Responses to Reviewer 1 comments.

Original comments in black, our responses and explanation of changes in red.

The present paper compares two different mathematical descriptions (Lagrangian and Eulerian frameworks) to address the cloud droplet diffusional growth in homogeneous isotropic turbulence. The manuscript shows interesting results with potential interest for Atmospheric Chemistry and Physics community. Nevertheless, the Reviewer has several comments/questions/suggestions that could make this paper even more useful for the community.

Specific comments:

-In order to have a more detailed (and useful) analysis, the Authors should report the final square radius distributions for all the Eulerian and Lagrangian cases. The theory shows that the pdf(R2) is Gaussian. Are they Gaussian or not in these cases? The referee expects a departure from the Gaussian distribution in high TKE SDS.26.

Motivated by this comment, we plotted radius squared (R^2) distributions for all simulations. These spectra are close to Gaussian, and some examples are included in the revised text (new section 5.3 included at the bottom of the document for the Reviewer's reference). We feel this addition significantly adds to the manuscript and we are thankful to the Reviewer for this suggestion.

-It is better to plot in the same graph the statistics from the bin and Lagrangian simulations for every single case so that we can directly compare one by one.

We appreciate this suggestion. However, such a change requires a significant modification of the manuscript structure. We prefer to avoid that. The way Lagrangian and Eulerian results are plotted allows a direct comparison without plotting the same droplet concentration cases on the same figure. Moreover, there are several lines for the bin results already and adding Lagrangian results would make such a plot very crowded.

However, per the Reviewer suggestion above, we added R^2 spectra that are now discussed in a separate section, with Lagrangian and Eulerian results next to each other. We feel the addition section allows a good comparison between the results.

-The Reviewer is not convinced that the results of the bin simulations differ so much due to the difficulty to describe the monodisperse delta distribution as an initial condition. The Authors can easily run the Lagrangian simulations for some iterations, extract the radius distribution pdf and use as an initial condition for the bin simulations (together with the same flow, temperature and humidity fields). Then they should be able to reduce the errors induced by wrong initial conditions and maybe to analyze better the effects of the bin resolutions or the effects of numerical diffusion.

We agree with this comment. However, as documented in the new section 5.3, the bin spectra are well resolved after some time when appropriately high bin resolution is applied. This is because the initially monodispersed spectrum evolves in R^2 Gaussian spectrum as the simulation progresses. We hope the added section 5.3 with R^2 spectra addresses this point. We include the new section at the end of this document.

That said, and motivated by the Reviewer 2 comments, we ran additional simulations and drafted an appendix that can be added to the manuscript. At the moment, we feel this is not needed, but we will be happy to include such an appendix in the revised manuscript. Please see the draft appendix at the end of our responses to the Reviewer 2 comments.

-Why the high TKE simulations are run just for few minutes compared with the low TKE cases? What happens for long times? Since the resolution and number of superdroplets/bins are the same, the computational efforts are precisely the same, so there are no problems to extend these cases in principle.

The low and high TKE cases were run for the same nondimensional time (6 eddy turnover times) and their computational effort is exactly the same. The nondimensional time is shown at the bottom of original figures 4 and 7. We feel this is long enough for the goal of this manuscript. Please also see the discussion in the new section 5.3 discussing the R^2 spectra.

-The Authors should describe with more equations and details the Lagrangian and the bin approaches, so far, everything is referred to previous papers.

Both Lagrangian and Eulerian approaches to cloud microphysics are fairly standard in the cloud physics literature. Thus, we feel including model equations would only add text that is not really needed. We prefer the style that we follow: discuss model details by words and refer to published papers where mathematical formulas are given.

However, as stated in our responses to the Reviewer 2 comments, we are open to include the equations if both Reviewers insist.

-For the bin simulations, a diffusion coefficient for the droplet distribution function should be given, how much is this value in the present results?

We are not sure what the diffusion coefficient the Reviewer has in mind. The two schemes apply the same droplet growth equations, so the molecular diffusion coefficient (implicit in the droplet growth equation) is the same for both schemes. For the numerics, the bin scheme does not use any diffusion coefficient per the ILES approach.

-Sedimentation and inertial terms are neglected in the Lagrangian simulations, what about in the Eulerian cases?

The Eulerian scheme excludes those as well. This is mentioned around line 405 of the original submission. This is appropriate for the spatial scales resolved by the simulations.

-Why does the term r0 appear in equation (9)? It is needed to avoid some singularities when r is small? Is r0 appearing also in the Lagrangian radius equation evolution? Why has r0 that specific value?

This is to include kinetic effects that play role at very small droplet radii. We added a reference to an old paper by Mordy (Tellus 1959) and a couple more recent papers that include such a formulation (by Clark and by Kogan). As stated in the original manuscript, the same droplet growth equation is used in both schemes (see the end of section 4).

-The value of the dissipation in Lanotte et al. (2009) is 10−3 not 10−4 m2/s3.

Corrected. Thank you.

-It is better to introduce a new Table with the case description.

We added a new table (Table 1) in section 4 with simulation details.

The new section:

5.3 Comparison of radius squared distributions between Eulerian and Lagrangian simulations.

This section compares radius squared (R^2) distributions at the end of the simulations, that is, after 6 turnover times, for both the low and high TKE simulations. As shown in Lanotte et al. (2009) and Sardina et al. (2015), an initial monodisperse distribution should evolve into a Gaussian R^2 spectrum because of the parabolic cloud droplet growth equation. Although the parabolic growth is only approximately valid because of the specific droplet growth equation (see Eq. 9), the Gaussian distribution is a good fit for simulation results discussed here as shown below.

Figure 9 shows the spectra for selected superdroplet simulations. The radius squared spectra are created by selecting R^2 bin size and binning superdroplet radii for a given simulation into the assumed bin grid. The bin size for the SDS.650/SDS.26 simulations (lower/upper panels in Fig.9) is 1/10 μm^2. There are two panels for each simulation, one with the linear vertical scale and the spectrum shown as a histogram, and the second one with the logarithmic vertical scale and using star symbols to show the spectrum. In addition, the logarithmic plots show the Gaussian distributions obtained with the mean and standard deviation calculated from the spectra.

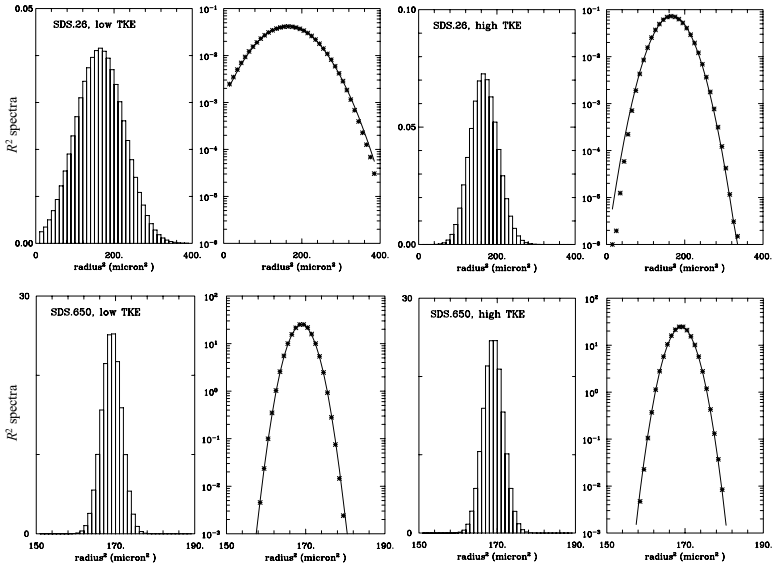

Figure 9. Results from simulations (upper panels) SDS.26 and (lower panels) SDS.650 superdroplet simulations. There are two panels for each simulation, the left one applying the linear vertical scale and the right one applying the logarithmic scale. The line in the logarithmic scale panels shows the Gaussian distribution with the mean and standard deviation calculated from the spectrum. Left/right pair in each row is for low/high TKE simulation.

For the SDS.650 simulations (lower panels in Fig. 9), the spectra at the end of low and high TKE simulations are practically the same. This agrees with the theoretical scaling and simulation results shown in Fig. 4 and 6. In contrast, results for SDS.26 differ drastically between the low and high TKE. The spectrum for the low TKE is wide, with some small droplets already evaporated because the spectrum is truncated at the low-radius end. Nevertheless, the Gaussian shape is still a good fit for the simulated spectrum. The high TKE SDS.26 spectrum is significantly narrower with small deviations from the Gaussian fit.

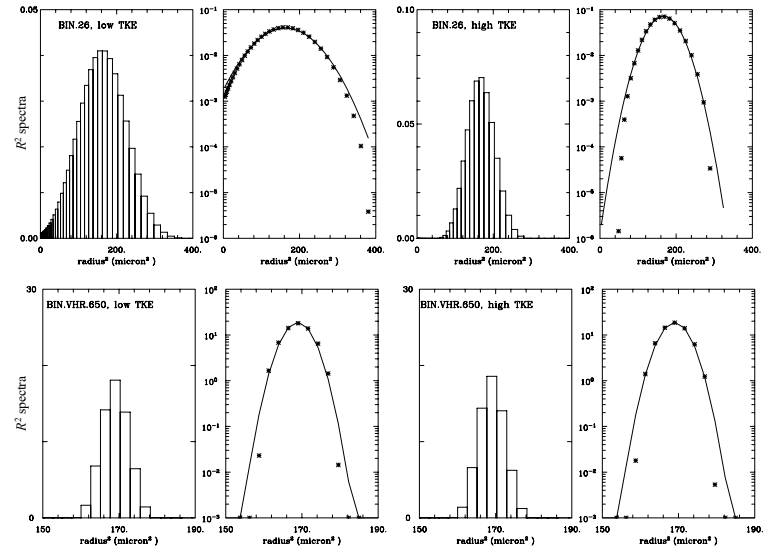

Figure 10. As Fig. 9, but for the bin (upper panels) BIN.26 and (lower panels) BIN.VHR.650 simulations.

Figure 10 shows the spectra for bin simulations similar to those in Fig. 9. Since bin simulations predict the spectra directly, the radius spectra are converted to R^2 spectra and then plotted at their native resolution in the R^2 space. This explains the change in the resolution along the horizontal axes evident in the upper panels. Overall, there are some similarities between Figs. 9 and 10. For instance, upper panels show spectra for the 26 cm^{-3} simulations with 0.5-μm bin width that are similar to those in superdroplet simulations. Spectra for 650 cm^{-3} simulations with 0.1-μm bin width (i.e., from the VHR set) are also similar between low and high TKE simulations, but their spectral widths are larger than in corresponding panels of Fig. 9. The impact of the bin resolution is further documented in Fig. 11 that shows results from the 650 cm^{-3} low TKE HR and SHR simulations, that is, with the bin width of 0.3 and 0.05 μm, respectively. Only the SHR simulation (i.e., the right panel in Fig. 11) resembles the spectra from the Lagrangian simulations shown in the lower panels of Fig. 9.

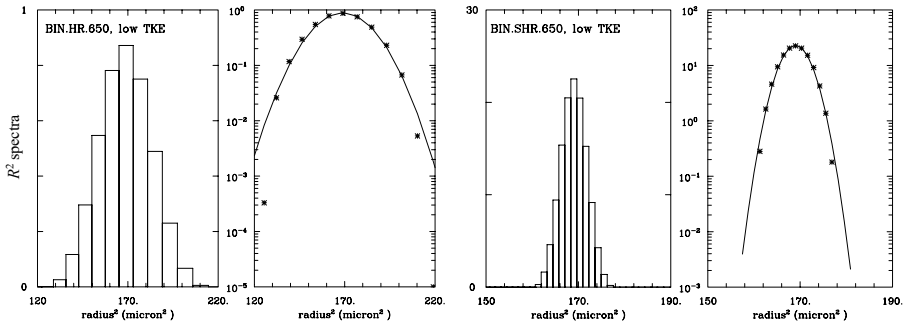

Fig. 11. As Figs. 9 and 10, but for the bin BIN.HR.650 and BIN.SHR.650 simulations. Note different horizontal range at the left and right pair of panels.

In summary, only extreme resolutions of the bin scheme (e.g., as in SHR, 0.05 μm bin width) allow good agreements between Lagrangian and Eulerian results for the concentration range considered here. Moreover, the ill-posed initial condition for the Eulerian scheme (i.e., the monodisperse initial droplet size distribution) seems irrelevant because the spectrum becomes well-resolved after some time during the simulation. With sufficiently high bin resolution, (e.g., 0.5 μm in the 26 cm^{-3} simulations or 0.05 μm for the 650 cm^{-3} simulations), the Eulerian and Lagrangian spectra compare well at the end of the simulations. This shows the benefit of the Lagrangian scheme as one does not have to worry about the bin size to obtain numerically converged solutions.