Atmospheric
Chemistry
and Physics
Discussions

Interactive comment on "Technical note: On comparing greenhouse gas emission metrics" *by* Ian Enting and Nathan Clisby

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Author comment: Proposed appendix

[Note: equation and section numbers refer to the version published in *At. Chem. Phys Discuss*].

The Laplace transform provides a natural formalism for analysing causal initial value systems. However Fourier transforms and Fourier analysis has wide familiarity and can be used to describe our results.

For a periodic variation with exponentially increasing amplitude, equation (5) generalises to

C1

$$\int_{-\infty}^{t} \exp(\alpha t' + i\omega t') R(t - t') \, dt' = \exp(\alpha t + i\omega t) \int_{0}^{\infty} R(t') \exp(-\alpha t' - i\omega t') \, dt'$$

For R_{CO2}, this relation requires $\alpha > 0$ in order to have the lower limit of the left-hand integral and the upper limit of the right hand integral defined. The $\alpha \to 0$ limit shows the relation between the Laplace transform and the Fourier transform, which, for functions with $R(t) = 0$ for $t < 0$, is given by the integral on the right.

Section 3 [as modified in response to review comments] noted that metric transformations defined by

$$a_{\text{CO2}} \, \tilde{S}_{\text{CO2-eq}}(p) = a_{\text{CH4}} \, \tilde{\Psi}(p) \, \tilde{S}_{\text{CH4}}(p)$$

can be assessed in radiative forcing terms by the accuracy of the approximation

$$a_{\text{CO2}} \, \tilde{R}_{\text{CO2}}(p) \, \tilde{S}_{\text{CO2-eq}}(p) = a_{\text{CH4}} \, \tilde{R}_{\text{CO2}}(p) \, \tilde{\Psi}(p) \, \tilde{S}_{\text{CH4}}(p) \approx a_{\text{CH4}} \, \tilde{R}_{\text{CH4}}(p) \, \tilde{S}_{\text{CH4}}(p)$$

which reduces to comparing

$$\tilde{R}_{\text{CO2}}(p) \, \tilde{\Psi}(p) \approx \tilde{R}_{\text{CH4}}(p)$$

where for FEI equivalence, the approximation becomes exact equality.

A frequency domain interpretation can be obtained by putting $p = 2\pi i f$. In these terms, the metric transformation is acting like a frequency equaliser in an audio system.

The phases of the complex numbers in the relations above capture the phase shifts for the various frequencies. For the present we show only the resulting amplitudes, given by the moduli, $|z|$ of the complex value, and ignore the phase (noting that the modulus of a product is the product of the moduli).

Figure 3 [now that old 3 becomes 2b] sets $p = 2i\pi f$ to evaluate the various cases considered in the paper, as functions of frequency f in cycles per year. It shows

- $|\tilde{R}_{\text{CH4}}(p)|$, the 'target' for FEI equivalence; the zero frequency value is the perturbation lifetime;

- $|\tilde{R}_{\text{CO}_2}(p)\tilde{\Psi}_{\text{GWP}}(p)|$, i.e. a multiple of the CO_2 response, growing indefinitely as frequency goes to zero;

- $|\tilde{R}_{\text{CO2}}(p)\tilde{\Psi}_{\text{Deriv}}(p)|$ which gives a better approximation over a wider range of frequencies;

- $|\tilde{R}_{\text{CO2}}(p)\tilde{\Psi}_{\text{Diff}}(p)|$ which gives a further improvement, but a notable discrepancy for cycles whose period is near the 20-year interval use in the difference calculation;

- $|\tilde{R}_{\text{CO2}}(p)\tilde{\Psi}_{\text{RM}}(p)|$ which gives a still closer fit of the range of frequencies shown.

Caption for Figure: Frequency response for the various cases of $|\tilde{R}_{\text{CO2}}(p)\,\tilde{\Psi}(p)|$ discussed above, compared to the actual frequency response, $|R_{CH4}(p)|$ to periodic CH_4 emissions (solid line), using $p = 2\pi i f$.
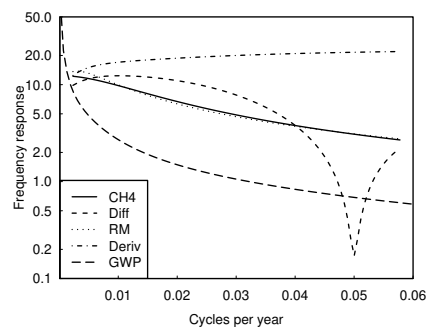
Fig. 1. See body of post for caption with Latex formatted,