Atmos. Chem. Phys. Discuss., doi:10.5194/acp-2016-405, 2016
Manuscript under review for journal Atmos. Chem. Phys.
Published: 17 May 2016

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A modified impulse-response representation of the global response to carbon dioxide emissions

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Abstract. Projections of the response to anthropogenic emission scenarios, evaluation of some greenhouse gas metrics and estimates of the social cost of carbon, often require a simple model that links emissions of carbon dioxide (CO_2) to atmospheric concentrations and global temperature changes. An essential requirement of such a model is to reproduce the behaviour of more complex models as well as an ability to sample their range of response in a transparent, accessible and reproducible form. Here we adapt the simple model of the Intergovernmental Panel on Climate Change 5th Assessment Report (IPCC-AR5) to explicitly represent the state-dependence of the CO_2 airborne fraction and reproduce several idealised experiments performed with more complex models. We find that a simple linear increase in 100-year integrated airborne fraction with cumulative carbon uptake and global temperature change is both necessary and sufficient to represent the response of the climate system to CO_2 on a range of timescales and under a range of experimental designs. Quantified ranges of uncertainty (analogous to current assessed ranges in Equilibrium Climate Sensitivity and Transient Climate Response) in integrated airborne fraction over the 21st century under a representative mitigation scenario, and an assessed range in how much this quantity may have changed relative to pre-industrial conditions, would be valuable in future scientific assessments.

1 Introduction

Future emissions of CO_2 over the remainder of the century are uncertain and a strong function of future climate policy (Van Vuuren et al., 2011). Future climate changes, and their associated impacts, will largely be determined by future cumulative carbon dioxide emissions (Matthews et al., 2009; Allen et al., 2009; Meinshausen et al., 2009), but linking specific CO_2 emission scenarios to future transient climate change requires a model of the interacting climate-carbon-cycle system. Comprehensive Earth System Models (ESMs) directly capture the physical processes that govern the coupled evolution of atmospheric carbon concentrations and the associated climate response (Friedlingstein et al., 2006). However, such models are typically highly computationally intensive and can therefore only be run for a few representative future emission scenarios (Taylor et al., 2012). For analysis of arbitrary emissions scenarios, as required for the integrated assessment of climate policy and calculation of the social cost of carbon, a computationally efficient representation of the Earth system is required (Marten, 2011).

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Simplified representations of the coupled climate-carbon-cycle system take many forms (Hof et al., 2012). A key test for simplified ESMs is whether they correctly capture the physics of the co-evolution of atmospheric CO_2 concentrations and global mean temperature under idealised settings. Following a CO_2 pulse emission of 100GtC under present-day climate conditions, ESMs (and Earth System Models of Intermediate Complexity – EMICs) display a rapid-draw down of CO_2 with the concentration anomaly reduced by approximately 40% from peak after 20 years and by 60% after 100 years, followed by a much slower decay of concentrations leaving approximately 25% of peak concentration anomaly remaining after 1000 years (Joos et al., 2013). The effect of this longevity of fossil carbon in the atmosphere, combined with the gradual "recalcitrant" thermal adjustment of the climate system (Held et al., 2010), is to induce a global mean surface temperature (GMST) response to a pulse emission of CO_2 of a rapid warming over approximately a decade to a plateau value of GMST anomaly (Joos et al., 2013). Warming does not noticeably decrease from this value over the following several hundred years, indicating that, short of artificial CO_2 removal (CDR) or other geoengineering methods, CO_2-induced warming is essentially permanent on human-relevant timescales.

A second important feature of more complex climate-carbon-cycle models is the increase in airborne fraction (the percentage of emitted CO_2 that remains in the atmosphere after a period of time) over time in scenarios involving substantial levels of emissions or warming (Millar et al., 2016). An emergent feature of the CMIP5 full-complexity ESMs appears to be that this increase approximately cancels the logarithmic relationship between CO_2 concentrations and radiative forcing, yielding an approximately linear relationship between cumulative CO_2 emissions and CO_2-induced warming (Matthews et al., 2009; Gillett et al., 2013).

In this paper we propose a simple extension of a standard impulse-response model of the carbon-cycle-climate system to reproduce the physical behaviour of the ESMs under a variety of idealised experiments. Our starting point is the impulse-response functions that are provided for the calculation of multi-gas equivalence metrics in IPCC-AR5 (Myhre et al., 2013), which we extend by coupling the carbon-cycle to the thermal response and to cumulative carbon uptake by terrestrial and marine sinks. This extension is necessary because the use of a state-insensitive impulse-response model cannot simultaneously reproduce the relationship between emissions, concentrations and temperatures seen over the historical period and the projected response over the 21^{st} century to both high-emission and mitigation scenarios estimated from more complex models. Section 2 describes the formalism of the model that we use. Section 3 then demonstrates the model's ability in replicating ESM behaviour under a set of idealised experiments, namely those of Gregory et al. (2009), Joos et al. (2013) and Herrington and Zickfeld (2014). We also compare the behaviour of our model with the simple models of Myhre et al. (2013), Glotter et al. (2014) and Meinshausen et al. (2011). Section 3.1 discusses how probabilistic assessments of climate response to CO_2 emissions could be made using our proposed model. Section 4 provides a concluding summary and discussion.

2 A "Finite Amplitude Impulse Response" (FAIR) model

The IPCC-AR5 proposed an idealised simple climate model for metric calculations, incorporating a "2-box" or "2-time-constant" model of the temperature response to radiative forcing with a "4-time-constant" (one of which is infinite) impulse-

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response model of the CO_2 response to emissions (Myhre et al., 2013). As proposed in IPCC-AR5, the carbon-cycle constants are not affected by rising temperature or CO_2 accumulation, and hence only represent the specific response to a particular perturbation scenario. In more comprehensive models, ocean uptake efficiency declines with accumulated CO_2 in ocean sinks (Revelle and Suess, 1957) and uptake of carbon into both terrestrial and marine sinks are reduced by warming (Friedlingstein et al., 2006). A state-insensitive impulse-response model is therefore unsuitable, unless modified, for calculations of, for example, the social cost of carbon against realistic baseline trajectories or long integrations with historical and projected emissions. We shall refer to the Myhre et al. (2013) combined (but non-interacting) carbon-cycle and temperature response, tuned to present-day climate conditions, as the AR5-IR model.

Here we attempt a minimal modification of the AR5-IR model to allow it to mimic the behaviour of more complex models in response to finite-amplitude CO_2 injections, which we call a Finite Amplitude Impulse-Response (FAIR) model. To introduce a state-dependent carbon uptake as simply as possible, we apply a single scaling factor α to all four of the time-constants (time units are in years) in the carbon-cycle of the AR5-IR model, such that the CO_2 concentrations in the 4 "carbon reservoirs" are updated thus:

$$\frac{\mathrm{d}R_i}{\mathrm{d}t} = a_i E - \frac{R_i}{\alpha\tau_i} \quad ; \quad i = 1,4 \tag{1}$$

where E are annual CO_2 emissions, converted to ppm/year. Atmospheric CO_2 concentrations are given by $C = C_0 + \sum_i R_i$, and radiative forcing by:

$$F = \frac{F_{2\mathrm{X}}}{\ln(2)} \ln\left(\frac{C}{C_0}\right) + F_{\mathrm{ext}} \quad , \tag{2}$$

where C_0 is the pre-industrial CO_2 concentration, $F_{2\mathrm{X}}$ the forcing due to CO_2 doubling, and F_{ext} the non-CO_2 forcing. GMST anomalies are computed thus:

$$\frac{\mathrm{d}T_j}{\mathrm{d}t} = \frac{c_j F - T_j}{d_j} \quad ; \quad T = \sum_j T_j \quad ; \quad j = 1,2 \tag{3}$$

with coefficients a_i, d_j and τ_i as given in AR5 Chapter 8, tables 8.SM.9 and 8.SM.10 (Myhre et al., 2013). The sole difference between this model and that used for metric calculations in AR5 is that the c_j are set to give an Equilibrium Climate Sensitivity (ECS) =2.75K and Transient Climate Response (TCR) =1.6K (corresponding to $c_1 = 0.46$ and $c_2 = 0.27$ (Millar et al., 2015)), closer to current best-estimate values than the ECS=3.9K and TCR=2.2K implied by the parameter-values given in Myhre et al. (2013), and the introduction of the state-dependent coefficient α.

To identify a suitable state-dependence, we focus on parameterising variations in the 100-year integrated impulse response function, iIRF_{100}. Focussing on the integrated impulse response, or average airborne fraction over a period of time, as opposed to the airborne fraction at a particular point in time, is more closely related to the impact of emissions on the global energy budget, and also to other metrics such as Global Warming Potential (Joos et al., 2013). With other coefficients fixed, this is a monotonic (but non-linear) function of α:

$$\mathrm{iIRF}_{100} = \sum_i \alpha a_i \tau_i \left[1 - \exp\left(\frac{-100}{\alpha\tau_i}\right)\right]. \tag{4}$$

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Following other simplified carbon-cycle models (Meinshausen et al., 2011; Glotter et al., 2014), we assume $iIRF_{100}$ is a function of accumulated perturbation carbon stock in the land and ocean, $C_{acc} = \sum_t E - (C - C_0)$, converted to GtC, and of GMST anomaly from pre-industrial conditions, T. A simple linear relationship appears to give an adequate approximation to the behaviour of more complex models:

$$iIRF_{100} = r_0 + r_C C_{acc} + r_T T. \quad (5)$$

Values of r_0=35 years, r_C =0.02 years/GtC and r_T =4.5 years/K, with ECS=2.7K and TCR=1.6K, give a numerically-computed $iIRF_{100}$ of 53 years for a 100 GtC pulse released against a background CO_2 concentration of 389ppm following a historical build-up, consistent with the central estimate of Joos et al. (2013).

For a prognostic model, we compute $iIRF_{100}$ at each time-step using C_{acc} and T from the previous time-step and equation 5, convert to a α using equation 4 and apply to the carbon-cycle equations (equation 1). This means the $iIRF_{100}$ is only exactly reproduced under constant background conditions with infinitesimal perturbations. Values of $iIRF_{100}$ larger than 100 years correspond to a net carbon source in response to a perturbation, and, as perturbations to the carbon stock in the atmosphere would grow indefinitely, makes the model unstable. In this regime there is no solution for α, so we set $iIRF_{100}$ to a maximum value of 95 years, corresponding with these parameters to α=65.4. This physically corresponds to a near-absence of carbon sinks in the Earth system following a very large injection, with very slow rates of decay of atmospheric concentrations.

3 Results

As a test of the behaviour of the FAIR model, we here conduct a set of experiments under idealised CO_2 emission and concentration scenarios that have also been conducted in the literature for more complex ESMs. Throughout this section, we contrast the performance of the FAIR model, described in the previous section, to the AR5-IR model, the MAGICC model (Meinshausen et al., 2011) and the BEAM model (Glotter et al., 2014), a simple carbon-cycle model that explicitly represents the physical effects of oceanic carbon uptake on ocean carbonate chemistry (Williams and Follows, 2011). We use a version of the BEAM model with no temperature dependence of model parameters, which has been shown to be small in Glotter et al. (2014).

Gregory et al. (2009) conducted a set of experiments with two different ESMs under specified CO_2 concentrations, followed up with a broader range of models by Arora et al. (2013). Concentrations were increased at 0.5%yr^{-1}, 1%yr^{-1} and 2%yr^{-1} respectively and consistent emissions were derived for different configurations of the ESMs: a "biogeochemically-coupled" experiment, where the carbon-cycle is only allowed to respond to the direct effect of increasing CO_2 concentrations and not to the resultant warming; a "radiatively-coupled" experiment in which the climate system is allowed to respond to the radiative forcing of CO_2 but the carbon-cycle is only allowed to respond to the simulated warming and not to increasing CO_2; and a "fully-coupled" experiment in which the carbon-cycle is allowed to respond to both warming and CO_2 concentrations. Within our simple model framework, we recreate the "biogeochemically-coupled" experiment by setting r_T =0. We approximate the "radiatively-coupled" experiment by evaluating the difference between the "fully-coupled" and "biogeochemically-coupled"

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experiments. Although Gregory et al. (2009) found that the relationship between the experiments was not simply a linear summation at high CO_2 concentrations, this serves as an adequate approximation for our purposes here, since our objective is the correct representation of aggregate feedbacks rather than a breakdown into specific contributions.

Figure 1 shows the response of the FAIR model (blue) described in section 2 under the three experiments described above. The responses of the AR5-IR (red) and BEAM models (green) are also shown. Figure 1a shows total carbon uptake by ocean and land, C_{acc} , as a function of time in the $1\%\text{yr}^{-1}$ increasing CO_2 experiment. As in the full-complexity ESMs shown in Arora et al. (2013) and Gregory et al. (2009), the coupling between temperature changes and the carbon-cycle in the FAIR model acts to suppress carbon uptake, shown by the difference between the thick and thin blue lines, a mechanism that is absent (by construction) in both AR5-IR and biogeochemical version of the BEAM model considered here. The coupling with cumulative carbon uptake in the FAIR model also increases airborne fraction in the later stages of the experiment relative to earlier stages (see figure 4), as illustrated by the approximately linear increase in C_{acc} in the "biogeochemically-coupled" experiment, also consistent with ESM responses. A constant airborne fraction necessarily gives an approximately quadratic increase in C_{acc} in this experiment, as illustrated by the AR5-IR model.

Figure 1b shows C_{acc} as a function of atmospheric CO_2 concentration: again, the FAIR model captures the concave-downward form of this diagnostic, in contrast to the AR5-IR model. Figure 1c shows the impact of GMST increase on cumulative uptake, or the difference between the biogeochemically coupled and fully coupled experiments shown in panel (a), as a function of warming. $1\%\text{yr}^{-1}$, $0.5\%\text{yr}^{-1}$ and $2\%\text{yr}^{-1}$ experiments all lie along the same line in panel (c). Panel (d) show cumulative airborne fraction increases after an initial decline, similar to the behaviour of the ESMs. In contrast, the IPCC-AR5 model shows a steady decrease in the cumulative airborne fraction with higher concentrations due to the state-invariant rates at which a pulse of carbon is removed from the atmosphere. The initial airborne fraction is higher in the AR5-IR model, as the carbon-cycle response parameters used here are those representative of present-day pulse-response experiments of Joos et al. (2013). An integration of this model under historical emissions therefore produces historical concentrations significantly in excess of those observed in the early 21^{st} century (see figure 4).

The BEAM model (run with parameters as given in Glotter et al. (2014), which are tuned for long time-scales) displays a very low cumulative uptake of carbon from the atmosphere relative to the other simple models considered here, associated with a cumulative airborne fraction initially around 0.9 which does not decline substantially with time. This model therefore also displays much higher concentrations than observed over the historical period when driven with estimates of historical emissions (not shown), although it must be noted again that the model was tuned to long time-scale responses of more complex models and not over the historical period. Although the BEAM model explicitly solves the equations of the ocean carbonate chemistry, it displays a roughly constant cumulative airborne fraction under the exponential concentration increase scenarios considered here, unlike the ESMs which behave more like the FAIR model constructed in this paper (see figure 3, 4, 5 and 6 in Gregory et al. (2009)). In addition, as BEAM focuses solely on oceanic feedback mechanisms and cannot capture saturation of land carbon sinks and the dependence of land carbon sinks on warming, an important part of the ESM feedbacks (Arora et al., 2013). Land carbon uptake can contribute an equal (or even greater) fraction of the total system carbon uptake in ESM models over time periods on the order of a century. However, it must be noted that physical mechanisms of the land carbon-cycle

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response to warming remain poorly understood compared to oceanic mechanisms and a wide uncertainty exists about their future responses. Due to this lack of potentially important feedback processes associated with the land carbon-cycle, we do not extend the comparison with the BEAM model in any of the subsequent analyses.

Figure 2 shows the response of the simple climate-carbon-cycle models to the emissions pulse experiments of Joos et al. (2013). In these experiments, a set of ESMs are used to derive emissions that are consistent with concentrations rising as observed historically they exceed 389ppm (achieved in 2012) and held constant thereafter. Figure 2a shows the emissions and warming consistent with the baseline (no additional pulse emission) experiment for the FAIR and AR5-IR models. A declining but sustained low level of diagnosed emissions are required following stabilisation of atmospheric concentrations in order to maintain them at a constant level. In a second experiment, a 100GtC pulse is added to these calculated emissions in the year that concentrations exceed 389ppm and the resulting concentration and temperature anomalies are compared with and without the pulse emission to isolate the coupled response to the pulse emission alone.

Figure 2b shows the concentration and warming response to a 100GtC pulse emission at 389ppm background. After 100 years the pulse in the concentration anomaly in the fully coupled FAIR model has decayed to 0.46 of its initial value, slightly greater than the multi-model average of the ESM responses of 0.41, but, the $iIRF_{100}$ of 53 years is consistent with the ESM multi-model mean of 52.4 years (Joos et al., 2013). Excluding temperature feedbacks on the carbon-cycle increases the decay of the temperature response to the pulse over the century following the pulse emission. The "fully-coupled" FAIR model shows temperature initially adjusting rapidly followed by near-constant temperature over the remainder of the century. The "biogeochemically-coupled" version of the FAIR model, in which temperature-induced feedbacks are suppressed, reduces the integrated 100-year airborne fraction by 11%.

Figure 2c and 2d also show the response to a 100GtC and a 5000GtC pulse respectively, applied in pre-industrial conditions. Similarly to the response shown by ESMs discussed in Joos et al. (2013), the 100GtC pre-industrial pulse decays faster than the present-day case, due to reduced saturation of the land and ocean carbon sinks. With these parameters, the $iIRF_{100}$ is approximately 30% lower in the pre-industrial case compared to the present day, consistent with corresponding ratio in the most detailed ESMs, with its value of 36 years within the 34-47 years range of the ESMs. The magnitude of temperature response is similar in both cases due to the increased radiative efficiency of a pulse of CO_2 at lower background concentrations counteracting the faster decay of carbon out of the atmosphere. The 89% increase of $iIRF_{100}$ in the 5000GtC pre-industrial pulse relative to the 100GtC pre-industrial, whilst smaller than the approximate doubling observed in the ESMs, shows that the FAIR model can capture the dependence of the pulse-response on pulse size as well as background conditions, whilst the AR5-IR model displays identical pulse response independent of pulse size or background conditions.

A difference between the FAIR model and the ESMs is that restricting temperature-induced feedbacks on the carbon-cycle does not result in a substantial reduction in the $iIRF_{100}$ for the pre-industrial 100GtC pulse experiment (the "fully-coupled" and "biogeochemically-coupled" experiments lie on top of each other in figure 2c), whereas a 13% reduction in $iIRF_{100}$ is observed for the ESMs (Joos et al., 2013). It is only for the 5000GtC pre-industrial pulse experiment that we see a reduction in the $iIRF_{100}$ associated with suppression of the temperature-induced feedbacks on the carbon cycle.

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Herrington and Zickfeld (2014) conducted several experiments with the UVic Earth System Model of intermediate complexity (Weaver et al., 2001). We here emulate the PULSE experiments of Herrington and Zickfeld (2014) by integrating the FAIR model with historical fossil fuel and land-use CO_2 emissions (as derived from historical concentrations using the MAGICC model, (Meinshausen et al., 2011)) together with estimates of the historical radiative forcing from non-CO_2 factors. Pulse emissions of various sizes were then applied over a two-year period from 2008 in order to restrict total all time cumulative emissions to specified totals (see Herrington and Zickfeld (2014) for details). Non-CO_2 forcings are held constant at 2008 levels after following RCP8.5 (Riahi et al., 2011) trajectories for 2005-2008.

Figure 3 shows the response of the FAIR model, as well as the AR5-IR model, to the experiments described above. For all pulse sizes (denoted with different linestyles) contrasting the fully coupled FAIR (thick blue) and the AR5-IR (red) models shows that including carbon-cycle feedbacks is essential to prevent a substantial decay in the temperature anomaly over the first 100 years following the pulse emission. Ricke and Caldeira (2014) used a version of the AR5-IR model to find that the maximum warming from a pulse emissions of CO_2 occurs approximately a decade after emission, but as shown here and as highlighted by Zickfeld and Herrington (2015), not accounting for feedbacks on the carbon-cycle fails to capture the plateau of CO_2-induced warming over the century following emission. At higher pulse sizes, the temperature response in the FAIR model fails to plateau as quickly as at lower pulses, where the balance between carbon-cycle cooling and long-timescale thermal warming takes centuries to reach balance (Figure 3 of Herrington and Zickfeld (2014)).

As a validation of the FAIR model over the historical period, Figure 4 shows the comparisons to historical data from Le Quéré et al. (2015). In figure 4 simulations are commenced from 1850, which is assumed to be a quasi-equilibrium state for the carbon cycle, in order to facilitate comparisons with the observed data (which is available from 1850 onwards). Panel (a) shows the concentration response to estimated global historical CO_2 emissions. The FAIR model replicates concentrations over the past several decades well, as opposed to the AR5-IR model, which has a bias of over 30ppm in 2011. Similarly, emissions derived from the time series of historical atmospheric CO_2 concentrations (Figure 4b), are lower in AR5-IR model, due to the slower decay of CO_2 from the atmosphere over the historical period. Whilst the time mean-value of the airborne fraction (defined as the fraction of emissions remaining in the air after one year) is captured well by the FAIR model, fluctuations are of much greater magnitude in the observed record, indicating short timescale processes and variability in the carbon cycle that is not captured by these simple models. The AR5-IR (which is tuned to the present-day response) displays a too large airborne fraction over the entire historical period and is less consistent with the observations than the FAIR model. Retuning the AR5-IR model to pre-industrial conditions would improve its fit to past emissions, but would then give a worse fit to the behaviour of more complex models under impulse-response experiments and future scenarios.

We also check the FAIR model's response to a pair of benchmark scenarios against that of the widely used MAGICC model (Meinshausen et al., 2011), a simplified box-model of the climate system response to emissions of various greenhouse gases that has commonly been used to assess climate mitigation scenarios (Clarke et al., 2014). Panels a) and b) of figure 5 show the CO_2 concentration from the FAIR model under RCP8.5 and RCP2.6 emissions respectively when forced with emissions diagnosed from the RCP concentration profiles using MAGICC, contrasted with the concentration timeseries from MAGICC

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(purple), which is by definition equal to the corresponding scenario-defined concentration profiles. Non-CO_2 forcing is the same in all cases.

The FAIR model compares well to MAGICC, particularly for the ambitious mitigation scenario (which is less well reproduced by other simplified climate-carbon-cycle models such as BEAM). There is some divergence after 2100 in the high emission scenario, but the behaviour of MAGICC (or indeed any other model) under these more extreme forcing scenarios has not been verified. Whilst comparing the performance of one simple model to another is not as rigorous a test of model performance as comparing directly to the behaviour of ESMs, it is encouraging that the FAIR model shows a close correspondence with a well-known and well-used simple model that has been used extensively to emulate the response of ESMs (Rogelj et al., 2012).

Temperature responses are shown in the lower left panel, and the relationship between temperature and cumulative carbon emissions in the lower right. Crucially, this modified impulse response model captures the straight-line relationship between cumulative carbon emissions and human-induced warming that was highlighted in the IPCC 5th Assessment, and is becoming an integral part of climate change policy analysis (Millar et al., 2016). When integrated under a $1\%\text{yr}^{-1}$ concentration increase scenario, the FAIR model, with parameter settings given in section 2, has a Transient Response to Cumulative Emissions (TCRE) =1.5K/TtC (see figure 6). A common +/- 10% perturbation to the parameters r_T , r_C and r_0 (combined with perturbations to c_1 and c_2 consistent with the IPCC-AR5 likely ranges for TCR: 1.0-2.5K and ECS: 1.5-4.5K) allow the IPCC-AR5 likely TCRE range of 0.8-2.5K/TtC to be spanned, as shown in figure 6. In contrast, the AR5-IR model, with a constant airborne fraction, shows a clear concave-downward shape in a plot of realised warming against cumulative carbon emissions, because the decline of the cumulative airborne fraction is unable to compensate (as it does in more complex models) for the logarithmic relationship between CO_2 concentration and radiative forcing (Millar et al., 2016). The FAIR model also displays some curvature at high cumulative emissions, consistent with Leduc et al. (2015).

3.1 Probabilistic parameter sampling within the FAIR model

Integrated assessment of climate change often requires probabilistic projections of the climate response to CO_2 emissions, partly in order to capture and assess the possibility of extreme, and highly costly, sensitivities within the Earth system (often called "fat-tailed" events) (Weitzman, 2011). Uncertainty in the global climate response to emissions of CO_2 is associated with several factors, which are each considered in turn here.

Uncertainty in the thermal response to radiative forcing typically tends to dominate uncertainty in the response of the global climate system to CO_2 emissions (Gillett et al., 2013). ECS and TCR co-vary in global climate models (Knutti et al., 2005; Millar et al., 2015), with TCR typically considered the more policy-relevant parameter and the parameter better constrained by climate observations to date (Frame et al., 2006; Gillett et al., 2013). Hence varying ECS alone in a probabilistic assessment risks introducing an implicit distribution for TCR that is inconsistent with available observations. Millar et al. (2015) observed that, within the coupled models of the CMIP5 ensemble, TCR and the ratio TCR/ECS (referred to as the Realised Warming Fraction or RWF) are approximately independent. IPCC-AR5 provided formally assessed uncertainty ranges for TCR and ECS

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(Collins et al., 2013) but not for their ratio. RWFs for the CMIP5 models lie within the range 0.45-0.7, while observationally-constrained estimates typically lie in the upper half of this range (Millar et al., 2015).

As IPCC-AR5 likely (>66% probability) ranges for a physical climate parameter attempt to capture structural uncertainties that might be present in all studies, therefore, IPCC-AR5 likely intervals are generally comparable to the 90% confidence intervals in the underlying studies. IPCC-AR5 gives no assessment of the shape of the distribution associated with structural uncertainty as, by definition, this encompasses "unknown unknowns" that are not included in any model or study available. For quantitative modelling purposes, likely ranges are best interpreted as 5-95 percentiles of input distributions for IPCC-AR5 assessed parameters, provided a similar "structural degradation" is applied to interpret the 5-95 percentiles of output quantities as corresponding only to a likely range, propagating the possibility of structural uncertainty in the assessed parameter through the study. We here assume a bounded (between 0 and 1) Gaussian distribution for RWF and a log-normal distribution for TCR, representative of a positive skew (a long, high response, tail to the probability distribution) (Pueyo, 2012) in many estimates. Convolving a bounded Gaussian RWF distribution (with 5-95 percentiles of 0.45-0.75) with a log-normal TCR distribution (with 5-95 percentiles of 1.0-2.5K), gives a corresponding ECS 5-95 percentile range of 1.6-4.5K, in good agreement with the IPCC-AR5 assessed likely range (1.5-4.5K). A sample of 300 ECS and TCR values drawn from these distributions are shown in figure 7a.

Another key uncertainty is the short thermal response timescale, d_1, an important determinant of the Initial Pulse-adjustment Time (IPT), the initial adjustment time of the temperature response to a pulse emission of CO_2 (on Assessing Approaches to Updating the Social Cost of Carbon, 2016). This can be approximated for the FAIR model as IPT=$d_1(1-a_3)$. Throughout this paper we have used the IPCC-AR5 default value for d_1 of 8.4 years, but this is longer than indicated by most climate models (Geoffroy et al., 2013). We therefore sample the short thermal response timescale using a Gaussian distribution with a median value of 4 years and a 5-95% probability interval of 2-8 years. This corresponds to an approximated median estimate of 2.8 years with 5-95 percentile range of 1.4-5.6 years for the IPT.

We consider uncertainties in the carbon cycle by sampling r_0, r_T and r_C with Gaussian distributions of 5-95% probability intervals equal to +/- 10% of their default value. Combined with the thermal response uncertainty sampling, the emergent 5-95% range (based on 300 draws from the input parameter distributions) for TCRE (figure 7c) of 1.0-2.4K/TtC is broadly consistent with the IPCC-AR5 *likely* range (0.8-2.5K/TtC).

Sampling these parameters independently, as described above, produces a range of responses to a 100 GtC pulse emissions in 2020 against the background of the RCP2.6 scenario (figure 7d). However, we consistently observe a rapid warming on the order of a decade followed by an approximate warming plateau (at differing values) that persists for a century or more. Such behaviour is broadly consistent, in all cases, with the range of pulse-response behaviour observed across the ensemble of ESMs in Joos et al. (2013).

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4 Conclusions

In this paper we have presented a simple Finite Amplitude Impulse Response (FAIR) carbon-cycle-climate model, which adjusts the carbon-cycle impulse-response function based on feedbacks from the warming of the climate and cumulative CO_2 uptake, through a parameterisation of the 100-year integrated impulse-response function, $iIRF_{100}$. This metric provides a potential parallel to those used to assess the thermal response to radiative forcing, namely the Transient Climate Response (TCR) and the Equilibrium Climate Sensitivity (ECS). Although a useful composite metric for the coupled climate-carbon-cycle system exists, the Transient Climate Response to Cumulative Emissions (TCRE), future studies of carbon cycle behaviour could report on ranges of $iIRF_{100}$, and importantly for carbon cycle feedbacks, the evolution of this metric over time under specific emissions scenarios, in order to isolate the changing response of the carbon cycle.

We have shown that including both explicit CO_2 uptake- and temperature- induced feedbacks are essential to capture ESM behaviour. Important dependences of the carbon-cycle response to pulse size, background conditions and the suppression of temperature-induced feedbacks are generally well captured with the FAIR model. As present-day pulse responses are an essential part of calculations of the social cost of carbon (Marten, 2011), the inclusion of climate-carbon-cycle feedbacks in the FAIR model offers an improvement on several simple and transparent climate-carbon-cycle models that have been proposed for policy analysis which either incorporate no feedbacks on the carbon-cycle or do not fully capture the operation of these feedbacks in ESMs.

We believe that the FAIR model could be a useful tool for offering a simple and transparent framework for assessing the implications of CO_2 emissions for climate policy analyses. It offers a structure that both replicates the essential physical mechanisms of the climate system's response to cumulative emissions, whilst at the same time can easily be modified to sample representative climate response uncertainty in either the thermal climate response component, the unperturbed carbon-cycle or the coupled climate-carbon-cycle response to anthropogenic CO_2 emissions.

Author contributions. RJM, ZRN and MRA developed the FAIR model formulation. RJM made the figures. MA conceived of the research. All authors contributed to the writing of the paper.

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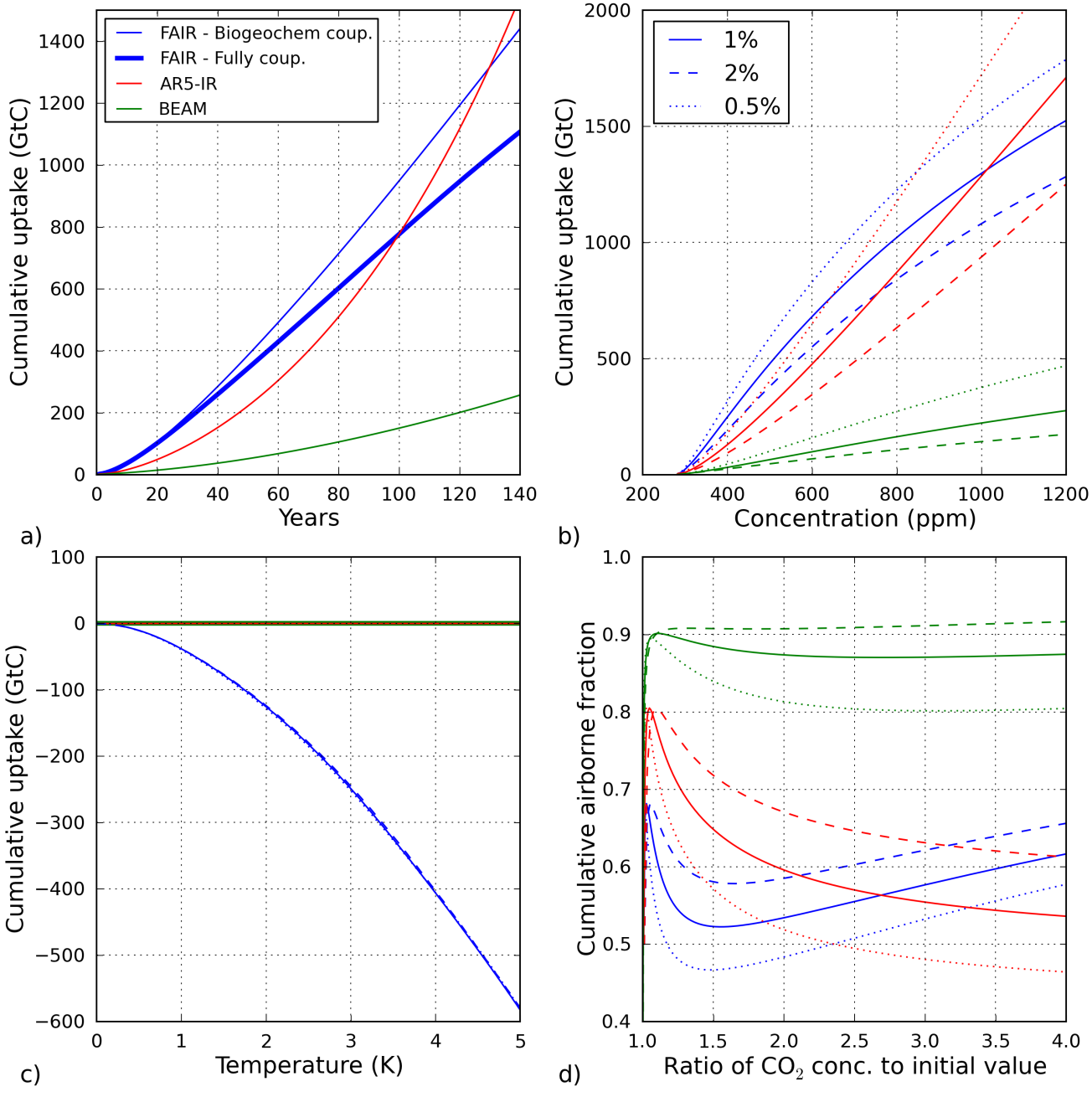

Figure 1. Response to idealised concentration increase experiments from Gregory et al. (2009) for the FAIR (blue), IPCC-AR5 (red) and BEAM (green) models. Panel a) shows the cumulative total carbon uptake over time in the "fully coupled" 1%yr^{-1} concentration increase scenario. Panel b) shows the evolution of cumulative total carbon uptake as a function of atmospheric concentration in the "biogeochemically coupled" experiment for 1%yr^{-1} (solid), 2%yr^{-1} (dashed) and 0.5%yr^{-1} (dotted) experiments. Panel c) shows the cumulative uptake as a function of temperature in the "radiatively coupled" experiment. Panel d) shows the evolution of the cumulative airborne fraction as a function of the proportional concentration increase for the "fully coupled" experiments.

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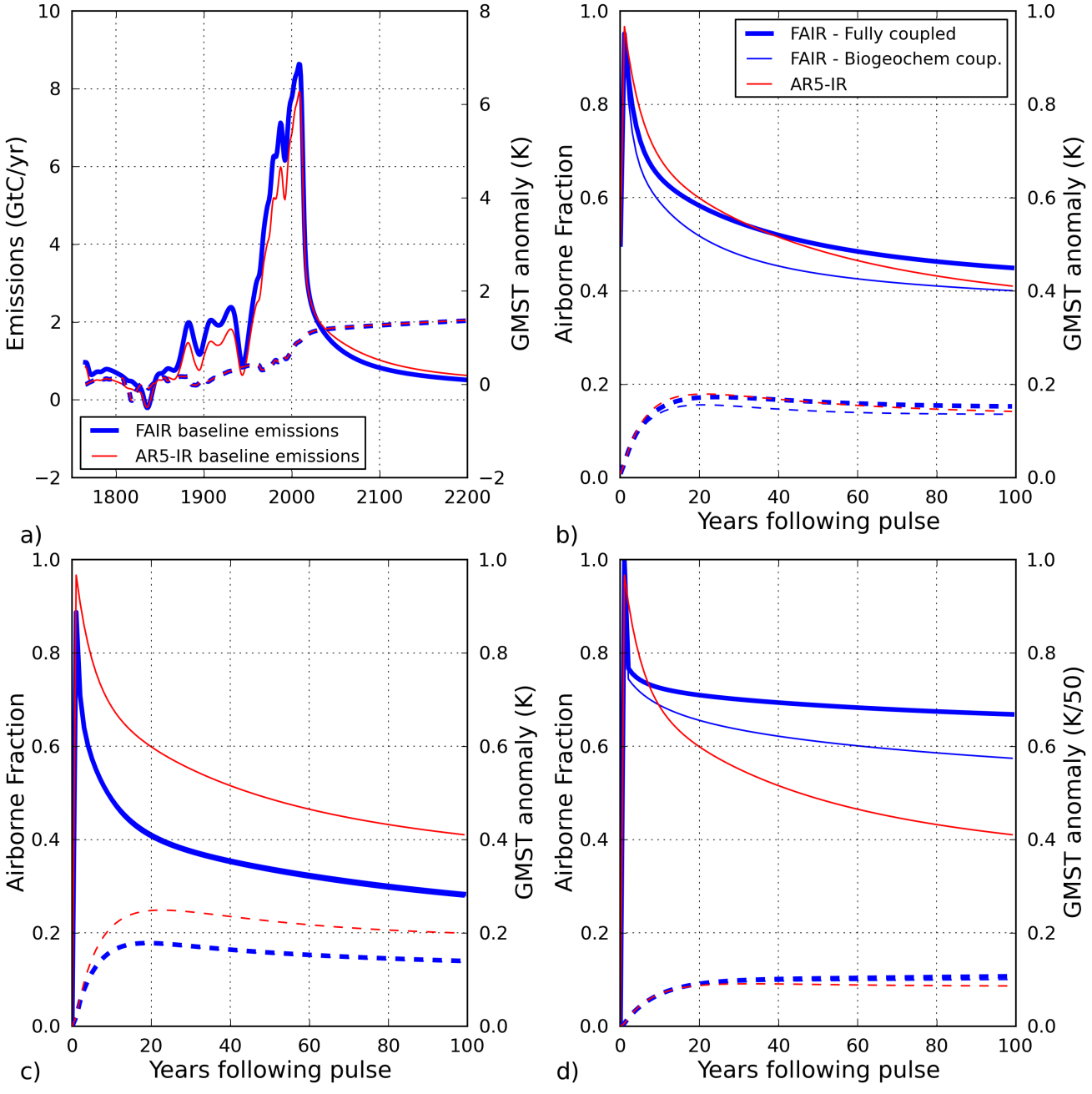

Figure 2. Response to pulse emission experiments of Joos et al. (2013). Panel a) shows the "baseline" emissions (left-hand axis, solid) and warming (right-hand axis, dashed) when concentrations are stabilised at 389ppm for the FAIR (blue) and AR5-IR (red) models. Panel b) shows the response to a 100GtC imposed on present-day (389ppm) background conditions. Panel c) shows the response to a 100GtC pulse in pre-industrial conditions. Panel d) shows the response to a 5000GtC pulse in pre-industrial conditions, with the warming normalised by the increase in pulse size between panels c) and d).

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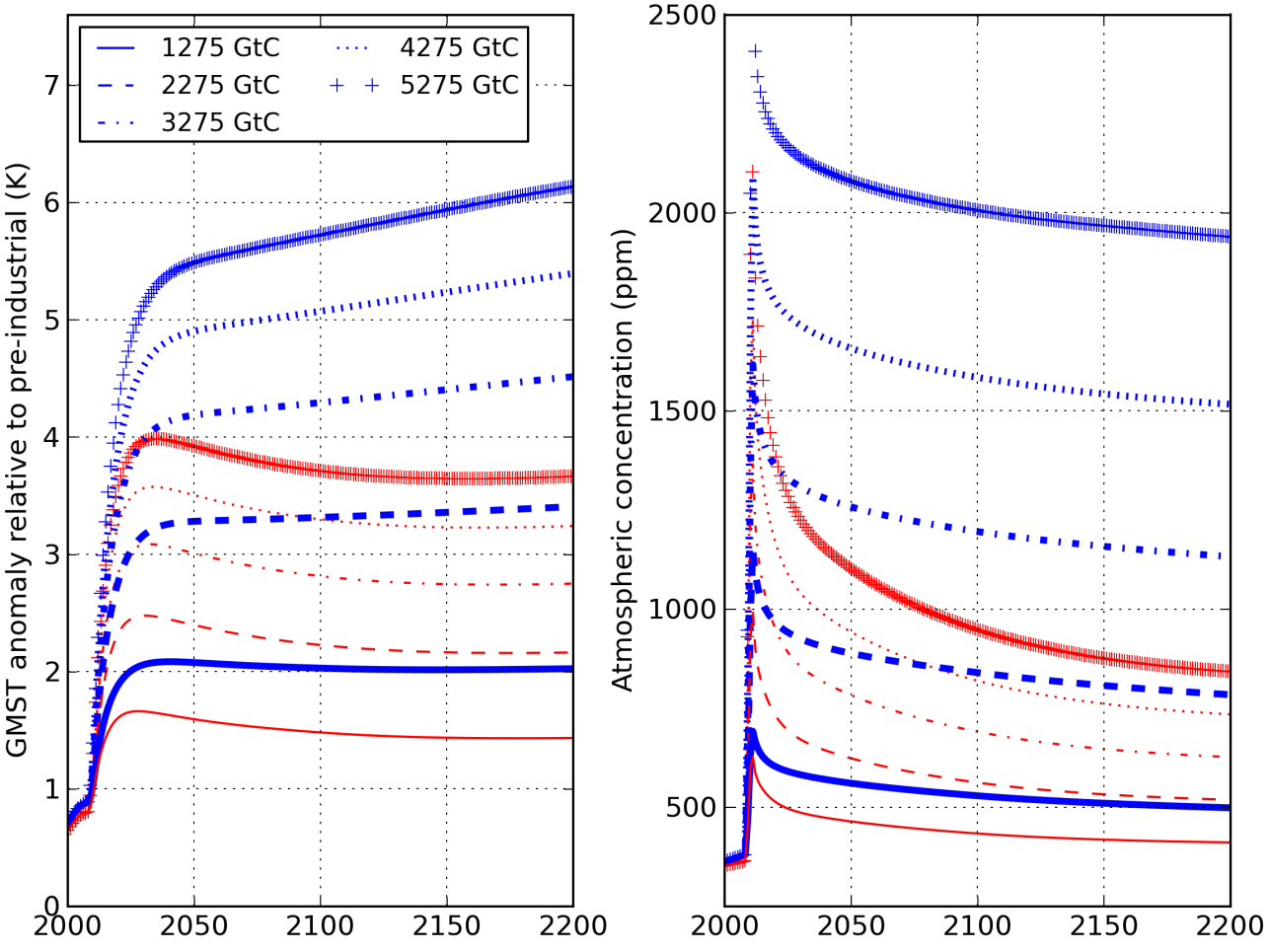

Figure 3. The left hand panel shows the global mean surface temperature (GMST) response to the pulse experiments of Herrington and Zickfeld (2014). Pulse emissions are applied over a 2-year period from 2008, with differing total cumulative carbon emissions denoted by different line styles. Responses are shown for the FAIR (blue) and AR5-IR (red) models. The right hand panel shows the corresponding concentration response.

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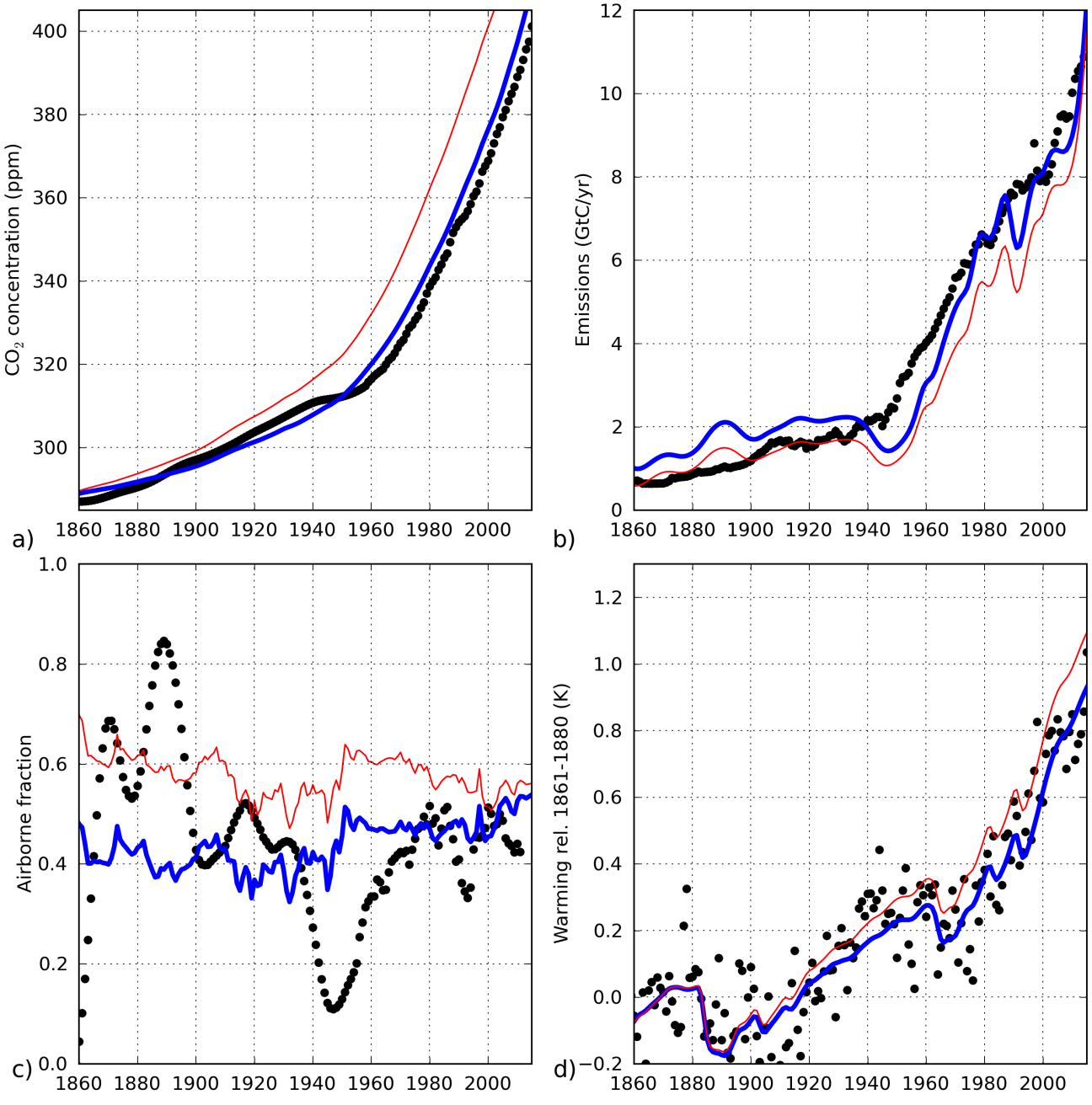

Figure 4. Panel a) shows the CO_2 concentration response when integrated under historical emissions (and non-CO_2 radiative forcing) for the FAIR (blue), AR5-IR (red) models. Panel b) shows the derived CO_2 emissions consistent with historical concentrations. Panel c) shows the evolution of annual airborne fraction (smoothed with a 7-year running mean for the observations) in the models when driven by historical emissions. Panel d) shows the warming anomaly in the models when driven by historical emissions. Historical observations are shown as black dots in all panels. Panels a), b) and c) all show data from Le Quéré et al. (2015) and panel d) shows the HadCRUT4 (Morice et al., 2012) dataset.

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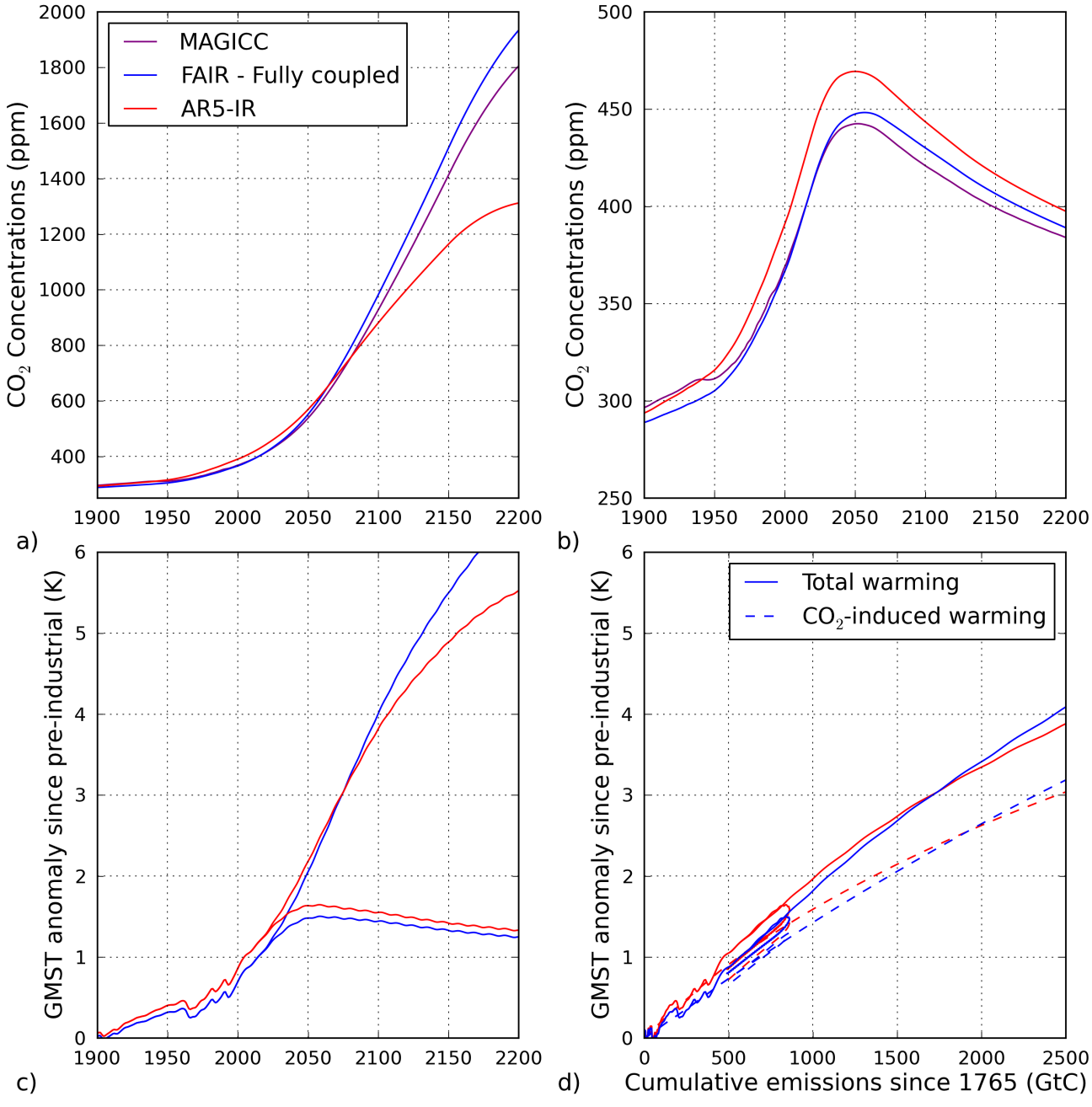

Figure 5. Panels a) and b) shows the CO_2 concentrations under RCP8.5 and RCP2.6 respectively for the FAIR (blue), IPCC-AR5 (red) and MAGICC (purple) models. Panel c) shows the temperature response under both RCP2.6 and RCP8.5. Panel d) shows the evolution of total warming (full) and CO_2-induced warming (dashed) as a function of cumulative carbon emissions.

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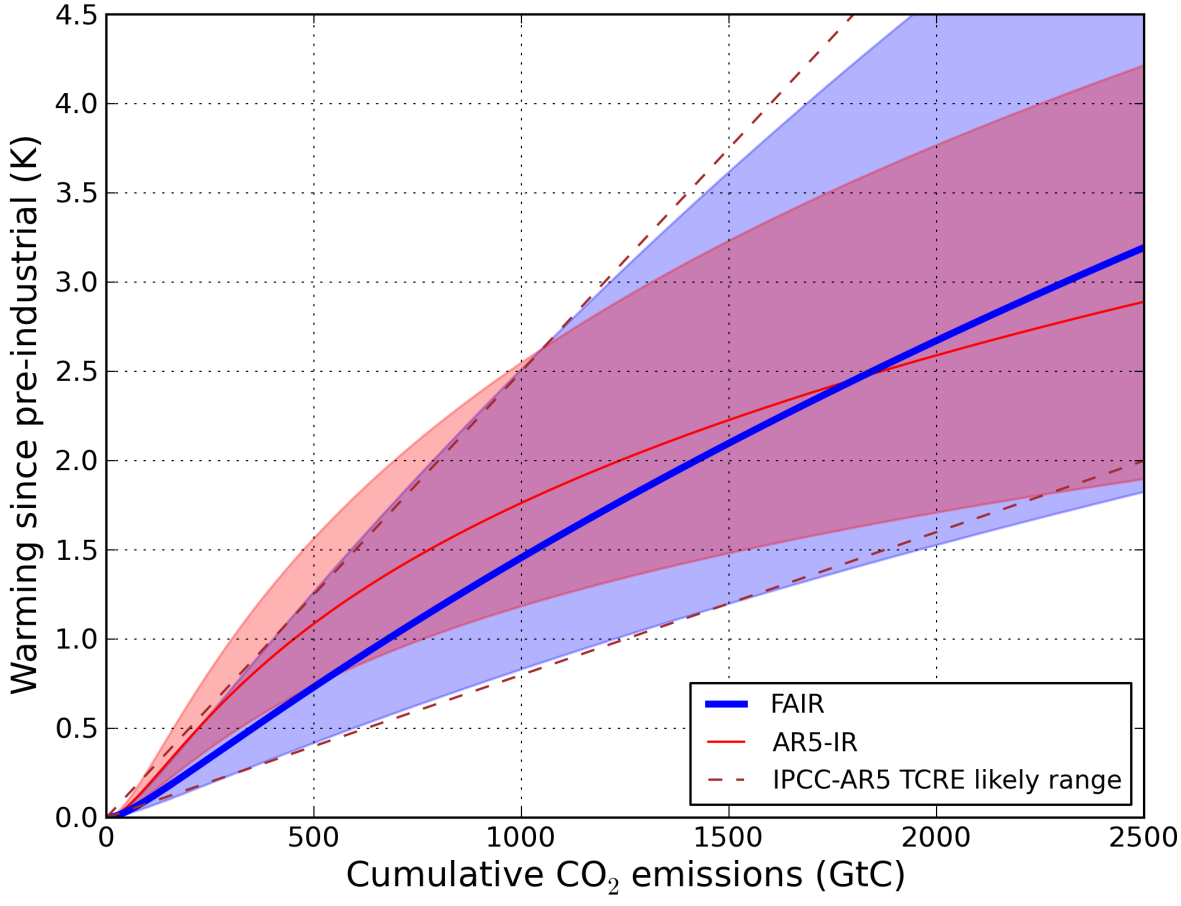

Figure 6. TCRE uncertainty in the FAIR model. Straight lines indicate a constant TCRE. Dashed brown lines show the IPCC-AR5 likely 0.8-2.5K/TtC assessed range for TCRE. The blue plume shows the response to 1%yr^{-1} increase in CO_2 concentrations for the IPCC-AR5 likely TCR and ECS ranges in the FAIR model, with an additional +/-10% perturbation to the r_0, r_T and r_C parameters for the high/low end the likely ranges respectively. The red plume shows the AR5-IR model response.

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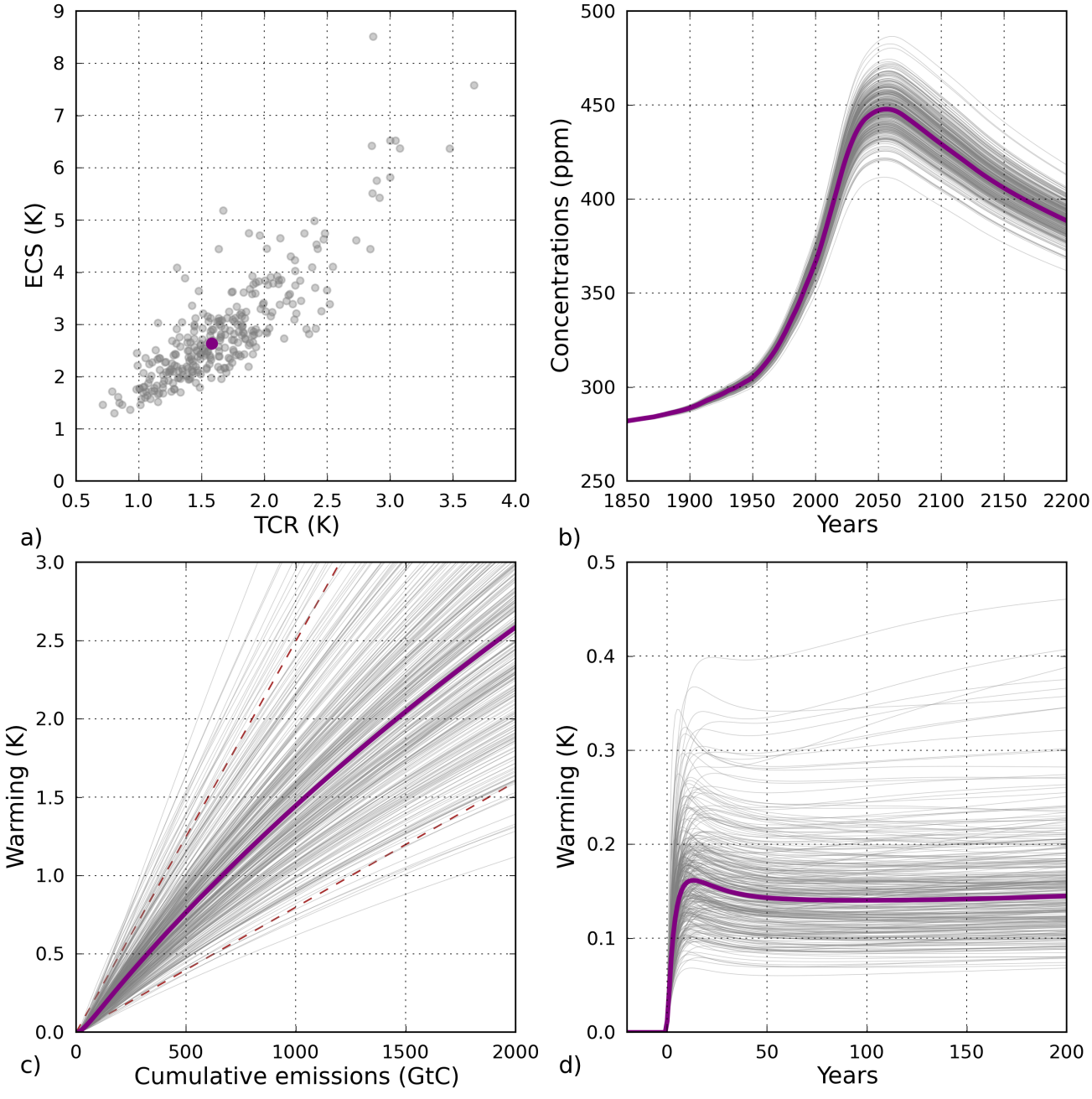

Figure 7. Probabilistic sampling in the FAIR model. Grey lines show 300 random draws from the input parameter distributions, as described in the text. Panel (a) shows the joint distribution of TCR and ECS. Panel (b), the concentration response under MAGICC-derived RCP2.6 emissions. Panel (c), warming as a function of cumulative emissions in the $1\%\mathrm{yr}^{-1}$ concentration increase experiment. The dashed brown lines represent the IPCC-AR5 likely TCRE range. Panel (d), the warming response to a 100GtC pulse emitted in 2020 on top of the MAGICC-derived RCP2.6 emissions. The purple line/dot represents the median estimate in all panels.