Supplement A:

Stochastic Model of Saltation in Turbulence

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We use a Lagrangian stochastic model for saltation in turbulent flow to examine the intensity of saltation bombardment. The model combines the equation of sand motion with a stochastic equation for fluid velocity fluctuations along the saltation trajectories. Following Thomson (1987, 1990), the turbulent motion of fluid elements can be modelled with

$$dU_i = a_i(U, X, t)dt + b_{ij}(U, X, t)d\omega_{ij} \qquad \text{(s1)}$$

$$dX_i = U_i(X, t)dt \qquad \text{(s2)}$$

where U is Lagrangian velocity (U_i its i component), X element position, a_i drift coefficient, b_{ij} diffusion coefficient and dω_{ij} increment of the Wiener process. Sand and fluid element follow different trajectories due to the trajectory-crossing effect (Yudine, 1959; Csanady, 1963).

The model is two dimensional, with x_1 aligned in the horizontal mean wind direction and x_3 in the vertical direction. We denote the sand position as $Y(t)$, sand velocity as $V(t)$, and the fluid velocity along $Y(t)$ as $U^*(t)$. The sand-to-fluid relative velocity is $V_R = V - U^*$.

The equation of sand motion can be written as

$$\frac{dV_i}{dt} = -\frac{V_{Ri}}{\tau_p} - \delta_{i3}g \qquad (i = 1, 3) \qquad \text{(s3)}$$

with τ_p being the sand response time (Morsi and Alexander, 1972). V_{Ri} is given by $V_{R1} = V_1 - \overline{U_1^*} - u_1^*$ and $V_{R3} = V_3 - u_3^*$, where $\overline{U_1^*}$ is the mean wind speed along the particle trajectory. The influences of turbulence on sand motion are embedded in u_1^* and u_3^*. These are calculated using a modified Thomson (1987) model. Note that $U = \overline{U} + u$ and $u = (u_1, u_3)$. Assume the mean wind being known, the fluid-element motion fluctuations (u_1, u_3) are calculated by using Equations (s1) and (s2). The diffusion coefficients b_{ij} there are given by

$$b_{ij} = \delta_{ij}\sqrt{C_0\varepsilon} \qquad \text{(s4)}$$

where δ_{ij} is Kronecker delta, C_0 a constant and ε the dissipation rate for turbulent kinetic energy. The determination of a_i uses the well-mixed condition of Thomson (1987), which leads to

$$a_i P = \frac{1_i}{2}\frac{\partial C_0 \varepsilon P}{\partial U_i} + \varphi_i \qquad \text{(s5)}$$

and

$$\frac{\partial \varphi_i}{\partial U_i} = -\frac{\partial P}{\partial t} - \frac{\partial U_i P}{\partial X_i} \quad \text{(s6)}$$

with P being the phase-space probability density function $P(U, X, t)$. The well-mixed condition of Thomson (1987) requires that P equals to the probability density function of the Eulerian velocity $U(x=X, t)$.

The increment du_i^* is expressed as

$$du_i^* = du_i + \delta u_i \quad \text{(s7)}$$

where du_i is the fluid-element velocity increment between t and $t+dt$, computed using Equation (s1), and δu_i the spatial velocity increment at $t+dt$ between the two points separated by $V_R\,dt$. While the structure function of du_i satisfies

$$\langle du_i du_i \rangle = C_0 \varepsilon dt, \quad \text{(s8)}$$

that of δu_i satisfies

$$\langle \delta u_i \delta u_i \rangle = C_1 \varepsilon^{2/3} V_R^{2/3} dt^{2/3}. \quad \text{(s9)}$$

Due to its fractional nature, δu_i is difficult to generate stochastically and it is in this study assumed to be

$$\langle \delta u_i \delta u_i \rangle = C_1 \varepsilon^{2/3} V_R l^{-1/3} dt \quad \text{(s10)}$$

with l being a fixed scaling length. Following Hanna (1981) and Stull (1988), $C_0 = 5$ and $C_1 = 2$.

Sand grains are randomly lifted from the surface with velocity (V_{1o}, V_{3o}). The PDF of V_{1o} is assumed to be Gaussian and that of V_{3o} Weibull (to avoid negative liftoff speed). The sand-grain liftoff angle is confined to 0^o and 180^o and Gaussian distributed with a mean liftoff angle of 55^o and a standard deviation of 5^o. The sand grains are allowed to rebound from the surface with the rebounding kinetic energy half the impacting kinetic energy and a mean rebounding angle of 40^o. If the kinetic energy of a sand grain becomes lower than a critical value, its motion is stopped.

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