Atmos. Chem. Phys. Discuss.,
doi:10.5194/acp-2017-10-RC1, 2017

Atmospheric Chemistry and Physics
Discussions
Open Access

Interactive comment on "Data Assimilation using an Ensemble of Models: A hierarchical approach" *by* Peter Rayner

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Received and published: 11 March 2017

Referee Report for
Data Assimilation using an Ensemble of Models:
A hierarchical approach

General comments:

In general I like this paper a lot. However, I find it extremely difficult to follow because of some type-o's and much notation with which I am not familiar. The notation seems inconsistent in distinguishing between fixed quantities and random ones, and indicating

where conditioning has taken place. I admit that I come from a different community, and at least some of my comments may be due simply to this notational difference. My comments below pertain to only the first part of the paper as I need these items clarified in order to proceed. Thanks for your patience in answering these questions.

Specific comments for the authors:

Section 2, through Section 2.1

1. Page 3, lines 6 and 7: Please define the random variables x and H_i. In what sense is $P(x|H_i)$ "the conventional data assimilation problem"?

2. Page 3, line 9: To what "linear model" are you referring? A linear transport model represented by H_i? What do you mean by "over enough of the relevant pdf's"? Or, do you mean "over enough of the support of the random variable H_i?

3. Page 3, line 12 and 13: Is $\mathbf{H}_i$ the same as H_i? $\mathbf{H}_1, \ldots, \mathbf{H}_N$ are defined here as Jacobian matrices corresponding to N different transport models "...with unknowns defined by the multivariate Gaussian $G(\mathbf{x}^b, B)$...". Which unknowns?

4. Page 3, line 15: "For each $\mathbf{H}_i$ our problem is the simple linear Gaussian inversion..." What does this mean? What is it you are trying to solve for or infer? Is it the flux that gave rise to the observed concentrations?

5. Page 3, line 16: "Most importantly for us $P(x^a|\mathbf{H}_i)$ is Gaussian." Please define x^a. Should it be $\mathbf{x}^a$?

6. Page 3, lines 16 and 17: $P(\mathbf{x}, \mathbf{H}_i)$ appears to be a joint distribution of two quantities: the vector-valued $\mathbf{x}$ and the matrix-valued $\mathbf{H}_i$. It's unclear from the notation

whether $\mathbf{H}_i$ is a random matrix or a fixed matrix. (On line 21, $\mathbf{H}_i$ is treated as random.) My guess is that it is fixed since the right side of the equal sign appears to show the pdf of just one variable; presumably $\mathbf{x}$. Is μ_i a vector or a scalar? Please define μ_i, $\mathbf{U}_i$, and W_i. The expression $P(\mathbf{x}, \mathbf{H}_i) = W_i G(\mu_i, \mathbf{U}_i)$ does not define a proper pdf unless $W_i = 1$ since the area under the pdf must equal one. A more precise definition of a mixture would be in terms of random variables: X = $\sum_{i=1}^{K} A_i \mathbf{X}_i, \quad A_i = \begin{cases} 1 \text{ with probability } w_i, \\ 0 \text{ otherwise} \end{cases}, \quad \sum_{i=1}^{K} w_i = 1,$ and $\mathbf{X}_i \sim G(\mu_i, \mathbf{U}_i)$.

7. Page 3, line 23: Either $\mathbf{x}$ should be bold, or not. Do not mix within the same equation. Also, the notation $G(\mu_i, \mathbf{U}_i)(\mathbf{x})$ seems is very confusing (to me, at least). Do you mean that $\mathbf{x}$ is an argument to the function G? Why not write $G(\mathbf{x}|\mu_i, \mathbf{U}_i)$?
8. Page 3, line 26: In this equation $\mathbf{H}_i$ is treated as a non-random quantity. Above in line 21 it was random. Have you conditioned on it? If so, this distribution should be written as a conditional distribution. If not, then W_i is a random variable, not a fixed weight.
9. Page 3, line 26: I can't check this equation because I can't follow the derivation in Appendix A. See below.
10. Page 3, line 27: I think there is an extra “v" at the end of this line.
11. Page 4, line 2: I assume that $\mathbf{x}_b$ has a prior distribution somewhere because it is being treated as both random and fixed in various places. What is the prior distribution?
12. Page 4, line 16: Type-o.
13. Page 4, Footnote is missing.

Appendix A, through page 12

1. Page 12, line 18: K was defined in the main text as a normalizing constant. What is $K(\mathbf{H}_i)$ here? Do you mean $P(\mathbf{H}_i)$?
2. Page 12, line 21: I am confused by this equation. G is a function that has an argument and parameters. What are the parameters and what are the arguments in this expression? The definitions from Section 2, lines 13 and 14 should be restated here and clarified as indicated earlier.
3. Page 12, line 23: Please define σ. Why is $\mathbf{x}$ in bold while dx is not?
4. Page 12, line 25: I find the use of $\mathbf{H}_i$ as both a Jacobian and an indicator of model identity to be very confusing. Why not let $\mathbf{H}_i$ be the Jacobian of model i, and introduce a model indicator, say Δ, an integer-value random variable taking values in $1, 2, \ldots, M$, where M is the number of models?