

Explanation: The red marks denote the changes to the equations which are marked in the proofreading file. These corrections do not change the results and conclusions.

- In eq (12), changes E to c . This is a misusing letter. Also, at line 25 in the proofreading, “The four parameters of E_{ext} indicate the extinction efficiencies ...” changed to “The four parameters of c_{ext} indicate the **Chebyshev polynomial values** of the extinction efficiencies ...”
- In eq (13), there is a type error which needs to exchange the subscripts of “01” and “10”. This exchange is also needed in a few following equations.
- In eq (15), there is a missing sum symbol.
- For the sentence between eq (16) and (17) in the proofreading, “Putting Eqs. (12) and (13) in to Eq. (11) leads to ” changed to “Putting Eqs. (12) and (13) in to Eq. (11) **and assuming the independence of scattering (absorbing) Chebyshev values on the imaginary (real) part** leads to ”

$$\delta c_{ext,z,k}(j) = \delta w_{00} \cdot c_{ext,00}(j) + \delta w_{01} \cdot c_{ext,01}(j) + \delta w_{10} \cdot c_{ext,10}(j) + \delta w_{11} \cdot c_{ext,11}(j) \quad (12)$$

$$\begin{aligned} \delta w_{00} &= (v-1)\delta u + (u-1)\delta v & \delta w_{10} &= (1-v)\delta u - u\delta v \\ \delta w_{01} &= (1-u)\delta v - v\delta u & \delta w_{11} &= u\delta v + v\delta u \end{aligned} \quad (13)$$

$$\begin{aligned} R_{mix,z,k} &= \sum_i^{n_{wet_aer}} R_i \cdot \frac{m_{i,z,k}}{\rho_i \cdot V_{wet,z,k}} & \delta R_{mix,z,k} &= \sum_i^{n_{wet_aer}} \frac{R_i}{\rho_i \cdot V_{wet,z,k}} \cdot \delta m_{i,z,k} \\ I_{mix,z,k} &= \sum_i^{n_{wet_aer}} I_i \cdot \frac{m_{i,z,k}}{\rho_i \cdot V_{wet,z,k}} & \delta I_{mix,z,k} &= \sum_i^{n_{wet_aer}} \frac{I_i}{\rho_i \cdot V_{wet,z,k}} \cdot \delta m_{i,z,k} \end{aligned} \quad (15)$$

$$\delta p_{ext,z,k} = [(v-1)\alpha_{sca,00} + (1-v)\alpha_{sca,10} - v\alpha_{sca,01} + v\alpha_{sca,11}]\delta u + [(u-1)\alpha_{abs,00} - u\alpha_{abs,10} + (1-u)\alpha_{abs,01} + u\alpha_{abs,11}]\delta v \quad (17)$$

$$\begin{aligned} \alpha_{sca,00} &= p_{sca,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot c_{sca,00}(j) & \alpha_{sca,01} &= p_{sca,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot c_{sca,01}(j) \\ \alpha_{sca,10} &= p_{sca,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot c_{sca,10}(j) & \alpha_{sca,11} &= p_{sca,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot c_{sca,11}(j) \end{aligned}$$

$$\begin{aligned}
\alpha_{abs,00} &= p_{abs,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot \mathbf{c}_{abs,00}(j) & \alpha_{abs,01} &= p_{abs,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot \mathbf{c}_{abs,01}(j) \\
\alpha_{abs,10} &= p_{abs,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot \mathbf{c}_{abs,10}(j) & \alpha_{abs,11} &= p_{abs,1,k} \cdot \sum_{j=1}^{n_{coef}} c_{ch}(j) \cdot \mathbf{c}_{abs,11}(j)
\end{aligned}
\tag{18}$$

$$\begin{aligned}
\frac{\delta\tau}{\delta m_{i,z,k}} &= \frac{\delta\tau_z}{\delta m_{i,z,k}} = \frac{\delta e_{ext,z,k} \cdot n_{z,k} \cdot H_z}{\delta m_{i,z,k}} + \frac{e_{ext,z,k} \cdot \delta n_{z,k} \cdot H_z}{\delta m_{i,z,k}} + \frac{e_{ext,z,k} \cdot n_{z,k} \cdot \delta H_z}{\delta m_{i,z,k}} = \\
&= \left[(v-1)\alpha_{sca,00} + (1-v)\alpha_{sca,10} - v\alpha_{sca,01} + v\alpha_{sca,11} \right] \cdot \frac{\pi \cdot r_{wet,z,k}^2 \cdot R_i \cdot n_{z,k} \cdot H_z}{\rho_i \cdot V_{wet,z,k} \cdot (R_{up,z,k} - R_{low,z,k})} + \\
&+ \left[(u-1)\alpha_{abs,00} - u\alpha_{abs,10} + (1-u)\alpha_{abs,01} + u\alpha_{abs,11} \right] \cdot \frac{\pi \cdot r_{wet,z,k}^2 \cdot I_i \cdot n_{z,k} \cdot H_z}{\rho_i \cdot V_{wet,z,k} \cdot (I_{up,z,k} - I_{low,z,k})} + \\
&+ \frac{3e_{ext,z,k} \cdot H_z}{4\pi \cdot r_{dry,z,k}^3 \cdot \rho_i} + \frac{e_{ext,z,k} \cdot n_{z,k} \cdot H_z}{m_{i,z,k}} \} \cdot \beta
\end{aligned}
\tag{20}$$

$$\begin{aligned}
\frac{\delta(\overline{e_{ext} \cdot n})}{\delta m_{i,z,k}} &= \frac{H_z}{\sum H_z} \cdot \frac{\delta(e_{ext,z,k} \cdot n_{z,k})}{\delta m_{i,z,k}} = \frac{H_z}{\sum H_z} \cdot \left[\frac{\delta e_{ext,z,k} \cdot n_{z,k}}{\delta m_{i,z,k}} + \frac{e_{ext,z,k} \cdot \delta n_{z,k}}{\delta m_{i,z,k}} \right] = \\
&= \left[(v-1)\alpha_{sca,00} + (1-v)\alpha_{sca,10} - v\alpha_{sca,01} + v\alpha_{sca,11} \right] \cdot \frac{\pi \cdot r_{wet,z,k}^2 \cdot R_i \cdot n_{z,k}}{\rho_i \cdot V_{wet,z,k} \cdot (R_{up,z,k} - R_{low,z,k})} + \\
&+ \left[(u-1)\alpha_{abs,00} - u\alpha_{abs,10} + (1-u)\alpha_{abs,01} + u\alpha_{abs,11} \right] \cdot \frac{\pi \cdot r_{wet,z,k}^2 \cdot I_i \cdot n_{z,k}}{\rho_i \cdot V_{wet,z,k} \cdot (I_{up,z,k} - I_{low,z,k})} + \\
&+ \frac{3e_{ext,z,k}}{4\pi \cdot r_{dry,z,k}^3 \cdot \rho_i} \} \cdot \beta \cdot \frac{H_z}{\sum H_z}
\end{aligned}
\tag{21}$$

$$\begin{aligned}
\frac{\delta(e_{sca,1,k} \cdot n_{1,k})}{\delta m_{i,1,k}} &= \{ [(v-1)\alpha_{sca,00} + (1-v)\alpha_{sca,10} - v\alpha_{sca,01} + v\alpha_{sca,11}] \cdot \frac{\pi \cdot r_{wet,1,k}^2 \cdot R_i \cdot n_{1,k}}{\rho_i \cdot V_{wet,1,k} \cdot (R_{up,1,k} - R_{low,1,k})} \\
&+ \frac{3e_{sca,1,k}}{4\pi \cdot r_{dry,1,k}^3 \cdot \rho_i} \} \cdot \beta
\end{aligned}
\tag{22}$$

$$\begin{aligned}
\frac{\delta(e_{abs,1,k} \cdot n_{1,k})}{\delta m_{i,1,k}} &= \{ [(u-1)\alpha_{abs,00} - u\alpha_{abs,10} + (1-u)\alpha_{abs,01} + u\alpha_{abs,11}] \cdot \frac{\pi \cdot r_{wet,1,k}^2 \cdot I_i \cdot n_{1,k}}{\rho_i \cdot V_{wet,1,k} \cdot (I_{up,1,k} - I_{low,1,k})} \\
&+ \frac{3e_{abs,1,k}}{4\pi \cdot r_{dry,1,k}^3 \cdot \rho_i} \} \cdot \beta
\end{aligned}
\tag{23}$$