Response to Dr. Foster's comments

The authors appreciate Dr. Foster's insightful review and many helpful comments. A point-by-point response is provided below.

Reviewer's comments:

Synopsis: This paper describes a numerical model that can be used to investigate how the hurricane boundary layer mean flow affects the resulting organized roll vortices. Observations suggest that rolls are a fundamental aspect of the hurricane boundary layer. Yet, no numerical model PBL parameterizations include their effects. In large part this is because quantitative information on how they affect the fluxes is lacking in observations. So, there have been attempts to capture their effects in LES-like numerical models. Given a simulation of rolls, their effects can be examined in isolation. The key question is whether or not the simulation properly represents the hurricane boundary layer.

Discussion: The method in this paper uses a relaxation methodology to impose a mean flow. It is clear that rolls are generated by a combination of shear and convective instabilities (dominated by the shear instability). The shape of the mean inflow pro- file (radial flow) controls the shear instability. However, the basic mean flow profiles are established by both the rolls and non-roll fluxes, which leaves a chicken-or-the-egg theoretical problem. Theory shows that the mean flow modifications due to rolls (roll-flux divergence effects) is somewhat subtle, so it makes sense to start with a reasonable non-roll flux (i.e. down-gradient, local fluxes), idealized PBL and examine the finite perturbations. This is the approach used in theoretical studies and it is essentially what has been done in this paper.

However, there is no clear way to select the down-gradient model and, as has been demonstrated elsewhere, the resulting mean flow profiles can vary wildly, as will the associated rolls. The value of this paper is that the relaxation methodology "generalizes" the LES technique toward the freedom allowed in theoretical studies to explore the parameter space associated with the selection of mean flow profiles. However, care must be taken to ensure that the target mean flow profiles are in fact realizable. There must be a consistent effective eddy-viscosity profiles that would produce the target mean flow profiles. This latent eddy viscosity associated with the target mean flow profile and the non-roll fluxes should be part of this paper.

Interesting questions arise. In some sense, current remote sensing capabilities are making it much easier to measure basic roll characteristics and mean surface winds than many other aspects of the turbulent hurricane boundary, especially turbulent fluxes (much less being able to separate the fluxes into roll and non-roll contributions). Can tools such as the model presented here (and theoretical models) combined with observations be used to find constraints on the various local "gradient-flux" methodologies used in numerical models?

How can the non-local roll fluxes be parameterized? The mass flux-like method pro- posed by Zhu has a fundamental limitation. Convective boundary layers are comparatively much simpler than mixed shear/convective boundary layers. The former is highly skewed and the nonlocal fluxes are largely vertical over the locally warmer perturbations. The roll PBL is much less skewed, the roll characteristics depend on the entire mean flow profile, and, the

updrafts are not vertical and co-located with the warmer temperature perturbations. (Zhu's lateral momentum entrainment parameter is an or- der of magnitude larger than that used for heat in mass flux models; this result has been replicated in theoretical models.) It might be simpler to attempt to modify the simple Mellor-Yamada-based parameterizations to include roll effects. However, the basic closure assumptions used in these models all assume near isotropy in the turbulence, so there may be fundamental inconsistencies.

In any case, the model that was developed for this paper stakes out an interesting middle path for exploring hurricane boundary layer rolls. In combination with the other tools and remote sensing capabilities that have been developed recently, we may be on the cusp of developing hurricane boundary parameterizations that correctly capture both the local and nonlocal contributions to the turbulent fluxes. As shown in this paper and elsewhere, the nonlocal fluxes are not a small contribution to the total. And they are inconsistent with standard down-gradient parameterization.

Response:

We agree with the reviewer on many points in the discussion comments, particularly on the parameterization of the momentum flux from both the roll and non-roll contributions. In our view, there are at least two scenarios in which the K closure would fail. The first is related to the nonlocal flux problem for which the flux is no longer locally downgradient as described by the closure theory. This scenario usually occurs when the mean wind profile has a maximum or minimum across which the vertical gradient changes sign while the momentum flux maintains the same sign. For example, the super-gradient wind in HBL may lead to this condition (Gao and Ginis 2016; referred to as GG16 hereafter). The second occurs when there is a large difference between the transfer coefficients for the radial and tangential winds, implying a single momentum flux transfer coefficient does not work for both directions simultaneously. Both scenarios have been examined by multiple authors (e.g. GG16; Green and Zhang 2015). Our simulation (H3) has indications of both scenarios, although they seem to be less robust than in other studies.

One of the advantages of this mean nudging approach is to provide more flexibility in choosing various idealized and realistic mean wind profiles for the HBL. This work has demonstrated some success of this approach. On the other hand, because of the strong nudging, the effects of the rolls on the mean wind profile are difficult to isolate. This drawback is briefly touched upon in the last paragraph of the summary section. One of the issues the reviewer raised is whether the target wind (and therefore the LES mean wind) is realizable. This is a good and valid point. While our target wind profiles are not derived directly from any balanced HBL model solutions, they are carefully chosen based on the normalized profiles of Foster (2005) and the observations of Morrison et al (2005). Our overall comment is that the target winds have the essential HBL features so that the simulated rolls are consistent with observations and other theoretical studies in many aspects. The choices of the target winds are justified in this idealized HBL study even though they may not necessarily be observable in the real world or derivable from some basic balance dynamic systems. The detailed response is as follows.

The relaxation is used to nudge the LES mean wind toward the target profiles so that the resultant equilibrium mean wind profiles have special characteristics for effective simulation comparisons. For example, the three simulations in either group H or L have a similar inflection point height but different wind shears. In general, group L has stronger wind shear and lower inflection point levels than those from group H. In addition, all the simulations have about the same surface wind speed. This is designed to investigate the impact of both the wind shear strength and the inflection point level associated with the shear on roll formation. Because the target winds can never be reached in simulations, it is not possible to derive the eddy viscosity associated with the target winds from the LES simulations. Although our target winds may be different from the basic-state mean wind profiles derived from the HBL momentum balance equations (e.g. Foster, 2005), they carry some essential features that are similar to the model derived and observed wind profiles, namely an inflection point in the radial wind profile, the super-gradient wind in HBL, and the gradient wind balance above the HBL. Therefore, we believe that the target wind profiles used in our study are relevant to real HBLs and as a result, the simulated rolls have broad similarities with those from observations and theoretical studies.

To clarify this issue, we have expanded the description and discussion about how the target wind profiles are formulated for different simulations in the text. Although we are not able to show the eddy-viscosity profile for the target wind, we present the eddy-viscosity profiles from three spectral groups for the LES mean winds of the simulation H3 in a new subsection 4.4. New discussions are also included to reflect the above response.

Modified text

"The target wind profiles are formulated based on the normalized typical hurricane wind profiles obtained from a dynamical model of Foster (2005) and from the observations by Morrison et al. (2005). The relaxation is used to nudge the LES mean wind toward the target profiles so that the resultant equilibrium mean wind profiles have special characteristics for effective simulation comparisons. We have experimented with dozens of LES simulations using a variety of the wind profiles. The two groups of the target wind profiles (i.e., H and L groups, see Fig. 1) are chosen from these additional trial simulations and they exhibit systematic variations in shear strength and infection point levels to serve our objectives. The target radial wind U_T of H2 and tangential wind V_T of group H generally follow those of Fig. 2 of Foster (2005) except for the HBL height. In addition, the super gradient wind shape is also included in V_T in accordance to Fig 3a of Morrison et al. (2005). The U_T profile of H2 is multiplied by 0.5 and 1.5 to provide U_T for H1 and H3 with the different shear strength but similar IPL, respectively. The target radial wind U_T of L2 is obtained by vertically suppressing U_T of H2 and increasing the near-surface value to 13 m s^{-1}. Then, U_T of L2 is multiplied by 0.5 and 1.5 to give U_T of L1 and L3, respectively. The target tangential wind profile V_T of group L is obtained by lowering the HBL height for V_T of group H." (Page 6, 1-12)

"While there is some quantitative difference between the target wind profiles defined above and the ones derived from the basic HBL balance equations such as those of Foster (2005), they carry some essential features that are similar to the model-derived or observed wind profiles such as an inflection point in the radial wind, the super-gradient wind in HBL, and the gradient wind balance above the HBL. Given our objective of investigating the impact of the wind shear (including both the shear strength and the inflection point level) on the roll structure, we believe that our choices of the target winds are justified in the sense that they retain the basic HBL mean wind features and provide a simple way to make a meaningful comparative study." (Page 6, line 23-29)

Please see the included subsection 4.4 in the response to the specific comment.

Specific suggestions:

1) Review the paper for English grammar. It is quite well written, but a few minor errors are present.

We have reviewed the paper very carefully.

2) On page 10, line 15-16 (and other places): I think you mean the vertical shear of the radial wind".

We have changed the wording according the reviewer.

" …increasing the vertical shear of the radial wind results in…"

3) Page 11 line 3: Might be clearer to use "r, t" subscripts instead of "x, y".

Because "x" and "y" are defined as the radial and tangential direction on page 3 (line 30) and page 4 (line 1-2) when the coordinate was introduced. For consistency, we still use "x, y" here. But the following sentence is added:

"(Note that the subscript "x" and "y" represents the radial and tangential direction, respectively, as defined in subsection 2.1.)"

4) I think you need to provide effective eddy viscosity profiles for the choices of the target mean flow profiles, especially the "mixed" ones. As a further benefit, this would allow direct comparisons with theoretical roll models.

As explained for the discussion comment, the target mean wind in the relaxation is only used to regulate the LES predicted mean wind to achieve the desired mean wind profiles. The target wind profiles are never reached in the simulations. Therefore, there is no effective eddy-viscosity for the target wind profiles from the simulations. We, however, present the eddy-viscosity profiles from three spectral groups for the LES mean winds of the simulation H3 in a new subsection 4.4. In our view, it is the LES mean wind (not the target wind) that directly generates the turbulence and roll circulations. Therefore, these eddy-viscosity and the mean wind profiles may be tested in the theoretical roll models. The entire new subsection is copied here.

"4.4 Momentum transfer coefficients

Momentum transfer coefficients, defined by the negative ratio of the momentum flux to the mean wind shear according to the K theory, play a central role in the representation of HBL. It has been shown and/or argued that the roll generated momentum flux cannot be represented by the local transfer theory because of the "large-scale" nature of the roll circulation in terms of its horizontal and vertical scales as compared to z_i (e.g., Foster, 2005; Zhu, 2008; and Gao and Ginis 2016). In this subsection, the issue of the transfer coefficient is discussed using the results from the spectral analysis. Because the momentum fluxes have been decomposed into three spectral groups, it is convenient to compute the transfer coefficient for each group. By definition, the transfer coefficient for the radial momentum flux from each spectral group K_u^i can be calculated by

$$K_u^i = -\frac{\overline{w'u'}^i}{\partial\overline{u}/\partial z}, \tag{3}$$

where superscript $i \in (s, l, r)$ represents the small-scale (< 1 km), large-eddy-scale (1 – 2.5 km), and roll-scale (> 2.5 km). The transfer coefficient for the tangential momentum flux K_v^i is computed in a similar equation. Because both the momentum fluxes (except for $\overline{w'u'}^r$) and the vertical gradient of the wind speed are very close to zero above 1400 m, all the values of the computed transfer coefficients are removed for $z \geq 1400$ m.

These transfer coefficients are shown in Fig. 12. The values of K_u^i change little with height from 200 m to 1.1 km, above which K_u^r increases significantly because of both the finite values of $\overline{w'u'}^r$ and the near-zero gradient of $\overline{u}$. The non-zero $\overline{w'u'}^r$ above HBL is likely caused by the internal gravity waves which are connected to the roll structure and have the same wavelength as the rolls as discussed previously (also see GG14). The transfer coefficients K_u^i are ill-defined around z = 200 m because $\partial\overline{u}/\partial z \approx 0$. Unlike the nearly constant K_u^i, the tangential transfer coefficients K_v^i increase with height from zero at surface to ~ 150 m^2 s^{-1} at 850 m. They then sharply increase near the HBL top where $\partial\overline{v}/\partial z \approx 0$, which results in both very large positive and negative values of K_v^i because $\overline{w'v'}^i$ is always negative while $\partial\overline{v}/\partial z$ changes sign. This behavior is contradictory to the downgradient transfer theory which assumes none negative K_v^i (e.g., Stull 1988, p. 108). This is similar to the result of the counter-gradient $\overline{w'v'}$ for the same reason from the two-dimensional roll model of Gao and Ginis (2016). The main difference is that their counter-gradient feature occurs in the mid-HBL where the momentum flux is significantly larger than that near the HBL top in our simulation of H3 as shown in Fig. 10b. This difference is mainly caused by the different mean tangential wind profiles obtained with different methods: the dynamic model approach of Gao and Ginis (2016) and the mean nudging in this work. Therefore, there is a need to apply the same mean wind profiles in both the 2-D roll and LES models for a more effective comparison. It is also worth noting that the sub-grid scale parameterized flux is not included in either $\overline{w'v'}^i$ or $\overline{w'u'}^i$. This inclusion would slightly change the small-scale transfer coefficient profiles K_v^s and K_u^s.

Overall, there are marked differences between K_u^i and K_v^i in the mid-HBL between 200 m and 850 m. The values of either K_u^i or K_v^i do not vary greatly between the spectral groups even though the differences are obvious. The counter-gradient feature occurs at the HBL top where $\partial\overline{v}/\partial z$ changes sign and $\overline{w'v'}^i$ remains negative. Its effect on the momentum flux parameterization would be likely negligible, because $\overline{w'v'}$ is very small near the HBL top in the simulation H3." (Page 15, line 6 – 27; Page 16, line 1 – 14)

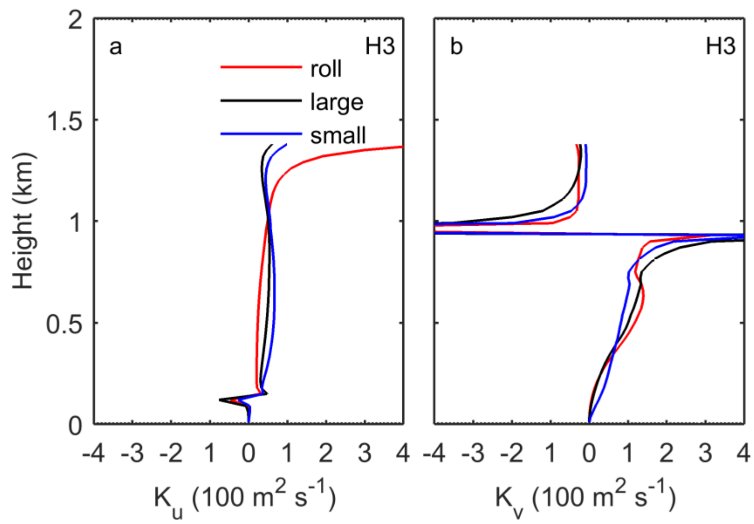

Fig. 13. The momentum transfer coefficients for three spectral groups from H3 for K_u (a) and K_v (b). The three spectral groups are small scale (< 1 km), large-eddy (1-2.5 km), and roll (> 2.5 km), respectively.

5) The roll effects are studied in detail and it took me a long time to puzzle through all of the figures. That isn't a criticism; there is a lot of information to digest. I wonder if some sort of "carton" drawing could help with the visualization and putting the results in context.

We understand the reviewer's frustration. It would be certainly nice to summarize these results in a "carton" drawing. We have made further effort to improve the text and figure captions. We have come up with several versions of "carton" to help summarizing the dynamics and roll characteristics, as suggested by the reviewer. Unfortunately, none of them are artistic and revealing enough to our satisfaction. It is probably, in part, because the work describes several aspects of the rolls such as the mean wind profiles, instantaneous roll patterns, roll spectral characteristics, etc. It is really difficult to put together these various aspects in a cartoon.