Supplement S1 – Minimizing criteria for regression methods applied in the manuscript.

The minimizing criteria are given as follows:

OLS minimizes the sum of squares vertical distances (residuals) between each point and the fitted line. OLS regression minimizes the following criterion:

$$C_{OLS} = \sum_{i=1}^{N} \left(y_i - \hat{\alpha}_{OLS} - \hat{\beta}_{OLS} x_i\right)^2$$

where $\hat{\alpha}_{OLS}$ and $\hat{\beta}_{OLS}$ refer to estimators calculated from the data, given by

$\hat{\beta}_{OLS} = \frac{S_x}{S_y}$, $\hat{\alpha}_{OLS} = \bar{x} - \hat{\beta}_{OLS}\bar{y}$

where $S_x = \sum_{i=1}^{N}(x_i - \bar{x})^2$, $S_y = \sum_{i=1}^{N}(y_i - \bar{y})^2$, $S_{xy} = \sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$

ODR (https://docs.scipy.org/doc/external/odrpack_guide.pdf, https://docs.scipy.org/doc/scipy/reference/odr.html, accessed 2018-07-27) minimizes the sum of the square of orthogonal distances between each point and the line, the criteria is given by

$$C_{ODR} = \sum_{i=1}^{N} \left(\left(x_i - \frac{y_i + x_i/\hat{\beta}_{ODR} - \hat{\alpha}_{ODR}}{\hat{\beta}_{ODR} + 1/\hat{\beta}_{ODR}} \right)^2 + \left(y_i - \hat{\alpha}_{ODR} - \frac{\hat{\beta}_{ODR} y_i + x_i - \hat{\alpha}_{ODR}\hat{\beta}_{ODR}}{\hat{\beta}_{ODR} + 1/\hat{\beta}_{ODR}} \right)^2 \right)$$

Where

$$\hat{\beta}_{ODR} = \frac{S_y - S_x + \sqrt{\left(S_y - S_x\right)^2 + 4S_{xy}^2}}{2S_{xy}} \text{ and } \hat{\alpha}_{ODR} = \bar{y} - \hat{\beta}_{ODR}\bar{x}$$

ODR takes into account the errors in both axes but not the values of their variances. Thus only the ratio between the two error variances is needed to improve the methodology. With notation of Francq and Govaerts (2014) this ratio is given by,

$$\lambda_{xy} = \frac{\sigma_y^2}{\sigma_x^2}$$

where the numerator of the ratio is the error variance in the Y-axis and the denominator is the error variance in the X-axis. The **Deming Regression (DR)** is the ML (Maximum Likelihood) solution of Eq. 1 when λ_{xy} is known. In practice, λ_{xy} is unknown and it is estimated from the variances of x and y calculated from the data.

The DR minimizes the criterion C_{DR} the sum of the square of (weighted) oblique distances between each point to the line

$$C_{DR} = \sum_{i=1}^{N} \left(\lambda_{xy} \left(x_i - \frac{y_i + \lambda_{xy} x_i/\hat{\beta}_{DR} - \hat{\alpha}_{DR}}{\hat{\beta}_{DR} + \lambda_{xy}/\hat{\beta}_{DR}} \right)^2 + \left(y_i - \hat{\alpha}_{DR} - \frac{\hat{\beta}_{DR} y_i + \lambda_{xy} x_i - \hat{\alpha}_{DR}\hat{\beta}_{DR}}{\hat{\beta}_{DR} + \lambda_{xy}/\hat{\beta}_{DR}} \right)^2 \right)$$

where

$$\hat{\beta}_{DR} = \frac{S_y - \lambda_{xy}S_x + \sqrt{\left(S_y - \lambda_{xy}S_x\right)^2 + 4\lambda_{xy}S_{xy}^2}}{2S_{xy}} \text{ and } \hat{\alpha}_{DR} = \bar{y} - \hat{\beta}_{DR}\bar{x}$$

Bivariate Least Square regression, **BLS**, is a generic name but here we refer to the formulation described in Francq and Govaerts (2014) and references therein. BLS takes into account errors and heteroscedasticity in both axes and is written usually in matrix notation. BLS minimizes the criterion C_{BLS}, the sum of weighted residuals given by:

$C_{BLS} = \frac{1}{W_{BLS}} \sum_{i=1}^{N} \left(y_i - \hat{\alpha}_{BLS} - \hat{\beta}_{BLS} x_i\right)^2$ with $W_{BLS} = \sigma_\varepsilon^2 = \frac{\sigma_y^2}{n_y} + \hat{\beta}_{BLS}^2 \frac{\sigma_x^2}{n_x}$

Estimators for the parameters are computed by iterations with the following formulas:

$$\frac{1}{W_{BLS}} \begin{pmatrix} N & \sum_{i=1}^{N} x_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \end{pmatrix} \begin{pmatrix} \hat{\alpha}_{BLS} \\ \hat{\beta}_{BLS} \end{pmatrix} = W_{BLS} \left(\sum_{i=1}^{N} \left(x_i y_i + \hat{\beta}_{BLS} \frac{\sigma_x^2}{n_x} \frac{\sum_{i=1}^{N} \left(y_i - \hat{\alpha}_{BLS} - \hat{\beta}_{BLS} x_i\right)^2}{W_{BLS}} \right) \right)$$

Where known uncertainties $\sigma_x^2$and σ_y^2 are in this study replaced with estimated variances S_x and S_y.

Second Bivariate regression used in this study is an implementation of the regression method described by **York** *et al.* (2004, Section III). See their description of the method for details.

The Principal Component Analysis based regression (**PCA**) can be applied for bivariate and multivariate cases.
For one independent and one dependent variable, the regression line is
$y = \hat{\alpha}_{PCA} + \hat{\beta}_{PCA} x$ where the error between the *observed* value y_i and *estimated* value $a+bx_i$ is minimum. For *n* points data, we compute *a* and *b* by using the method of least squares that minimizes:

$$C_{PCA} = \sum_{i=1}^{N} \left(y_i - \hat{\alpha}_{PCA} - \hat{\beta}_{PCA} x_i\right)^2$$

This is a standard technique that gives regression coefficients α and β.

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{\begin{bmatrix} S_x & -\bar{x} \\ -\bar{x} & 1 \end{bmatrix}}{S_x - \bar{x}^2} \begin{bmatrix} \bar{y} \\ S_{xy} \end{bmatrix}$$

Bayes EIV regression estimate applies Bayesian inference using the popular Stan software tool (http://mc-stan.org/users/documentation/, accessed 2018-07-27), which allowed the use of prior information of the model parameters. We assumed

β_0 ~ student_t(5, 0.0, 100.0)
β_1 ~ student_t(5, 0.0, 100.0)
x_{true} ~ lognormal(μ_x, σ_x)

where μ_x, and σ_x are the mean and standard deviation of x_{true} and are treated unknown. Also

x_{obs} ~ normal(x_{true}, $\sigma_{rel,x}$*x_{true} + $\sigma_{abs,x}$);

The Stan tool solved 1000 iterations of regression fitting and provided a posteriori distributions for the model parameters β_0 and β_1. In our regression analysis we finally utilized the maximum a posteriori estimates for β_0 and β_1.