Figure S1: Geographic distribution of the opaque height (z\_opaque), where the lidar beam is fully attenuated, for CALIPSO-GOCCP observations (2007-2016, nighttime). The magenta contour denotes the regions wherein  $\omega_{500}$  (ERAI reanalysis for the observations, Dee et al., 2011) is greater than 10 hPa/d. Note that this contour is well correlated with low values of z\_opaque and that z\_opaque is lower than 3 km within the  $\omega_{500}$  contour.



Figure S2: Vertical profiles of cloud fraction (a, in %) and cloud fraction interannual change due to SST variations (b, in % K<sup>-1</sup>) as observed by CALIPSO-GOCCP observations (black and circle line) and as simulated by the models (colored lines). Dashed lines correspond to the constrained models while the full lines are the unconstrained models. Unconstrained models either simulate a small decrease of the cloud fraction (smaller magnitude than 1 % K<sup>-1</sup>, grey dotted line) or an unrealistic increase of the cloud top (larger than 0.5 % K<sup>-1</sup>, grey dotted line).



Figure S3: Geographic distribution of low cloud cover (LCC, %) for CALIPSO-GOCCP observations (2007-2016, nighttime) along with regions of stratocumulus clouds (blue rectangles) and trade cumulus regions (green rectangles) used in section 4.3.



Figure S4: Same as Fig. 5a but declined for (a) all, (b) trade cumulus and (c) stratocumulus cloud means. The means for all SST datasets are represented in a different color to show that CALIPSO-GOCCP mean is consistently larger in magnitude than the other datasets in the all clouds case.



Figure S5: Global mean of GISS-E3 LCC over tropical oceans in regime of subsidence using no simulator (blue) the ISCCP simulator (red) the ISCCP + Qu et al. (2015) correction method (yellow) and the sum of low- and mid-cloud cover of ISCCP simulator (purple). Note that the correction method derived from Qu et al. (2015) generates a small overestimation of the LCC as originally simulated by the model (i.e., without simulator).

