Response to Anonymous Referee #2
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Title: A new Description of Probability Density Distributions of Polar Mesospheric Clouds (PMC)
Author(s): Uwe Berger et al.
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Introductory remarks:
We greatly appreciate the comments from the reviewer. We have taken his/her suggestions for improvements into account when preparing the revised version of the manuscript. In the following we respond to the reviewer's comments point by point.
We have marked the changes in the tracked version of the manuscript. Author responses are in italics. Line numbers refer to the first paper version. In the new tracked version deleted sequences are marked red. New text is marked in blue.
We want to thank the reviewer for the detailed review with many useful ideas and suggestions which, we think, have significantly increased the quality of the manuscript.

GENERAL COMMENTS: This is an interesting and generally well written article dealing with probability density functions of various noctilucent cloud (NLC) parameters such as particle radius, cloud backscatter, ice particle density and ice mass density. While the backscatter is found to follow an exponential distribution, this is not the case for the other parameters considered. The NLC parameter database employed is based on the well-known Alomar LIDAR dataset. I do not have major objections against the publication of this article but ask the authors to consider the comments listed below. In addition, I have the following general comment: The LIDAR backscatter measurements, like all other optical measurements, are quite insensitive to particles with radii below a certain threshold. This is related to the finding that radii below about 20 nm are very infrequent in the data set, despite the fact that there are typically many more small particles than large particles. I think this aspect should be discussed in the paper, because it also (qualitatively) explains some of the differences between the PDFs of the different parameters.

The Referee #2 is correct that optical instruments can not observe the smallest particles in PMC. However the cutoff at r > 20 nm is caused by the threshold of max beta > 3 and the fact that the limit the analysisto the peak of the layer.
In this paper we discuss 3 maximum backscatter signals that relate to a certain height within the PMC column (the height where backscatter maximizes). These are our measurements. From these 3 measurements we derive some more ice parameters as mean ice radius, standard deviation and ice mass density. These values refer exclusively to this specific height where the PMC shines brightest. Assuming a threshold in max beta of 3 produce the measured histograms in ice radius, ice mass density, and number density as shown in the plots. So it is not the question that the lidars are insensitive to small ice particles, a case that happens at the nucleation zone near the mesopause (87-90 km) where ice formation starts and up to several thousand ice particles exist (r<10 nm). But our lidar analysis takes into account only measurements from faint to strong brightness levels near the bottom (at height of beta max, 83 km) of the vertical PMC ice column.

We have extended the description of the retrieval of particle sizes in section 2 'Discription of ALOMAR lidar':

...After separation of the ice particle and molecular backscatter signal, we extract three vertical profiles of so-called backscatter ratios which are a measure of height dependent brightness of the ice cloud. From each backscatter height profile we estimate three maximum backscatter (MBS) values. We assume that at the altitude of MBS, typically located near 83 km, the actual shape of the ice particle distribution can be described by a Normal-distribution. Then we derive from the three measured MBS values the characteristics of the Normal-distribution with mean ice radius, ice number density and variance. Finally, we also estimate from these ice parameters the actual ice mass density (IMD) at the MBS height...

This paper is well-written. Some suggestions and comments related to specific items are provided below.

SPECIFIC COMMENTS
Page 1, line 1 and line 15: "of Polar Mesospheric Clouds (PMC) and noctilucent clouds (NLC)."
This sounds like the two are different clouds. I suggest changing this sentence.
Done: The expression NLC was considered redundant and was removed everywhere. Now, only the expression PMC is used throughout this paper.

Page 1, line 5: "previously statistical methods" -> "previous statistical methods" or "previously used statistical methods"
Done

Page 1, line 6: "probability statistic"
Does "statistic" exist?
Done: replaced by 'distributions'

Page 1, line 12: "that facilitate" -> "that facilitates", because "facilitates" refers to "assessment",
right?
Done

Page 2, line 3: "many .. analysis" -> "many .. analyses"
Done

Page 2, line 8: "analysis have used" -> "analyses have used"
Done

Page 2, line 17 and line 18: "statistic" ?
Done: statistics

Page 2, line 31: "From each backscatter height profile we estimate a maximum backscatter (MBS) signal which corresponds to mean height of maximum brightness"
I don't fully understand this sentence. It mixes "signal" and "height" in a way, which makes it difficult to understand. Can you clarify, please?
Done: ... From each backscatter height profile we estimate three maximum backscatter (MBS) values. We assume that at the altitude of MBS, typically located near 83 km, the actual shape of the ice particle distribution can be described by a Normal-distribution. Then we

derive from the three measured MBS values the characteristics of the Normal-distribution with mean ice radius, ice number density and variance. Finally, we also estimate from these ice parameters the actual ice mass density (IMD) at the MBS height.

Page 3, line 15: “exponential distributed“ -> "exponentially distributed”
Done

Page 3, line 30: “mode” is not a really frequently used term and I suggest briefly explaining it. It is explained on the next page and I suggest moving the explanation here.
Done

Page 4, line 19: “in a semi-logarithm scale”. I suggest replacing this by “in a semilogarithmic diagram” (a scale can be linear or logarithmic, but not semi-logarithmic)
Done

Page 4, line 22: “Consequently, the relative error is rather small”
Please explain briefly how this relative error is determined.
Done: The good quality of the fit is characterized by a small relative error of 6.5 % that is calculated as a sum of 100%· SUM(j,M) |E j −X j | for x >3 with theoretical exponential frequencies E j and normalized frequencies X j of data x per class j with a total of M classes.

Page 5, line 5: “ . . . as expected”
It’s not entirely clear, what you consider to be expected. Do you expect that these other parameters also follow an exponential distribution or do you not? Please clarify.
Done: ‘as expected’ has been deleted.

Page 5, line 6: “in a semi-logarithmic scale” -> “in a semi-logarithmic diagram”
Done

Page 5, line 9: “significant smaller“ -> "significantly smaller”
Done

Page 6, Caption Fig. 2, line 2: “least square fit” -> “least squares fit”
Done

Page 7, Fig. 3: I suggestion mentioning in the Figure caption what the dashed lines are.
Done: The solid line shows the regression defined by regression points and corresponding (c,d)-values. Dashed lines result from regression analysis of y(x) : x ⇒ y and x(y) : x ⇒ y.

Page 7, line 7: “Linearity between maximum backscatter (MBS) and ice mass density (IMD), ice radius r and ice number density n data is a necessary and sufficient condition that also IMD, r and n data samples are exponentially distributed”
I’m not sure you would really expect that MBS scales linearly with, e.g. radius. The intensity of the backscattered radiation does certainly not scale linearly with particle radius, right? Why should the maximum backscatter depend linearly on radius? If there are other indications etc. for that, please discuss. Considering that you use a power law to describe the relationship between two parameters, you don’t really assume linearity, right? I think the term “linearity” should be replaced and then all is fine.

This section is titled 'Test on linearity between maximum backscatter and ice mass density, ice radius, ice number density data'. Here we show in a first step that there exist no linearity between (MBS, r), (MBS, n), (MBS,IMD), and also other pairs as e.g. (n,r).
This gives a first theoretical hint why IMD,n, and r do not follow an exponential distribution as has been tested empirically in the section before (section 3.1.2.).
Or vice versus: In section 3.1.2 we plot exponential fit functions to all data samples (IBS,IMD,r, and n) and see for IMD,n, and r large statistical uncertainties for exponential fits that indicate that there is 'something wrong'!. Section 3.2 investigates this hypothesis in a different second way with the method of correlation and regression pairs.
Now we find a second reason for missing exponential fits of IMD,n, and r, since also linear regression fails. These are two independent different ways, and both saying that a g-function can't be the universal pdf for all four ice parameters.

This is the motivation why we introduce in the following sections a new z-pdf which now fits equally to ALL four parameters MBS,IMD,n, and r.

Page 8, line 1: ". . . also relate to the half width of the angle"
Can you mention how the regression points "relate to" the half width of this angle? To me this is not obvious, but perhaps I'm missing something.
Let us assume two data samples x and y. First, we calculate the means (x_mean, y_mean) and standard deviaitons s_x and s_y of x and y, plus the Pearson correlation coefficient R between x and y. Then a linear regression from x to y means to construct a linear fit with:
y = y_mean + R(s_y/s_x) * (x - x_mean) -> dashed line y(x)*

A linear regression from y to x means to construct a linear fit with:
x = x_mean + R(s_x/s_y) * (y - y_mean)*
*This equation can be transformed to y as y = y_mean + 1/R * (s_y/s_x) * (x - x_mean) and plotted as a second dashed line named x(y).*

For R unequal to one the two regression lines are different. Then the best estimate is a 'mean' regression line as shown in Figure 3 (solid line) that cut the angle of the two classical regression lines (y(x),x(y))in half. The Formula is y = y_mean + R s_y/s_x *(x – x_mean) setting R=1.*

Page 8, line 5: "criteria" -> "criterion"
Done

Page 8, line 6: "which is far away from unity"
This is not surprising at all, because the backscatter does not scale linearly with radius.
But perhaps this is discussed below.
At this point we simply show (prove) that, in fact, our lidar measurements of radius and max. backscatter do not scale linearly. Consequently, this non-linearity forces a modification of the exponential distribution (g-function) called Z-approach in our paper. The derivation and the characteristics of new Z-pdf and its connection to the former g-function are discussed in the following sections 4,5, and 6.

Page 9, last line: Something is missing in this equation. "C" is not defined and it is neither required here. The integral can be explicitly evaluated, but I find that b>1 is a requirement for the integral being 1. Please check.

Done: "C" was the constant of integration that has been deleted. Instead we use a vertical bar in order to show the integration boundaries of the antiderivative. This change has been also applied in the appendix.

Page 10, line 10: Meaning of "Only, " at beginning of sentence not clear, at least to me. Without the comma it would make sense.
Done

Page 12, line 26: "should be here possible too" -> "should be possible here too"
Done

Page 13, line 15: Suggest replacing ", and resulting" by ", resulting in "
Done

Page 16, line 8: "in turns" -> "in turn"
Done

Page 16, line 12: "of an practical example" -> "of a practical example"
Done

Page 16, line 25: "Now the Z-distribution approach offers a more general possibility to derive artificial data samples without any knowledge of correlation and regression coefficients."
I'm not sure I fully agree with this statement, because information on the power law relationship between the two quantities is required, right?
No: no information on the relationship between the two quantities is required. This is precisely the advantage of the new method. See text in section 6.1.

The statement suggests (or may suggest) that no prior information on the relationship between the two quantities is needed, which is certainly not the case, because you assume that a_r and b_r are known.
Sure this is true that a_r and b_r are assumed to be known. As we explain in detail in the text, the constants a_r and b_r are calculated from a Z-analysis of the sole radius data sample. No relationship between a pair of parameters is needed. We had already described these two assumptions one sentence later:
First, we assume that a data sample of x=MBS of number N exists and also its Z-distribution Z(x,a x ,b x) with a x = 0.140 and b x = 0.931 is well known, see Figure 5a. Secondly, we assume that we know a priori the form of the Z-distribution Z(r,a r ,b r) ...

Page 18, line 19: "of an satellite" -> "of a satellite"
Done

Page 19, line 7: I think "for r_i > 37.5 nm" should be "for r_i < 37.5 nm"
Done, yes a typing error.

Page 19, line 10: "signal result" -> "signals result"
Done

Page 19, line 11: "of an spherical ice particle" -> "of a spherical ice particle"
Done

Page 19, line 11/12: you assume a fixed relationship between LIDAR backscatter signal and particle radius. In reality, the power will decrease with increasing particle radius. For your estimation this certainly does not have to be considered, but it's perhaps worth mentioning.
In reality, the power will sharply ***increase*** *with increasing particle radius (~r**5.8). We use this relationship explicitly in section 6.2 when calculating the integrals.*

Page 21, last line: "We present two numericalLY stable .."
Done

Page 21, line 6: "The Z-distribution approach offers a more general possibility to derive artificial data samples without any knowledge of correlation and regression coefficients" OK, but the approach requires a priori knowledge on the Z-distribution parameters, right?
Comment: This sentence is related to a brief summary of section 6.1 (Construction of artificial data). In section 6.1 we state clearly for several times the assumption of a priori knowledge on the Z-distribution parameters.