

# 1 Supplement

## 2 Derivation of Equation (6)

$$N_{CCN}(AOP) = (a_{ss}BSF + b_{ss})\sigma_{sp} = ((a_1 \ln(SS\%) + a_0)BSF + b_1 \ln(SS\%) + b_0)\sigma_{sp}$$

$$R_{CCN/\sigma} = \frac{N_{CCN}(AOP)}{\sigma_{sp}} = a_{ss}BSF + b_{ss} = (a_1 \ln(SS\%) + a_0)BSF + b_1 \ln(SS\%) + b_0$$

Linear regressions of the coefficients in Table 2 yield

$$a_0 \approx (2.38 \pm 0.06)a_1, b_0 \approx (2.33 \pm 0.03)b_1, b_1 \approx -(0.097 \pm 0.013)a_1 + (6.4 \pm 5.9)$$

$\Rightarrow$

$$a_1 \ln(SS\%) + a_0 \approx a_1 \ln(SS\%) + (2.38 \pm 0.06)a_1 \approx a_1 (\ln(SS\%) + (2.38 \pm 0.06))$$

$$b_1 \ln(SS\%) + b_0 \approx b_1 \ln(SS\%) + (2.33 \pm 0.03)b_1 = b_1 (\ln(SS\%) + (2.33 \pm 0.03))$$

$$\approx (-(0.097 \pm 0.013)a_1 + (6.4 \pm 5.9))(\ln(SS\%) + (2.33 \pm 0.04))$$

$\Rightarrow$

$$R_{CCN/\sigma} = (a_1 \ln(SS\%) + a_0)BSF + b_1 \ln(SS\%) + b_0$$

$$\approx a_1 (\ln(SS\%) + (2.38 \pm 0.06))BSF + (-(0.097 \pm 0.013)a_1 + (6.4 \pm 5.9))(\ln(SS\%) + (2.33 \pm 0.03))$$

Approximation, since  $(2.33 \pm 0.03) \approx (2.38 \pm 0.06)$

$\Rightarrow$

$$R_{CCN/\sigma} \approx a_1 (\ln(SS\%) + (2.38 \pm 0.06))BSF - (0.097 \pm 0.013)a_1 (\ln(SS\%) + (2.38 \pm 0.07)) + (6.4 \pm 5.9)(\ln(SS\%) + (2.38 \pm 0.06))$$

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$$\approx a_1 (\ln(SS\%) + (2.38 \pm 0.06))(BSF - (0.097 \pm 0.013)) + (6.4 \pm 5.9)(\ln(SS\%) + (2.38 \pm 0.06))$$

$$\approx (\ln(SS\%) + (2.38 \pm 0.06))(a_1(BSF - (0.097 \pm 0.013)) + (6.4 \pm 5.9))$$

$$\approx (\ln(SS\%) - \ln(0.093 \pm 0.006))(a_1(BSF - (0.097 \pm 0.013)) + (6.4 \pm 5.9))$$

$$\approx \ln\left(\frac{SS\%}{0.093 \pm 0.006}\right)(a_1(BSF - (0.097 \pm 0.013)) + (6.4 \pm 5.9))$$

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## 5 Derivation of Equation (7)

$$N_{CCN}(AOP) \approx \ln\left(\frac{SS\%}{0.093 \pm 0.006}\right)(a_1(BSF - BSF_{\min}) + C)\sigma_{sp},$$

where C is an unknown constant.

If  $BSF = BSF_{\min}$

$$\Rightarrow a_1(BSF - BSF_{\min}) = 0$$

$$\Rightarrow N_{CCN}(AOP, BSF_{\min}) \approx \ln\left(\frac{SS\%}{0.093 \pm 0.006}\right)C \cdot \sigma_{sp}$$

$$\Leftrightarrow C \approx \frac{1}{\ln\left(\frac{SS\%}{0.093 \pm 0.006}\right)} \frac{N_{CCN}(AOP, BSF_{\min})}{\sigma_{sp}} \approx \frac{1}{\ln\left(\frac{SS\%}{0.093 \pm 0.006}\right)} R_{\min}$$

$\Rightarrow$

$$N_{CCN}(AOP) \approx \ln\left(\frac{SS\%}{0.093 \pm 0.006}\right) \left( a_1(BSF - BSF_{\min}) + \frac{1}{\ln\left(\frac{SS\%}{0.093 \pm 0.006}\right)} R_{\min} \right) \sigma_{sp}$$

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$$\approx \left( \ln\left(\frac{SS\%}{0.093 \pm 0.006}\right) a_1(BSF - BSF_{\min}) + R_{\min} \right) \sigma_{sp}$$