Supplement to manuscript: Bayesian inverse modeling and source location of an unintended I-131 release in Europe in the fall of 2011

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1 Parameters for model selection

The moments of the respected distributions are denoted with the hat symbol, $\widehat{\theta}$.

The terms related to $\mathrm{E}\left[\ln p(\mathbf{y},\mathbf{x},\Upsilon,L,\boldsymbol{\psi},\omega,M_i)\right]$ are:

$$\ln p(\mathbf{y}|\mathbf{x},\omega) = -\frac{p}{2}\ln(2\pi) + \frac{p}{2}\widehat{\ln\omega} - \frac{1}{2}\widehat{\omega}\left(\mathrm{tr}\left(\widehat{\mathbf{x}\mathbf{x}^T}M^TM\right) - 2\mathbf{y}^TM\widehat{\mathbf{x}} + \mathbf{y}^T\mathbf{y}\right), \quad (1)$$

$$\ln p(\omega) = -\ln\Gamma(\vartheta_0) + \vartheta_0\ln\rho_0 + (\vartheta_0 - 1)\widehat{\ln\omega} - \rho_0\widehat{\omega}, \quad (2)$$

$$\ln p(\mathbf{x}|\boldsymbol{\upsilon},L) = -\frac{n}{2}\ln(2\pi) + \frac{1}{2}\widehat{\ln|\Upsilon|} - \frac{1}{2}\left(\mathbf{x}^T(\widehat{L\Upsilon L^T})\mathbf{x}\right) \quad (3)$$

$$= -\frac{n}{2}\ln(2\pi) + \frac{1}{2}\sum_{j=1}^{n}\widehat{\ln\upsilon_j} - \frac{1}{2}\mathrm{tr}\left(\mathbf{x}^T(\widehat{L\Upsilon L^T})\mathbf{x}\right) \quad (4)$$

$$= -\frac{n}{2}\ln(2\pi) + \frac{1}{2}\sum_{j=1}^{n}\widehat{\ln\upsilon_j} - \frac{1}{2}\mathrm{tr}\left(\widehat{L^T\mathbf{x}\mathbf{x}^TL}\widehat{\Upsilon}\right) \quad (5)$$

$$\ln p(x_j|\boldsymbol{\upsilon},L) = \ln\sqrt{2} - \frac{1}{2\sigma_{x_j}}\widehat{x_j^2} - \ln\sqrt{\pi\sigma_{x_j}} - \ln\left(1 - \mathrm{erf}\left(\frac{0}{\sqrt{2\sigma_{x_j}}}\right)\right) \quad (6)$$

$$\sigma_{x_j} \rightarrow \left((\widehat{L\Upsilon L^T})^{-1}\right)_{j,j} \quad (7)$$

$$\ln p(\boldsymbol{\upsilon}) = -n\ln\Gamma(\alpha_0) + n\alpha_0\ln\beta_0 + \sum_{j=1}^{n}(\alpha_0 - 1)\widehat{\ln\upsilon_j} - \sum_{j=1}^{n}\beta_0\widehat{\upsilon}_j, \quad (8)$$

$$\ln p(\mathbf{l}_j|\boldsymbol{\varsigma}_j) = -\frac{rs}{2}\ln(2\pi) + \sum_{m=1}^{rs}\frac{1}{2}\widehat{\ln\varsigma_{j,m}} - \frac{1}{2}\left(\mathbf{l}_j^T\widehat{\mathrm{diag}(\boldsymbol{\varsigma}_j)}\mathbf{l}_j - 2\mathbf{l}_{0,j}^T\widehat{\mathrm{diag}(\boldsymbol{\varsigma}_j)}\widehat{\mathbf{l}}_j + \mathbf{l}_{0,j}^T\widehat{\mathrm{diag}(\boldsymbol{\varsigma}_j)}\mathbf{l}_{0,j}\right), \quad (9)$$

$$\ln p(\boldsymbol{\varsigma}_j) = -rs\ln\Gamma(\zeta_0) + rs\zeta_0\ln\eta_0 + \sum_{m=1}^{rs}(\zeta_0 - 1)\widehat{\ln\varsigma_{j,m}} - \sum_{m=1}^{rs}\eta_0\widehat{\varsigma_{j,m}}. \quad (10)$$

The terms related to $\mathrm{E}\left[\ln \tilde{p}(\omega)\right] - \mathrm{E}\left[\ln \tilde{p}(\mathbf{x})\right] - \mathrm{E}\left[\ln \tilde{p}(\Upsilon)\right] - \mathrm{E}\left[\ln \tilde{p}(L)\right] - \mathrm{E}\left[\ln \tilde{p}(\boldsymbol{\psi})\right]$ are:

$$\ln \tilde{p}(\mathbf{x}|\mathbf{y}) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|\Sigma_{\mathbf{x}}| - \frac{1}{2}\left(\mathbf{x}^T(\widehat{L\Upsilon L^T})\mathbf{x}\right) \tag{11}$$

$$\ln \tilde{p}(x_j|\mathbf{y}) = \ln\sqrt{2} - \frac{1}{2\sigma_{x_j}}\left(\widehat{x_j} - \mu_j\right)^2 - \ln\sqrt{\pi\sigma_{x_j}} - \ln\left(1 - \operatorname{erf}\left(\frac{0 - \mu_{x_j}}{\sqrt{2\sigma_{x_j}}}\right)\right) \tag{12}$$

$$\ln \tilde{p}(\boldsymbol{\upsilon}|\mathbf{y}) = -\sum_{j=1}^{n}\ln\Gamma(\alpha_j) + \sum_{j=1}^{n}\alpha_j\ln\beta_j + \sum_{j=1}^{n}(\alpha_j - 1)\widehat{\ln\upsilon_j} - \sum_{j=1}^{n}\beta_j\widehat{\upsilon_j}, \tag{13}$$

$$\ln \tilde{p}(\mathbf{l}_j|\mathbf{y}) = -\frac{rs}{2}\ln(2\pi) - \frac{1}{2}\ln|\Sigma_{\mathbf{l}_j}| - \frac{1}{2}\left(\left(\widehat{\mathbf{l}_j} - \mu_{\mathbf{l}_j}\right)^T\Sigma_{\mathbf{l}_j}^{-1}\left(\widehat{\mathbf{l}_j} - \mu_{\mathbf{l}_j}\right)\right) \tag{14}$$

$$\ln \tilde{p}(\boldsymbol{\varsigma}_j|\mathbf{y}) = -\sum_{m=1}^{rs}\ln\Gamma(\zeta_m) + \sum_{m=1}^{rs}\zeta_m\ln\eta_m + \sum_{m=1}^{rs}(\zeta_m - 1)\widehat{\ln\varsigma_{j,m}} - \sum_{m=1}^{rs}\eta_m\widehat{\varsigma_{j,m}}, \tag{15}$$

$$\ln \tilde{p}(\omega|\mathbf{y}) = -\ln\Gamma(\vartheta) + \vartheta\ln\rho + (\vartheta - 1)\widehat{\ln\omega} - \rho\widehat{\omega}.$$