

Referee’s Report: “Data Assimilation Using an Ensemble of Models: A Hierarchical Approach” by Peter Rayner, revised for *Atmospheric Chemistry and Physics*.

I found that the overall presentation of this paper has improved a lot, and most of my comments have been addressed. The author has changed the writings in a few places and some additional comments are as follows:

1 Major Comments

1. In Equation (1), the author introduced the notation of observed observations, \mathbf{y}^o . I am wondering whether the conditional probability $p(\mathbf{y}|\mathbf{y}^o)$ in the rhs of Equation (1) should be $p(\mathbf{y}^o|\mathbf{y})$, which is the data model when \mathbf{y} denotes the underlying true data process. In the meanwhile, to be clearer, $p(\mathbf{x}, \mathbf{y})$ may be written as $p(\mathbf{x}, \mathbf{y}|\mathbf{x}^b, \mathbf{y}^o)$.
2. Page 3, Line 26: the sentence “Combing Equation 1 and Equation 3” is not necessary, since Equation (4) can be directly obtained based on the chain rule of probabilities.
3. Page 4, Line 16: in Equation (6), the proportional symbol \propto should be $=$, since Equation (4) holds and $p(\mathbf{x}|\mathbf{y}, H_i) = G(\mathbf{x}, \mathbf{x}_i^a, \mathbf{A}_i)$ (Equation (5)).
4. Page 4, Line 20: In Equation (7), $p(\mathbf{H}_i|\mathbf{y})$ should be $p(\mathbf{H}_i|\mathbf{y}^o)$, since \mathbf{y}^o appears in the rhs of Equation (7), and Equation (7) is derived based on $p(\mathbf{H}_i|\mathbf{y}^o) \propto \int p(\mathbf{y}^o|\mathbf{H}_i, \mathbf{x}, \mathbf{R}) \cdot p(\mathbf{H}_i|\mathbf{x}) \cdot p(\mathbf{x}|\mathbf{x}^b, \mathbf{B})d\mathbf{x}$.
5. For Equation (7), I guess that the constant K should have a subscript i , since K depends on the prior distribution of different parameter models, $p(\mathbf{H}_i)$; also it is better to mention that K is a constant in the context.
- 6 In Figure 2 and Figure 5, I still noticed that the titles of different boxes are **repeated**, and they are not the distinct names for the 22 land-ocean regions of the TRANSCOM inter-comparison. The author needs to double check whether these results are correctly produced.

2 Minor Comments

1. Page 1, Line 22: The “PDF” needs to be explained.
2. Page 4, Line 9, the covariance B should have a bold font.
3. Page 4, Line 11, the brackets of the reference “Rayner et al., 2018, Section 6.4” should be removed.
4. Page 5, Line 26: The AIC should be $2M + \chi^2$ with the penalty factor 2.
5. Page 9, Line 29: In Equation (14), the second $(\mathbf{H}\mathbf{x}_j^b - \mu_j^b)$ should have a transpose, $(\mathbf{H}\mathbf{x}_j^b - \mu_j^b)^T$; similarly, in Equation (15), the second $(\mathbf{H}\mathbf{x}_j^a - \mathbf{y}_i)$ should be $(\mathbf{H}\mathbf{x}_j^a - \mathbf{y}_i)^T$.