Response to: "Frequently asked questions about Lagrangian Descriptors" by Ana Mancho

Alfonso Ruiz-Herrera*

September 27, 2016

Next I provide a reply to Mancho's comment in order to avoid misleading conclusions in the literature.Vague comments (without external peer review process) are not adequate to clarify a methodology.

1 What are Lagrangian Descriptors?

The M function does not detect the invariant manifolds in many simple dynamical systems, e.g. $x' = x, y' = -2y$ or $x' = f(x), y' = -yf'(x)$ with $f(x) = \tanh x$. In fact, in a neighbourhood of the y-axis (the stable manifold of the origin) the contour lines of the M function are horizontal lines in the previous systems.

2 What are singular features of the M function?

Mancho, Wiggins, and their co-workers always use an unclear definition for the concept of singularity (even it looks rigorous). The method of Lagrangian Descriptors aims to detect mathematical objects with mathematical tools. The sentence: "Singular features are defined as abrupt changes in M which are quantified by discontinuities in the derivative of M along a specific direction crossing the manifold" is meaningless from a mathematical point of view. Please, give a formal definition. For instance, everyone knows that $f(x) = |x|$ is not smooth at 0 because $\lim_{x \longrightarrow 0} \frac{f(x)-f(0)}{x-0}$ does not exist.
Two remarks are in order:

- We have proved that the contour structure of the M function has no dynamical significance in the detection of invariant manifolds (independently of the definition under consideration).
- As mentioned in my previous report, the theorems presented in *(Lopesino et al. 2015)* are a consequence of the diagnostic itself because MD_p is non-smooth if, for some iteration,

$$x_{i+1} = x_i \quad or \quad y_{i+1} = y_i. \tag{2.1}$$

*Departamento de Matemáticas, Universidad de Oviedo, Spain (alfonsoruiz@dma.uvigo.es, ruizalfonso@uniovi.es).

3 What is novel in the method of Lagrangian Descriptors with respect to previous work based on time averages along trajectories?

Analyzing the novelty of a diagnostic that leads to inaccurate responses does not seem very interesting. Mancho, Wiggins, and their co-workers have emphasized that their method is computationally cheap. However, in all global Lagrangian (diagnostic) computations of invariant manifolds in general data sets, the main computational cost from the advection of a large number of initial conditions. So the M function is no less (or more) computational than, say, the FTLE analysis for the same level of spatial resolution.

4 What is the objectivity property discussed in the literature? Is it important for practical purposes that LDs satisfy that property?

The notion of objectivity refers to the independence of the observer, see

- *Non-objectivity of the M function and other thoughts*, Interactive discussion in Nonlinear Processes in Geophysics about the paper *Detecting and tracking eddies in oceanic flow fields by Rahel Vortmeyer-Kley, U. Grave, and U. Feudel.*

Invariance under Galilean transformations is much weaker. However, I agree with Ana Mancho that the method of Lagrangian Descriptors is not objective (even in this weak sense). The message of Haller's comment was: The M-function is not objective. Mancho has now a different opinion on this because she stated in her reply to Haller's comment that the M-function was objective. (See Remarks on the comment Non-objectivity of the M function and other thoughts.) As emphasized in Haller's comment, one can simply point out that this diagnostic is not objective and hence cannot possibly to capture anything intrinsic about material transport. End of the discussion. We mention that Mancho's reply was posted without external peer-review process because of she is in the editorial board of Nonlinear Processes in Geophysics (the journal that contains most papers on Lagrangian Descriptors).

5 Do Ruiz-Herrera (Chaos 2015) results disqualify the use of Lagrangian Descriptors in geophysical flows?

As mentioned in my previous report, Ana Mancho and her co-workers in (Balibrea 2015) misrepresent what they have done. I have provided a detailed list of their contradictions. For instance, Figure 1 in *(Ruiz-Herrera arxiv)* is exactly the same as Figure 1 (c) and 2(a) in *(Mancho et al 2013).* However, they have introduced the following misleading comment in *(Balibrea-Iniesta 2016)*, (see caption in figure 1):
This figure should be compared with figure 1 of the comment of Ruiz-Herrera.

The performance of the method of Lagrangian descriptors has been discussed theoretically just in the trivial system

$$\begin{cases} x' = x \\ y' = -y \end{cases} \tag{5.1}$$

In Mancho's applications and Manney-Lawrence work, the geophysical flows are not similar in any remote sense to (5.1). Of course, the performance in this trivial system is not enough to provide a effective diagnostic for any flow. It is clear that the M function always creates patterns when plotted over the initial conditions. However, as emphasized in my work, this output has no dynamical significance.

The message of my work is that the method of Lagrangian Descriptors can fail in simple systems. There are many counter-examples to the method of Lagrangian Descriptors, for instance, $x' = f(x), y' = -y$ with f bounded; $x' = 2x, y' = -y$ or $x' = f(x), y' = -yf'(x)$ with $f(x) = \tanh x$. More pathologies and counter-examples are discussed in (Ruiz-Herrera Chaos 2015) and (Ruiz-Herrera in press). Therefore, we can not expect reliable responses in complex systems.

As mentioned in my previous comment, the M-function was used for the first time in the pioneering work

- A.J. Jimenez Madrid and A.M. Mancho, *Distinguished trajectories in time dependent vector fields,* **Chaos 19** (2009), 013111.

In two papers published in the same journal,

- A. Ruiz-Herrera, *Some examples related to the method of Lagrangian Descriptors,* **Chaos 25** (2015), 063112,

- A. Ruiz-Herrera, *Performance of Lagrangian Descriptors and Their Variants in Incompressible Flows*, **Chaos** (in press),

I have shown that the contours of the M-function have no significance in the detection of barriers to transport (under any consideration). *Chaos: An Interdisciplinary Journal of Nonlinear Science* offers the possibility of comments to regular papers, see

- http://scitation.aip.org/content/aip/journal/chaos.

Mancho, Wiggins and their co-workers submitted a reply to *(Ruiz-Herrera 2015)* the last year. Please, submit your new critiques to the journal in order to avoid misleading conclusions in the literature.