, 1990) and so the FEI curve can be defined as the ratio of asymptotic airborne fractions.

The plot in Figure 1 describes the specific case of methane, CH_4, referenced to carbon dioxide, CO_2. The solid line, denoted FEI, can be interpreted in several different, but mathematically equivalent, ways:

- it gives the ratio that leads to FEI-equivalence in the special case of exponentially growing emissions;
- it is the ratio of asymptotic airborne fractions for exponential growth, shown as a function of growth rate;
- it is the Laplace transform of an operator that acts on methane emission functions to produce FEI-equivalent CO_2 emissions.

3 Comparison of metrics

The examples given here compare four different metrics, again for the case of CH_4 referenced to CO_2, benchmarking them against FEI. In these calculations, the response used for CO_2 is the multi-model mean from (Joos et al., 2013, Table 5) and the response of CH_4 described by a 12.4 year perturbation lifetime. In each case, these represent the response to small perturbations about current conditions, reflecting our interest in the use of metrics for trade-offs, reporting and target-setting.

The calculations were developed for methane emissions from active biological sources. For fossil methane, an additional CO_2 contribution from the oxidation of CH_4 should be included.

3.1 Global Warming Potential

The Global Warming Potential (GWP) with time horizon H defines an equivalence (denoted $\underset{\rm GWP}{\equiv}$) for component Y given by

$$\Delta S_{\rm CO2}(t) \underset{\rm GWP}{\equiv} \mathbf{GWP}_H\,\Delta S_{\rm Y}(t) \tag{8}$$

where

$$\mathbf{GWP}_H = \frac{a_{\rm Y}}{a_{\rm CO2}}\frac{H^{-1}\int_0^H R_{\rm Y}(t')\,dt'}{H^{-1}\int_0^H R_{\rm CO2}(t')\,dt'} \quad \text{for gas Y} \tag{9}$$

Although (9) is usually written without the H^{-1} factors, in the form above the numerator and denominator correspond to the airborne fractions of Y and $\mathrm{CO_2}$, averaged over the time horizon H, and multiplied by the factor $a_\mathrm{Y}/a_\mathrm{CO2}$ which corresponds to GWP_0, the $H \to 0$ limit of GWP_H. This factor can be called the instantaneous GWP.

GWP_{100}, the GWP with the time horizon $H = 100$ years, has become the standard for greenhouse gas equivalence in international agreements.

For $\mathrm{CH_4}$, the equivalence is

$$\Delta S_\mathrm{CO2}(t) \underset{\mathrm{GWP:100}}{\equiv} \mathsf{GWP}_{100}\,\Delta S_\mathrm{CH4}(t) \tag{10}$$

where all use of GWP in what follows will specifically refer to $\mathrm{CH_4}$. Relation (10) corresponds to using

$$\tilde{R}_\mathrm{CH4}(p)/\tilde{R}_\mathrm{CO2}(p) \approx \mathsf{GWP}_{100}/\mathsf{GWP}_0 \tag{11}$$

However, this definition of equivalence has long been known to be poor (e.g. Reilly et al., 1999), especially for emission profiles approaching stabilisation of concentrations.

For $H > 100$ the approximation

$$\tilde{R}_\mathrm{CH4}(p)/\tilde{R}_\mathrm{CO2}(p) \approx \mathsf{GWP}_{H=1/p}/\mathsf{GWP}_0 \tag{12}$$

is quite close, suggesting that the appropriate time horizon should match the e-folding time of emissions (Enting, 2018).

3.2 Derivative

Several studies (Smith et al., 2012; Lauder et al., 2013) suggested that for short-lived gases such as $\mathrm{CH_4}$, changes in emissions in the short-lived gases should be related to one-off $\mathrm{CO_2}$ emissions. This suggests a metric of the form:

$$\Delta S_\mathrm{CO2}(t) \underset{\mathrm{DERIV}}{\equiv} 100\,\mathsf{GWP}_{100}\frac{d}{dt}\Delta S_\mathrm{CH4}(t) \tag{13}$$

or (as a Laplace transform):

$$\tilde{R}_\mathrm{CH4}(p)/\tilde{R}_\mathrm{CO2}(p) \approx 100\,p\,\mathsf{GWP}_{100}/\mathsf{GWP}_0 \tag{14}$$

Subsequently, the search for an improved metric, termed GWP*, has been the subject of extensive studies undertaken by Allen and co-workers: (Allen et al., 2016, 2018; Jenkins et al., 2018; Cain et al., 2019; Collins et al., 2019; Lynch et al., 2020). These studies have included cases defined by linear combinations of the derivative metric and the GWP. Such cases are not shown in the transform domain illustrated in Figure 1, but correspond to linear functions of p that do not pass through the origin.

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3.3 Difference

A recent proposal for an improved GWP* (Cain et al., 2019) proposes the equivalence:

$$\Delta S_{\mathrm{CO2}}(t) \underset{\mathrm{DIFF}}{\equiv} \mathsf{GWP}_{100}\left[4\Delta S_{\mathrm{CH4}}(t) - 3.75\Delta S_{\mathrm{CH4}}(t-20)\right] \tag{15}$$

The Laplace transform, as shown in Figure 1, is derived using the generic result that a time-shift by T corresponds to multiplying the Laplace transform by $\exp(-pT)$, giving:

$$\tilde{R}_{\mathrm{CH4}}(p)/\tilde{R}_{\mathrm{CO2}}(p) \approx \mathsf{GWP}_{100}/\mathsf{GWP}_{0} \times [4 - 3.75\exp(-20p)] \tag{16}$$

3.4 Reduced model

When, as is done here, the response functions are expressed as a sum of exponentially decaying functions of time, the Laplace transform becomes a sum of partial fractions of the form $\alpha/(p+\beta)$ so that the combination is a ratio of polynomials in p. Thus the FEI ratio will also be a ratio of polynomials which can in turn be re-expressed as a sum of partial fractions, giving an exact, but complicated, form for the FEI relation. Studies in a number of fields such as electronic engineering (e.g. Feldman and Freund, 1995) have noted that such expressions can often be usefully approximated by lower order expressions. For emission equivalence, it is only practical to use very low order approximations for such a reduced model.

As shown in Figure 1, a close fit to FEI can be obtained with the reduced model (RM) given by

$$\tilde{R}_{\mathrm{CH4}}(p)/\tilde{R}_{\mathrm{CO2}}(p) \approx \frac{p}{p+b} \tag{17}$$

with $b = 0.035$.

This gives an equivalence:

$$\frac{a_{\mathrm{CH4}}}{a_{\mathrm{CO2}}}\frac{p}{p+b}\Delta\tilde{S}_{\mathrm{CH4}}(p) \underset{\mathrm{RM}}{\equiv} \Delta\tilde{S}_{\mathrm{CO2}}(p) \tag{18}$$

In the time domain, (18) becomes:

$$\frac{a_{\mathrm{CH4}}}{a_{\mathrm{CO2}}}\int_{0}^{t}\exp(-b(t-t'))\Delta\dot{S}_{\mathrm{CH4}}(t')\,dt' + \frac{a_{\mathrm{CH4}}}{a_{\mathrm{CO2}}}\Delta S_{\mathrm{CH4}}(t=0)\exp(-bt) \underset{\mathrm{RM}}{\equiv} \Delta S_{\mathrm{CO2}}(t) \tag{19}$$

where $\Delta\dot{S}_{\mathrm{CH4}}$ denotes the rate of change in the perturbation to CH_4 emissions.

This expresses the CO_2-equivalent of CH_4 as a weighted average of the CH_4 emission growth rate. Consequently, the metric retains the property that constant emissions of CH_4 are treated as equivalent to zero CO_2 emissions as in ‘derivative’ metrics (Smith et al., 2012; Lauder et al., 2013). The parameter b can be chosen to match other metrics. The value $b = 0.035$ is chosen so that for emissions with 1% per annum growth rate the RM metric closely matches the 100-year GWP.

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For specific calculations it may be more appropriate to represent this metric as

$$\frac{a_{\mathrm{CH4}}}{a_{\mathrm{CO2}}}\left[\Delta S_{\mathrm{CH4}}(t) - b\int_0^t \exp(-b(t-t'))\Delta S_{\mathrm{CH4}}(t')\,dt'\right] \underset{\mathrm{RM}}{\equiv} \Delta S_{\mathrm{CO2}}(t) \tag{20}$$

Relation (20) is derived from (19) using integration by parts (or equivalently by putting $p/(p+b) = 1 - b/(p+b)$). It has the advantage that it is expressed in terms of emissions rather than their rates of change.

Equation 20 defines the reduced model equivalence as a difference between present emissions and a weighted average of past emissions. When considered in terms of frequency f (by setting $p = 2\pi f \times \sqrt{-1}$) this avoids the frequency aliasing that occurs with the 'difference' metric for periods of 20 years or integer fractions thereof (see supplementary information).

The equivalence relation (18) can also be re-written as

$$p\Delta\tilde{S}_{\mathrm{CH4}}(p) \underset{\mathrm{RM}}{\equiv} \frac{a_{\mathrm{CO2}}}{a_{\mathrm{CH4}}}(p+b)\Delta\tilde{S}_{\mathrm{CO2}}(p) \tag{21}$$

This defines an equivalence between the rate of change of CH_4 emissions and a combination of rate of change of CO_2 emissions (as in GWP) and current CO_2 emissions (as in the derivative-based equivalences suggested by Smith et al. (2012) and Lauder et al. (2013)).

4 Comparisons in the time domain

Many previous studies of metrics have concentrated on global-scale calculations over the long term. When metrics are used for emissions trading, the behaviour at shorter timescales becomes important. This can be analysed by taking a notional CH_4 emission profile and calculating the resulting CH_4 concentrations. This is then compared to the CO_2 concentrations that result from the notionally equivalent CO_2 emissions.

Figure 2 shows a CH_4 source perturbation with a rapid increase from zero to a fixed emission rate, and the CO_2-equivalent emissions as determined by the various equivalence metrics. Figure 3 shows the CH_4 concentration resulting from the methane emission and the CO_2 concentration resulting from the various CO_2-equivalent emissions. In Figures 2 and 3, the relative scaling of the axes is given by $a_{\mathrm{CH4}}/a_{\mathrm{CO2}}$ so that forcing can be compared directly.

The results clearly show the failings of the 100-year GWP for defining emission equivalence in this type of context. The forcing from GWP-equivalent CO_2 initially lags well behind the actual forcing from CH_4 but in the long term it continues to increase indefinitely long after the forcing from on-going CH_4 emissions has stabilised. Compared to this behaviour, the 'derivative' metric based on rates of change of CH_4 emissions is a great improvement. However, the CO_2-equivalent forcing initially exceeds the actual forcing from CH_4 and in the long-term drops below the CH_4 forcing. The difference metric from Cain et al. (2019) provides a CO_2-equivalent forcing that follows the actual CH_4 forcing more closely with only a slight shortfall in the longer term. The increase after several centuries reflects a contribution to the metric that corresponds to 0.25 times the 100-year GWP.

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The CO_2-equivalence derived from the reduced model follows the actual CH_4 forcing particularly closely as would be expected given the close agreement when the relations are expressed as Laplace transforms.

The nature of the FEI relation precludes close matches in forcing from instantaneous relations between CH_4 and CO_2 emissions. The 'difference' and 'reduced model' metrics relate CO_2 equivalents to the past history of CH_4 emissions. For a specific case, Lauder et al. (2013) suggested an approximate equivalence to changes in methane emissions balanced by an ongoing future CO_2 uptake from growing trees.

We briefly note that there are trade-offs between different metrics that are difficult to balance. The goal of defining emissions equivalence is to allow for emissions of different greenhouse gases to be substituted for each other, so that a given radiative forcing target can be achieved for the least economic cost. If the metric of emissions equivalence is too complex, as it is for FEI, then it may be difficult or impossible for an effective trading scheme to be implemented. If the metric is inaccurate at the relevant timescales, as is the case for GWP100, then the 'least cost' emissions pathway may overshoot the radiative forcing target, especially as stabilisation in radiative forcing is approached.

5 Concluding summary

FEI-equivalence is defined by equivalent radiative forcing at all times. Applying this to different gases constrains emissions over all time.

In the special case of exponentially growing emissions, FEI-equivalence can be achieved when the emissions are scaled by the instantaneous (0 time horizon) GWP, multiplied by the ratio of the asymptotic airborne fractions.

This ratio depends on the e-folding growth rate. Various emission metrics can be compared in terms of how well they match this ratio at the range of relevant timescales.

GWP treats this ratio as a constant, defining GWP_H as the instantaneous GWP multiplied by the ratio of average airborne fractions over the time horizon, H. For CH_4, referenced to CO_2, this means that GWP over-estimates the CH_4 contribution for growth rates less than $1/H$ and under-estimates the CH_4 contribution from shorter timescales.

Metrics relating CO_2-equivalence to rates of change of CH_4 emissions are treating the ratio of airborne fractions as proportional to the e-folding rate. This can provide a good representation of long-term behaviour relevant for stabilisation, but over-estimates the role of CH_4 on the shorter timescales relevant for emission trading

The metric proposed by Cain et al. (2019) matches the FEI requirement over a wide range of timescales, from decades to millennia, by comparing CH_4 emissions over a 20 year interval.

Simple metrics that give closer fits can be obtained as reduced model approximations to FEI-equivalence. This is achieved at the expense of comparisons involving longer time periods.

Code availability. An annotated listing of the R code used to perform the calculations and generate the figures is included as supplementary information.

Appendix: Notation

Laplace transforms are denoted by the tilde notation with $\tilde{R}(p)$ as the Laplace transform of $R(t)$.

Equivalence relations are denoted by $\equiv$ with particular cases identified, e.g. $\underset{\text{GWP}}{\equiv}$.

a_{X} Radiative forcing per unit mass of constituent X.

b e-folding time in reduced model equivalence relation.

$F_{\mathrm{X}}(t)$ Radiative forcing of constituent X.

GWP, **GWP**$_H$ Global warming potential for CH_4 (unless otherwise specified), for time horizon H.

H Time horizon for GWP.

$M_{\mathrm{X}}(t)$ Atmospheric content of constituent X. Perturbation is $\Delta M_{\mathrm{X}}(t)$.

p Argument of Laplace transform. Equivalent to e-folding rate when comparing exponentially growing emissions.

$R_{\mathrm{X}}(t)$ Atmospheric response function for constituent X.

$S_{\mathrm{X}}(t)$ Anthropogenic emission of constituent X. Perturbation is $\Delta S_{\mathrm{X}}(t)$.

t Time

X,Y Labels for constituent. Specific cases CO_2, CH_4.

α e-folding rate of exponentially growing emissions.

$\delta(t)$ Delta 'function'. Instantaneous unit pulse. The notional derivative of unit step function.

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References

Allen, M. R., Fugelstvedt, J. S., Shine, K. P., Reisinger, A., Pierrehumbert, R. T., and Forster, P. M.: New use of global warming potentials to compare cumulative and short-lived climate pollutants, Nature Climate Change, 6, 773–777, 2016.

Allen, M. R., Shine, K. P., Fugelstvedt, J. S., Millar, R. A., Cain, M., Frame, D. J., and Macey, A. M.: A solution to the misrepresentation of CO_2-equivalent emissions of short-lived climate pollutants under ambitious mitigation, npj: Climate and Atmospheric Science, 2018.

Cain, M., Lynch, J., Allen, M. R., Fugelstvedt, J. S., Frame, D. J., and Macey, A. M.: Improved calculation of warming-equivalent emissions for short-lived climate pollutants, npj: Climate and Atmospheric Science, 2:29, https://doi.org/10.1038/s41612-019-0084-4, 2019.

Collins, W., Frame, D. J., Fugelstvedt, J. S., and Shine, K. P.: Stable climate metrics for emissions of short- and long-lived species — combining steps and pulses, Environ. Res. Lett. (in press), 2019.

Enting, I. G.: Ambiguities in the calibration of carbon cycle models, Inverse Problems, 6, L39–L46, 1990.

Enting, I. G.: Metrics for greenhouse gas equivalence, in: Encyclopedia of the Anthropocene, edited by Dellasala, D. A. and Goldstein, M. I., pp. 467–471, Elsevier, 2018.

Feldman, P. and Freund, R. W.: Efficient linear circuit analysis by Pade approximation via the Lanczos process, IEEE Trans Computer aided design of integrated circuits and systems, 14, 639–649, 1995.

Jenkins, S., Millar, R. J., Leach, N., and Allen, M. R.: Framing climate goals in terms of cumulative CO_2-forcing-equivalent emissions, Geophys. Res. Lett., 45, 2795–2804, https://doi.org/10.1002/2017GL076173, 2018.

Joos, F., Roth, R., Fuglestvedt, J. S., Peters, G. P., Enting, I. G., von Bloh, W., Brovkin, V., Burke, E. J., Eby, M., Edwards, N. R., Friedrich, T., Frölicher, T. L., Halloran, P. R., Holden, P. B., Jones, C., Kleinen, T., Mackenzie, F., Matsumoto, K., Meinshausen, M., Plattner, G.-K., Reisinger, A., Segschneider, J., Shaffer, G., Steinacher, M., Strassmann, K., Tanaka, K., Timmermann, A., and Weaver, A. J.: Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis, Atmos. Chem. Phys., 13, 2793–2825, 2013.

Lauder, A. R., Enting, I. G., Carter, J. O., Clisby, N., Cowie, A. L., Henry, B. K., and Raupach, M.: Offsetting methane emissions — An alternative to emission equivalence metrics, International Journal of Greenhouse Gas Control, 12, 419–429, 2013.

Lynch, J., Cain, M., Pierrehumbert, R., and Allen, M.: Demonstrating GWP* as a means of reporting warming-equivalent emissions that captures the contrasting impacts of short- and long-lived climate pollutants, Environ. Res. Lett., 15(4), 044 023, https://doi.org/10.1088/1748-9326/ab6d7e, 2020.

Reilly, J., Prinn, R., Harnisch, J., Fitzmaurice, J., Jacoby, H., Kicklighter, D., Melillo, J., Stone, P., Sokolov, A., and Wang, C.: Multi-gas assessment of the Kyoto Protocol, Nature, 401, 549–555, 1999.

Smith, S. M., Lowe, J. A., Bowerman, N. H. A., Gohar, L. K., Huntingford, C., and Allen, M. R.: Equivalence of greenhouse-gas emissions for peak temperature limits, Nature Climate Change, 2, 535–538, 2012.

Wigley, T. M. L.: The Kyoto Protocol: CO_2, CH_4 and climate implications, Geophys. Res. Lett., 25, 2285–2288, 1998.

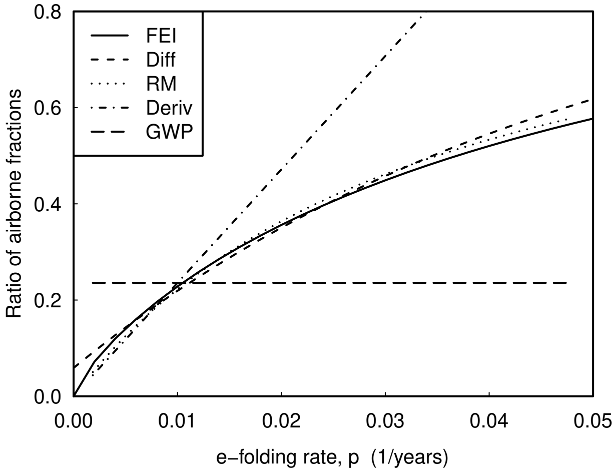

Figure 1. Ratio of airborne fractions for CH_4 relative to CO_2 as defined or assumed for various metrics. The solid curve shows the FEI which acts as a reference. The GWP line treats this ratio as independent of timescale (eqn 11); the chain line for the 'Deriv' case treats the timescale dependence as proportional to the inverse timescale (eqn 14); the shorter dashes of the Diff curve (eqn 16) more closely approximate FEI. The dotted line, 'RM', is an empirical 'reduced model' approximation (eqn 18) to FEI. These curves can also be interpreted as the Laplace transforms of the operations that define the equivalence in the time domain.

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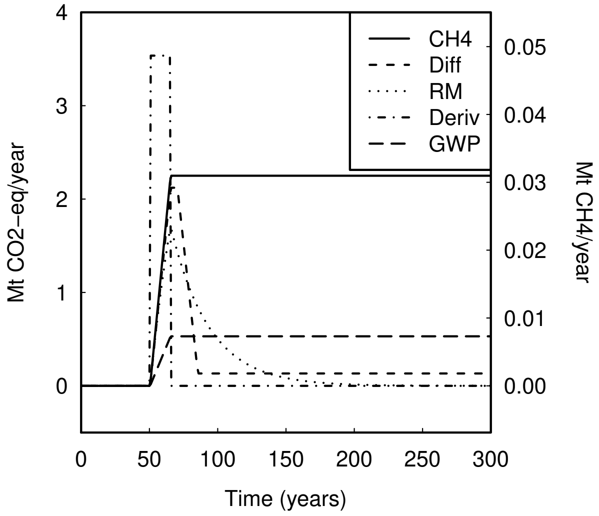

Figure 2. A CH_4 source representing an increase, over 15 years, from zero to a constant (solid line) and the CO_2-equivalent sources as defined by the various metrics described in Section 3. The relative scaling of the CH_4 and CO_2 axes is $a_{\mathrm{CH4}}/a_{\mathrm{CO2}}$.

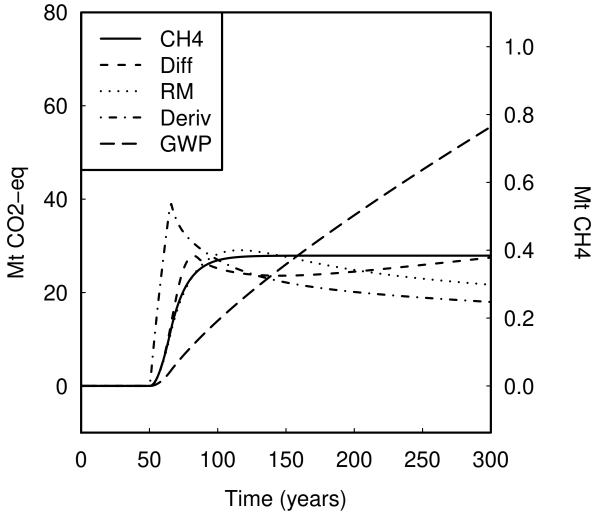

Figure 3. CH_4 concentrations from source shown in Figure 2 (solid line) and the CO_2 concentrations resulting from the CO_2 concentrations resulting from the equivalent CO_2 sources, as shown in Figure 2. The relative scaling of the axes is $a_{\mathrm{CH4}}/a_{\mathrm{CO2}}$ so that the radiative forcing can be compared directly.