

Supplement of

A mass-weighted atmospheric isentropic coordinate for mapping chemical tracers and computing inventories

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S1 Computation of equivalent potential temperature (θ_e) and dry air mass (M) of atmospheric field

Saturation vapor pressure ($P_{s,v}$, mbar) is computed following:

$$P_{s,v} = 0.06122 \cdot e^{\frac{17.67 \cdot T}{T+243.5}} \quad (S1)$$

10 where T is the temperature of air in unit of °C.

Water vapor mixing ratio (w, kg water vapor per kg dry air mass) is computed following Bolton (1980) as:

$$w = RH \cdot 0.622 \cdot \frac{P_{s,v}}{P - P_{s,v}} \quad (S2)$$

where RH is relative humidity in kg kg⁻¹, and P is barometric pressure in mbar.

Equivalent potential temperature, θ_e , is computed following Stull (2012) as:

$$15 \quad \theta_e = \left(T + \frac{L_v(T)}{C_{pd}} \cdot w \right) \cdot \left(\frac{P_0}{P} \right)^{\frac{R_d}{C_{pd}}} \quad (S3)$$

where T (K) is the temperature of air, R_d (287.04, J kg⁻¹ K⁻¹) is the gas constant for air, C_{pd} (1005.7 J kg⁻¹ K⁻¹) is the specific heat of dry air at constant pressure, P_0 (1013.25, mbar) is the reference pressure at the surface, and $L_v(T)$ is the latent heat of evaporation at temperature T. $L_v(T)$ is defined as 2406 kJ kg⁻¹ at 40 °C, and 2501 kJ kg⁻¹ at 0 °C and scales linearly with temperature.

20 The mass of air (M, kg) for each grid cell x is then calculated by using pressure (P), latitude (Φ), longitude (λ), radius of the earth (R, 6371 km) and gravity constant (g) following

$$M_x = \frac{\Delta P}{g} \cdot |\Delta \sin(\Phi) \cdot \Delta \lambda| \cdot R^2 \quad (S4)$$

where Δ represents the difference between two boundaries of each grid cell.

The gravity constant (g, kg m⁻²) is computed following Arora et al. (2011) as:

$$25 \quad g(\text{lat}, p) = g_0 \cdot (1 + 0.0053 \cdot \sin^2(\text{lat}) - 0.000006 \cdot \sin^2(2 \cdot \text{lat})) - 0.000003086 \cdot h \quad (S5)$$

where the reference gravity constant (g_0) is assumed to be 9.78046 m s⁻², and the height (h) in unit of m is computed from

$$P = P_0 \cdot e^{-\frac{h}{H}} \quad (S6)$$

where H is the scale height of the atmosphere and assumed to be 8400 m.

S2: Contribution of each heating term to the overall time variation of M_{θ_e}

The fractional contributions from different heating terms to the temporal variation of M_{θ_e} on seasonal and synoptic scales are computed by using a vector projection method (Graven et al., 2013). In this method, each heating term

35 $(\frac{\partial}{\partial t} M_{\theta_e}^i(\theta_e, t))$ is projected onto the sum of all the heating terms $(\frac{\partial}{\partial t} M_{\theta_e}(\theta_e, t))$ via:

$$x_i = \frac{\sum_t \left[\frac{\partial}{\partial t} M_{\theta_e}^i(\theta_e, t) \cdot \frac{\partial}{\partial t} M_{\theta_e}(\theta_e, t) \right]}{\sum_t \left[\frac{\partial}{\partial t} M_{\theta_e}(\theta_e, t) \cdot \frac{\partial}{\partial t} M_{\theta_e}(\theta_e, t) \right]} \quad (S7)$$

with

$$\frac{\partial}{\partial t} M_{\theta_e}(\theta_e, t) = \sum_i \frac{\partial}{\partial t} M_{\theta_e}^i(\theta_e, t) \quad (S8)$$

where the sum is over all time steps, and the mean of each $\frac{\partial}{\partial t} M_{\theta_e}^i(\theta_e, t)$ has been pre-subtracted (i.e.,

40 $\sum_t \frac{\partial}{\partial t} M_{\theta_e}^i(\theta_e, t) = 0$). The sum over x_i equals 1, but individual x_i can be either positive or negative and the absolute value can be either larger or smaller than 1.

Table S1: Number of data points of each airborne campaign transect for each simulation

Airborne Transect	Original	Equator to 30 °N	Poleward of 30 °N	Surface – 600 mbar	600 mbar – Trop.	Pacific Only	Medusa Coverage	Random 10 %	Random 5 %	Random 1 %
HIPPO1 SB	4837	1454	3383	1794	3043	4837	76	484	242	48
HIPPO2 SB	4665	1510	3155	1945	2720	4665	82	451	233	45
HIPPO2 NB	5508	2428	3080	2159	3349	5508	93	543	275	54
HIPPO3 SB	4439	1371	3068	2038	2401	4439	88	427	222	43
HIPPO3 NB	4086	1135	2951	1790	2296	4086	84	399	204	40
HIPPO4 SB	5491	1602	3889	2340	3151	5491	81	534	275	53
HIPPO4 NB	6411	3134	3277	3142	3269	6411	124	626	321	63
HIPPO5 SB	5538	1678	3860	2569	2969	5538	78	548	277	55
HIPPO5 NB	4715	1705	3010	2066	2649	4715	86	392	236	39
ATom1 SB	9832	2333	7499	3186	6646	9832	83	455	492	46
ATom1 NB	10685	3186	7499	3665	7020	0	59	893	534	89
ATom2 SB	11372	3909	7463	4057	7315	11372	84	1109	569	111
ATom2 NB	10741	3284	7457	3792	6949	0	91	1042	537	104
ATom3 SB	15143	3751	11392	4817	10326	15143	87	1460	757	146
ATom3 NB	14039	4173	9866	4764	9275	0	92	1362	702	136
ATom4 SB	13554	3683	9871	5249	8305	13554	84	1327	678	132
ATom4 NB	11995	3626	8369	4130	7865	0	89	1187	600	119

45 **References**

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