

## Response to reviews and short comments

Responses to the reviews and the short comment received are included below in the form of in-line response for clarity. A list of changes is not included (and most likely not of any use) given of the restructuring in response to one of the reviews. Specific changes can be identified from the tracked changes version of the manuscript included here.

### Response to Reviewer 1

Despite the reviewers recommendation we believe that the points raised can be relatively simply dealt with and would hope the making the changes outlines through the in-line responses to the specific points raised (red, italic) below would change the reviewers recommendation.

*The authors motivate the need for their modified impulse response function model by claiming “This extension is necessary because the use of a state-insensitive impulseresponse model cannot simultaneously reproduce the relationship between emissions, concentrations and temperatures seen over the historical period and the projected response over the 21st century to both high-emission and mitigation scenarios estimated from more complex models.” (p. 2, lines 23ff). While it is true that the Joos et al. (2013) model is not accounting for a state dependence, it got not really clear to me from the paper whether this inability of ‘simultaneous reproduction’ is true or not.*

Figure 4 of the original manuscript shows that the AR5-IR model (red) fails to reproduce observed concentrations over the historical period when integrated with historical emissions (panel a) and fails to reproduce historical emissions when emissions consistent with historical concentrations are derived from the model (panel b). A substantial error of nearly 30 ppm is seen in figure 4a, representing a large difference from the correct climate state. We also show in figure 5 how the AR5-IR model fails to produce similar behavior to the MAGICC model under the RCP8.5 and RCP2.6 scenario. The MAGICC model is approximately consistent with the response of the ESMs in CMIP5. We agree with the reviewer that this point could be made more prominently and have emphasized this point more in the revised version of the manuscript.

*“it is encouraging that the FAIR model shows a close correspondence with a well-known and well-used simple model [=MAGICC] that has been used extensively to emulate the response of ESMs” (p. 8, lines 7f) (‘encouraging’ is nice but not convincing).*

We agree with the reviewer that agreement between two simple models of the climate system is only encouraging and is not, and cannot ever be, a sufficient evaluation of model skill. This is the point we wished to convey here and deliberately chose not to use a stronger word precisely for this reason. We have clarified why we compare to MAGICC (in order to produce a single set of emissions for the RCP scenarios that all simple models can be driven by) in the revised manuscripts discussion of this figure.

*In particular, it is not well specified which model simulations are meant to be emulated by FAIR.*

We intend for the FAIR model to reproduce features of the climate response to CO<sub>2</sub> as shown in ESMs (e.g. p. 2, l. 19-20). We have often referred to “more complex/comprehensive models” in the text to indicate that we are attempting to emulate behavior simulated by both ESMs and EMICs (e.g. p. 2 l. 4) but we agree that this wording is unhelpfully ambiguous, particularly in the abstract, and has been rectified in a revised version of the manuscript.

The reviewer raises several points about the method of comparison of our model to other simple models and ESMs. We explicitly didn’t set out to do a full model tuning exercise, in which the consistency of the FAIR model with other ESM simulations is evaluated through defined metrics, as this paper was intended to be a simple demonstrate on how with this parameter set (which hasn’t been formally derived in any way) approximate consistency with the historical record and behavior of ESMs could be attained. A full parameter fitting exercise would be worthwhile with the FAIR model, but as there are fewer free parameters within FAIR than constraints, the output of such a tuning procedure would depend on exactly which consistencies are of most interest to the users of the model and would therefore need to be undertaken on a case-by-case basis depending of the desired usage. We have added a new figure (Figure 4 of the revised manuscript that shows a possible tuning exercise with the FAIR model to reproduce

particular aspects of ESM behaviour.

*the results of several impulse experiments are discussed in section 3, resulting in questionable claims on the quality of their new model like “consistent with corresponding ratio in the most detailed ESMs” (p. 6, lines 23f) (what means ‘most detailed’?)*

We intended consistency to refer to the quantitative statement in the following clause, namely, “with its value of 36 years within the 34-47 years range of the ESMs”. We agree with the reviewer that the word ‘detailed’ adds no extra information, introduces confusion and has been removed in the revised version of the manuscript. We have added ranges of the pulse responses in ESMs from Joos et al (2013) onto the original figure 3 to display this comparison visually.

*“the FAIR model can capture the dependence of the pulse-response on pulse size” (p. 6, line 28) (what means ‘capture’? In comparison to what?)*

The black lines showing the range of response from the Joos et al (2013) models to the pulse-response experiments figure (Figure 3 of the revised manuscript) allow this dependency and the ability to reproduce it to be seen.

## Response to Reviewer 2

We enclose responses to the points raised by reviewer 2 (red) in-line below...

First, the authors must provide a better literature review that contains brief descriptions of the other models to which their FAIR model is compared, including the IPCC simple model and the BEAM model, which is introduced rather abruptly in the results section.

We provide a lengthier introduction to the simple carbon cycle models in the revised version of the manuscript

At the end of the introduction, the authors propose to "extend" on the IPCC-AR5 model, but I don't think they have fully introduced this model, or described why they think it is deficient.

We introduce this model more thoroughly to set the modified version of this model (FAIR) in better context in the revised manuscript.

Second, the authors must provide a better description of their own FAIR model. The time constants should be better described, the carbon reservoirs used in the model should be named, and all the variables used in equations 1-5 should be described and the appropriate units should be listed.

The impulse-response formulation of the AR5 and FAIR models are empirical models that are based on the fits to the response of more complex models as done in Joos et al. Therefore the specific timescales and carbon reservoirs in the model do not necessarily have simple physical interpretations and as such it is impossible to assign specific names to them. This point is added to the revised version of the manuscript. However, approximate correspondence exists between physical processes and decay timescales, which is now commented on in the revised version of the manuscript (table 1 – which documents parameters and their units).

Third, the discussion of the figures, which is the bulk of the manuscript, needs to be enhanced. The Results section reads like a laundry list of the figures, so you might consider structuring your

results around the scientific questions each figure answers rather than beginning each paragraph with "Figure N shows...". Given the current state of the Results section, I have a hard time knowing whether you think the model shows good agreement with some benchmark or plausible agreement with other model output. I also have no idea why you think the benchmarks you selected are the best ones to use (and whether they are sufficient), or which models you are trying to show that yours agrees with.

We agree that the results section could have been better structured around the scientific questions raised by each figure. We have re-organized and rewritten the results section as appropriate and have included ESM/EMIC data in the figures where appropriate to be clear on the success and limitations of our model evaluation.

Fourth, I think that Fig. 4c merits a bit more attention. The authors contrast their FAIR model with the BEAM model by stating that FAIR includes some parameters for terrestrial uptake, rather than just marine uptake. Variations in terrestrial uptake largely drive variations in the airborne fraction, yet the FAIR model shows variability in the airborne fraction that is maybe 25% of the observations. So is FAIR really capturing climate-terrestrial uptake interactions? If variations don't show much semblance to reality at shorter timescales, how can we trust the longer timescales? Given high profile recent papers that have tried to use interannual variability in the CO<sub>2</sub> growth rate in ESM ensembles to constrain long-term temperature sensitivity of terrestrial uptake, I think this merits a bit more discussion.

As FAIR (and the other impulse-response models considered in this paper only represent the externally-forced response of the climate-carbon-cycle system they are likely to be structurally unable to reproduce the observational variations in airborne fraction (which are likely to be driven by internal variability in a large fraction). We have added this to the paper and indicated that more complex models are required to understand the drivers of these variations and their implications for future carbon-cycle response.

The paper would also benefit from paying a bit more attention to details in the figures. For example, the legends for the figures are

often incomplete and rely on information buried in the caption. Please help your reader by including this in the figure panels. The legend in Fig. 2 is incomplete (should include solid vs dash trendlines), as is the legend in Fig. 3 (should include red vs blue). There is no legend for Fig. 4. For Fig. 5, please consider using different colors since the purple and red are hard to distinguish. For Fig. 7 – I am not sure what are the brown dashed lines. It is possible that the purple shade and the brown shade are also too close.

We have improved the presentation of our figures in the revised version of the manuscript.

In the revised paper, I think it is necessary to include more discussion of how do we know this simplified model is "good enough". The figures show comparisons against other, "wrong" models. Why is this model sufficient? Perhaps better discussion of the variables that comprise the model itself would and their physical significance/relationship to variables that exist in full-physics and full-BGC models would accomplish this, but the authors might also consider adding an additional section to the paper.

We are not exactly sure what the reviewer is asking for here, but we interpret 'how do we know the model is good enough/sufficient' as asking whether this model structure is capable of spanning the range of responses seen in ESMs. We hope the addition of the new figure and the inclusion of ESM/EMIC data on the figures addresses this point by showing that the FAIR model can act as an effective emulator of the range of response in the Joos et al models with perturbations to only a subset of the parameters within the FAIR model.

I would also like to see some discussion of how the FAIR model can be improved/ extended in the future. Will variables simply be re-tuned when AR6 models come back with different emergent responses? Are there clear steps that would better emulate the physics, biology, and chemistry that govern the airborne CO<sub>2</sub> feedbacks that can be added independent on next generation ESM simulations?

We do see FAIR as a model emulator framework in which

the parameters can be tuned to reproduce CMIP5/CMIP6 behaviour whilst maintaining the physically understood dependencies of the response on background conditions and pulse size. We have revised the manuscript to reflect this perspective more clearly including a new figure in which we emulate the response of Joos et al models to pulses emissions of CO<sub>2</sub> with a set of parameters appropriate for both present-day and pre-industrial pulse responses.

## Response to SC1 by Elisabeth Moyer

Professor Moyer raises some interesting points in her short comment on our ACPD paper, which influenced our revisions and has enhanced the revised paper. We address some of the main points raised (red,italic) in the comment in-line here:

*The authors write: "We find that a simple linear increase in 100-year integrated airborne fraction with cumulative carbon uptake and global temperature change is both necessary and sufficient to represent the response of the climate system to CO<sub>2</sub> on a range of timescales and under a range of experimental designs." But, the airborne fraction does not increase linearly over long timescales in most realistic emissions scenarios.*

We parameterize the *integrated* airborne fraction as linearly increasing with warming and cumulative uptake. This does not imply that the instantaneous airborne fraction would be expected to increase linearly with time. Indeed the FAIR model displays a similar temporal evolution of the instantaneous airborne fraction under the A2+ scenario as shown by the UVic EMIC (shown in Figure 1 of this comment which shows the same data as Figure 1 of SC1 with the FAIR model). After emissions stop in this scenario (2300) atmospheric concentrations decay away with four characteristic timescales back toward the pre-industrial equilibrium. The most relevant timescales for the several hundred years is associated with equilibration of the full-depth of the ocean. We see no conflict between the FAIR formulation and these features of the UVic/CLIMBER models under the A2+ scenarios.

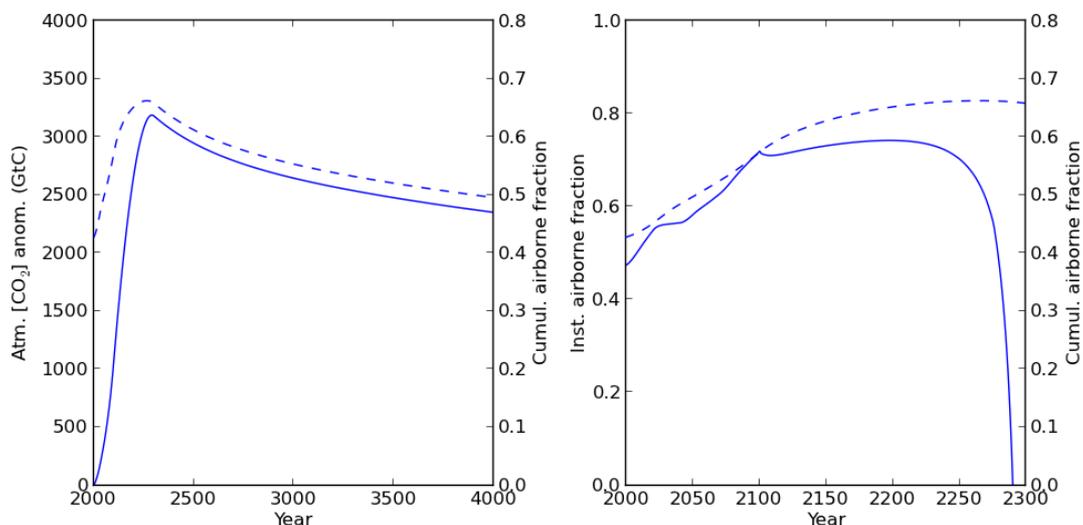


Figure 1: The response of the FAIR model to the A2+ emissions scenario. The dashed lines correspond to the right hand axis.

*Figure 1d does seem to imply a decrease in airborne fraction over time in certain experiments, though followed by a subsequent increase. This decrease is not explained well, but is different from the long-term decrease that comes in realistic emissions scenarios when emissions slow*

Figure 1d shows the *cumulative* airborne fraction, and as highlighted in SC1, the cumulative airborne fraction represents the integrated evolution of the instantaneous airborne fraction over previous years. The initial decrease in cumulative airborne fraction, followed by a subsequent increase, demonstrated by the FAIR model is a feature of the response of many ESMs under a 1%/yr increasing CO<sub>2</sub> scenario. Figure 2 of this comment shows a new version of Figure 1 of the original manuscript that shows pastel-coloured lines associated with the ESMs analysed in Arora et al. (2013) under a 1%/yr CO<sub>2</sub> increase scenario. The initial decrease in cumulative airborne fraction followed by subsequent increase can be understood in terms of the saturation of carbon sinks. If atmospheric anomalies of carbon decay with invariant timescales (as in the AR5-IR model case), then instantaneous airborne fraction remains constant in time, which necessarily means that cumulative airborne fraction must decline in time (as emissions from previous years decay further, so the cumulative fraction of the emitted carbon continually decays from the instantaneous airborne fraction), this behaviour is shown by the AR5-IR model in figure 1d of the main text and figure 2 here. However, if carbon sinks become saturated, the instantaneous airborne fraction would be expected to increase with time (this is represented in the FAIR model by increases to the decay timescales through the parameterised increase in iIRF100). As more recent emissions (which are of increasing magnitude under the 1%/yr scenario) have a higher instantaneous airborne fraction, the cumulative airborne fraction decrease stops and then begins to increase again as this accelerating saturation becomes the dominant effect. We have provided a more in-depth discussion of this in the revised manuscript to help better communicate the results shown in figure 1d.

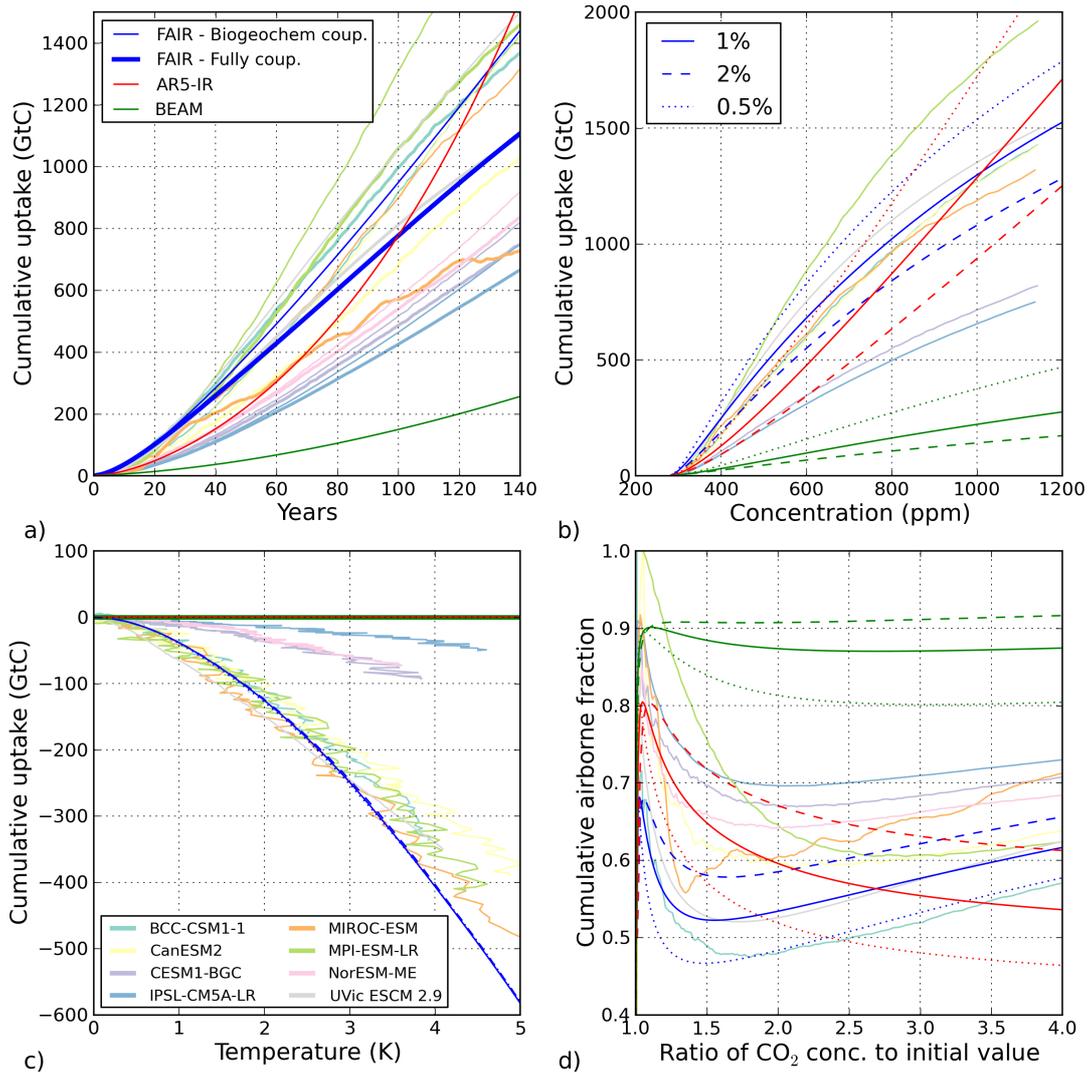


Figure 2: As for figure 1 in the main text, but new coloured lines included correspond to the response of the ESMs analysed in Arora et al (2013) under the 1%/yr CO<sub>2</sub> increase scenario.

*Figure 1 is introduced before either the model or the experiments has been described, which makes it hard to understand.*

We begin our discussion of Figure 1 with the statement “Figure 1 shows the response of the FAIR model (blue) described in section 2 under the three experiments described above.” so we do not believe that this to be the case. However, our discussion of Figure 1 evidently needed to be clearer and we believe we have improved this in our revised manuscript to hopefully make it clearer for readers to follow.

*We see BEAM only as a framework that can capture the response of more complex models, and we would strongly prefer*

*that parameters be chosen appropriate to the models being compared.*

We used the default parameter settings from Glotter et al. (2014) as those were the only consistent set that were documented in the paper. However, we agree that in general simple models should be tuned to specific more complex models. The version of BEAM given in Glotter et al (2014) is clearly a good approximation of the response of the UVic and CLIMBER EMICs under the A2+ scenario as shown in the figures of Glotter et al (2014). We feel that documenting a method for tuning the BEAM model, and the results of the results of that process, would be a worthy investigation in its own right but not something that would be possible within the constraints on this paper here, where the focus is on the FAIR model. On reflection, as our main point in the paper is demonstrating the need to include state-dependence within impulse-response carbon-cycle models, we feel that inclusion of BEAM as a comparison model for just one figure in the paper (as was the case in the original manuscript) does little to enhance the paper and maybe adds unnecessary baggage to communication of its core message. We have therefore removed explicit discussion and comparison to BEAM in the revised manuscript and instead focus solely on comparisons to state-independent impulse-response climate-carbon-cycle models. We would however be interested in doing an extended comparison of FAIR with BEAM as a separate study, including tuned versions of the BEAM parameters, which we would be delighted to collaborate with Professor Moyer on in the future.

*We had assumed that the primary use for BEAM would be in simple Integrated Assessment Models that consider long timescales and require relatively crude representations of the physical climate system. The Bolin and Eriksson values seemed acceptable for this purpose, as resulting temperature differences between BEAM and the ESMs are no more than 0.23 K in the first 100 years, and thereafter the two ESMs bracket BEAM temperatures*

We would argue that because of the economic practice of discounting future damages in conventional integrated assessment activities, the response of a carbon-cycle model to a pulse emission of carbon over the first 100 years is of most importance for Integrated Assessment Models. Even at moderate discount

rates of about 2.5%/yr the weighting for climate damages driven by physical changes are only 0.08 of today's weighting. Whilst this relative over-weighting of the multi-century scale response may be an undesirable feature of the economic paradigm, it therefore currently remains most important to correctly represent the response of the climate system over the first 100 years in a physical model that will be used as part of Integrated Assessment Models and particularly in calculations of the social cost of carbon. This is why we chose to structure the FAIR model in terms of an explicitly parametrised iIRF100 to allow this timescale of the model's response to be mapped to the behavior of a range of ESMs as transparently as possible.

*I am confused about the author's definition of "cumulative uptake" and "cumulative airborne fraction" in Figure 1. In this figure BEAM output is shown as beginning with ~300 ppm and zero cumulative uptake. But the initial conditions suggested in Glotter et al 2014 begin BEAM after historical emissions from 1800-2000, so that starting atmospheric CO2 is over 380 ppm, and substantial emissions and uptake have occurred already. In addition, given those initial conditions, the starting "cumulative airborne fraction" is ~0.5 and rises only slowly over time even when ocean uptake is small and instantaneous airborne fraction is high. Here the cumulative airborne fraction is shown as reaching 0.9 nearly immediately.*

Cumulative uptake is defined as the total amount of the carbon previously emitted into the atmosphere that has been removed from the air by the carbon-cycle system. Cumulative airborne fraction is the fraction of the previously emitted carbon that still remains in the atmosphere at a point in time. We have tried to make this clearer in the introduction of the FAIR model in section 2 of the revised manuscript. The definitions used in the manuscript are consistent with those used in SC1. The confusion mentioned above may arise in part because Figure 1 represents scenarios that begin from pre-industrial conditions with concentrations increasing by 1%/yr, 0.5%/yr and 2%/yr. Glotter et al (2014) also suggested a pre-industrial initial condition for BEAM (Table 4, Appendix A.3) which corresponds to a pre-industrial concentration of 280ppm. It is these initial conditions that were used in Figure 1 of the original manuscript.

*Finally, I was confused by statements implying that different emissions scenarios can be captured by a model that represents airborne fraction as a function of cumulative emissions (and temperature). Again the writing is confusing and I may have misunderstood, but airborne fraction seems quite sensitive to the emissions scenario (Figure 3 below). It seems that a figure is needed to explicitly validate this assertion.*

We agree that the response of a carbon-cycle-climate model is dependent on the emissions scenario. The long dashed and short dashed lines in figure 2 of this comment show that the FAIR model can capture dependencies of the response on the emissions scenario, similarly to as shown for two ESMs in Gregory et al (2009) (Figures 4, 5, 6 of that paper). We have additionally reproduced Figure 3 of SC1, with the FAIR model (Figure 3 of this comment) that demonstrates that the scenario dependencies of the BEAM model under variants of the A2+ emissions scenario can be similarly represented with FAIR.

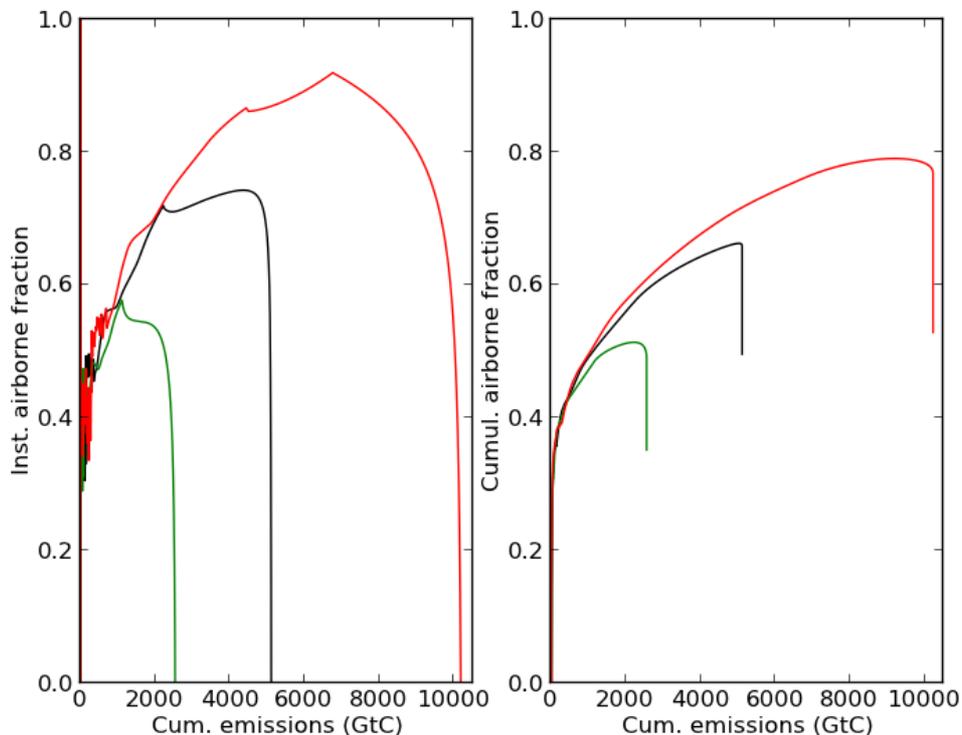


Figure 3: As in Figure 3 of SC1 but for the FAIR model. The red line represents a 2xA2+ emissions scenario, the black the standard A2+ emissions scenario and the green a 0.5xA2+ emissions scenario.

# A modified impulse-response representation of the global response to carbon dioxide emissions

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**Abstract.** Projections of the response to anthropogenic emission scenarios, evaluation of some greenhouse gas metrics and estimates of the social cost of carbon, often require a simple model that links emissions of carbon dioxide (CO<sub>2</sub>) to atmospheric concentrations and global temperature changes. An essential requirement of such a model is to reproduce the behaviour of more ~~complex models~~ [Earth System Models](#) as well as an ability to sample their range of response in a transparent, accessible and reproducible form. Here we adapt the simple model of the Intergovernmental Panel on Climate Change 5th Assessment Report (IPCC-AR5) to explicitly represent the state-dependence of the CO<sub>2</sub> airborne fraction and reproduce ~~several idealised experiments performed with more complex models~~ [the range of behaviour shown in full and intermediate complexity Earth System Models under several idealised carbon-cycle experiments](#). We find that a simple linear increase in 100-year integrated airborne fraction with cumulative carbon uptake and global temperature change is both necessary and sufficient to represent the response of the climate system to CO<sub>2</sub> on a range of timescales and under a range of experimental designs. Quantified ranges of uncertainty (analogous to current assessed ranges in Equilibrium Climate Sensitivity and Transient Climate Response) in integrated airborne fraction over the 21st century under a representative mitigation scenario, and an assessed range in how much this quantity may have changed relative to pre-industrial conditions, would be valuable in future scientific assessments.

## 1 Introduction

Future emissions of CO<sub>2</sub> over the remainder of the century are uncertain and a strong function of future climate policy (Van Vuuren et al., 2011). Future climate changes, and their associated impacts, will largely be determined by future cumulative carbon dioxide emissions (Matthews et al., 2009; Allen et al., 2009; Meinshausen et al., 2009), but linking specific CO<sub>2</sub> emission scenarios to future transient climate change requires a model of the interacting climate-carbon-cycle system. Comprehensive Earth System Models (ESMs) ~~directly capture~~ [explicitly simulate](#) the physical processes that govern the coupled evolution of atmospheric carbon concentrations and the associated climate response (Friedlingstein et al., 2006). However, such models are typically highly computationally intensive and can therefore only be run for a few representative future emission scenarios (Taylor et al., 2012). For analysis of arbitrary emissions scenarios, as required for the integrated assessment of climate policy

and calculation of the social cost of carbon, a computationally efficient representation of the Earth system is required (Marten, 2011).

Simplified representations of the coupled climate-carbon-cycle system take many forms (Hof et al., 2012). A key test for simplified ESMs is whether they correctly capture the physics of the co-evolution of atmospheric CO<sub>2</sub> concentrations and global mean temperature under ~~idealised settings~~both idealised settings and under possible projections of future emissions scenarios. Following a CO<sub>2</sub> pulse emission of 100GtC ~~under-in~~ present-day climate conditions, ESMs (and Earth System Models of Intermediate Complexity – EMICs) display a rapid-draw down of CO<sub>2</sub> with the concentration anomaly reduced by approximately 40% from peak after 20 years and by 60% after 100 years, followed by a much slower decay of concentrations leaving approximately 25% of peak concentration anomaly remaining after 1000 years (Joos et al., 2013). The effect of this longevity of fossil carbon in the atmosphere, combined with the gradual “recalcitrant” thermal adjustment of the climate system (Held et al., 2010), is to induce a global mean surface temperature (GMST) response to a pulse emission of CO<sub>2</sub> ~~of characterised by~~ a rapid warming over approximately a decade to a plateau value of GMST anomaly (Joos et al., 2013). Warming does not noticeably decrease from this value over the following several hundred years, indicating that, short of artificial CO<sub>2</sub> removal (CDR) or ~~other-geoengineeringmethods~~active geoengineering, CO<sub>2</sub>-induced warming is essentially permanent on human-relevant timescales.

As computations of the social cost of carbon require the discounted summation of future climate change-induced economic damages associated with an additional pulse emission of CO<sub>2</sub> above a baseline scenario, the correct representation of the temporal evolution of the warming response to the pulse emission is required from computationally-simple climate-carbon-cycle models. As simple climate-carbon-cycle models are not explicitly evaluated in terms of their pulse-response behaviour, it is unclear how well this robustly simulated physics is represented in such models.

A second important feature of more complex climate-carbon-cycle models is the increase in airborne fraction (the percentage of emitted CO<sub>2</sub> that remains in the atmosphere after a period of time) over time in scenarios involving substantial levels of emissions or warming (~~Millar et al., 2016~~)(Friedlingstein et al., 2006; Millar et al., 2016). An emergent feature of the CMIP5 full-complexity ESMs appears to be that this increase in airborne fraction approximately cancels the logarithmic relationship between CO<sub>2</sub> concentrations and radiative forcing, yielding an approximately linear relationship between cumulative CO<sub>2</sub> emissions and CO<sub>2</sub>-induced warming (Matthews et al., 2009; Gillett et al., 2013). This relationship has given rise to the concept of an all-time cumulative ‘carbon budget’ to restrict warming to a certain level (Rogelj et al., 2016), which has quickly become an important tool in evaluating the required energy-system transitions that are needed to limit warming to below particular thresholds (Davis and Socolow, 2014; Pfeiffer et al., 2016). As simple climate-carbon-cycle models are often used to compute particular carbon budgets in integrated assessment scenarios (e.g. Meinshausen et al. (2009)), the ability to reproduce the approximate linearity of the relationship between warming and cumulative emissions is a desirable property.

~~In this paper we propose a simple extension of a standard impulse-response model of the carbon-cycle-climate system to reproduce~~In this paper we show that the physical behaviour of the ESMs under a variety of idealised experiments. Our starting point is the impulse-response functions that are provided for the calculation of multi-gas equivalence metrics in IPCC-AR5 (Myhre et al., 2013), ~~which we extend by coupling the carbon-cycle to the thermal response and to cumulative~~

~~carbon uptake by terrestrial and marine sinks. This extension is necessary because the use of a simple and easy to use coupled climate-carbon-cycle model, are insufficient to fully capture these emergent responses of the climate-carbon-cycle system. Such a state-insensitive impulse-response model cannot simultaneously reproduce the relationship between emissions, concentrations and temperatures seen over the historical period and the projected response over the 21<sup>st</sup> century to both high-emission and mitigation scenarios estimated from more complex models, as simulated by ESMs. We therefore propose a simple extension of the standard IPCC-AR5 impulse-response model, coupling the carbon-cycle to the thermal response and to cumulative carbon uptake by terrestrial and marine sinks in order to reproduce the behaviour of the ESMs under a variety of idealised experiments and future emissions scenarios.~~

Section 2 describes the formalism of the ~~model that we use. Section 3 then demonstrates the model's~~ models that we contrast throughout this paper. We then describe, in section 3.1, why a state-dependence modification to the IPCC-AR5 carbon-cycle impulse-response function is required, motivating the modified model described in section 2. Section 3.2 then evaluates these models' ability in replicating ~~ESM~~ the dependencies of the response to a pulse-emission on background conditions and pulse size shown in ESMs and EMICs. Section 3.3 evaluates the models' behaviour under a set of idealised experiments ; namely those of Gregory et al. (2009), Joos et al. (2013) and Herrington and Zickfeld (2014). We also compare the behaviour of our model with the simple models of Myhre et al. (2013), Glotter et al. (2014) and Meinshausen et al. (2011a) in which CO<sub>2</sub> concentrations are increased by a fixed percentage each year starting from pre-industrial values. Section 3.4 discusses uncertainty in the modified simple model and how probabilistic assessments of climate response to CO<sub>2</sub> emissions could be made using our proposed model. Section 4 provides a concluding summary and discussion.

## 2 A “Finite Amplitude Impulse Response” (FAIR) model ~~Model descriptions~~

### 2.1 The IPCC AR5 Impulse-Response (AR5-IR) model

The IPCC-AR5 proposed an idealised simple climate model for metric calculations, incorporating a “2-box” or “2-time-constant” model of the temperature response to radiative forcing with a “4-time-constant” ~~(one of which is infinite)~~ impulse-response ~~model of the~~ model of the CO<sub>2</sub> response to emissions (Myhre et al., 2013). This model represents the evolution of atmospheric CO<sub>2</sub> by partitioning emissions of anthropogenic CO<sub>2</sub> between four different reservoirs (all of which are empty in pre-industrial equilibrium) of atmospheric carbon anomaly that each decay with a fixed time constant. The impulse-response function for a unit emission at time  $t = 0$  is therefore give as,

$$\frac{dR_i}{dt} = a_i E - \frac{R_i}{\tau_i} \quad ; \quad i = 1, 4 \quad (1)$$

where  $E$  are annual CO<sub>2</sub> emissions, in units of ppm/year (1 ppm = 2.12GtC). Atmospheric CO<sub>2</sub> concentrations are given by  $C = C_0 + \sum_i R_i$ , and radiative forcing by:

$$F = \frac{F_{2X}}{\ln(2)} \ln\left(\frac{C}{C_0}\right) + F_{\text{ext}} \quad , \quad (2)$$

where  $C_0$  is the pre-industrial  $\text{CO}_2$  response to emissions (Myhre et al., 2013). As proposed in IPCC-AR5, the concentration,  $F_{2x}$  the forcing due to  $\text{CO}_2$  doubling, and  $F_{\text{ext}}$  the non- $\text{CO}_2$  forcing. GMST anomalies are computed thus:

$$\frac{dT_j}{dt} = \frac{c_j F - T_j}{d_j} \quad ; \quad T = \sum_j T_j \quad ; \quad j = 1, 2 \quad (3)$$

with coefficients  $a_i$ ,  $d_j$  and  $\tau_i$  as given in AR5 Chapter 8, tables 8.SM.9 and 8.SM.10 (Myhre et al., 2013).  $c_j$  are set to give an Equilibrium Climate Sensitivity (ECS) = 2.75K and Transient Climate Response (TCR) = 1.6K (corresponding to  $c_1 = 0.46$  and  $c_2 = 0.27$  (Millar et al., 2015)), indicative of a typical mid-range climate response to radiative forcing in ESMs (Flato et al., 2013). The four carbon pools, each with a fixed decay time constant, are determined to be sufficient to empirically represent the response of atmospheric  $\text{CO}_2$  concentration anomalies following a pulse emission of 100GtC, above a specified background concentration of 389ppm, over the 1000 years following the pulse (Joos et al., 2013). As the fraction of carbon emissions entering each reservoir ( $a_i$ ) and the decay time constant ( $\tau_i$ ) are determined empirically, they do not in themselves correspond to individual physical processes and instead represent the combined effect of several carbon-cycle mechanisms. However, the distinct range of decay timescales indicates specific physical processes that are strongly associated with the evolution of each carbon reservoir. These are summarised in table 1.

We use two versions of the AR5-IR model in this paper, one calibrated to the present-day (AR5-IR) and one calibrated to the pre-industrial climate response to a pulse emission (PI-IR) respectively. The AR5-IR model is used for the calculation of absolute Global Temperature Potentials (aGTPs) in IPCC-AR5 and has carbon-cycle coefficients that best represent the evolution of a 100GtC pulse emission under approximately present-day conditions. The PI-IR model uses an alternative set of coefficients that are selected to represent the evolution of a 100GtC pulse emission in pre-industrial conditions for the multi-model mean of the ensemble of ESMs and EMICs in Joos et al. (2013) (see table 1 for parameter values).

## 2.2 A “Finite Amplitude Impulse Response” (FAIR) model

In the AR5-IR model the carbon-cycle constants are not affected by rising temperature or  $\text{CO}_2$  accumulation, and hence only represent the specific response to a particular perturbation scenario. In more comprehensive models, ocean uptake efficiency declines with accumulated  $\text{CO}_2$  in ocean sinks (Revelle and Suess, 1957) and uptake of carbon into both terrestrial and marine sinks are reduced by warming (Friedlingstein et al., 2006). A state-insensitive impulse-response model is therefore unsuitable, unless modified, for calculations of, for example, the social cost of carbon against realistic baseline trajectories or long integrations with historical and projected emissions. We shall refer to the Myhre et al. (2013) combined (but non-interacting) carbon-cycle and temperature response, tuned to present-day climate conditions, as the AR5-IR model.

Here we attempt to capture some of these dynamics within the simple impulse-response model structure, we here attempt a minimal modification of the AR5-IR model to allow it to mimic the behaviour of more complex models in response to finite-amplitude  $\text{CO}_2$  injections, which we call a Finite Amplitude Impulse-Response (FAIR) model. To introduce a state-dependent carbon uptake as simply as possible, we apply a single scaling factor  $\alpha$  to all four of the time-constants (time units are in years) in the carbon-cycle of the AR5-IR model, such that the  $\text{CO}_2$  concentrations in the 4 “carbon reservoirs” are

Parameter	Value - AR5-IR	Value - PI-IR	Value - FAIR	Processes
$a_0$	0.2173	0.1266	0.2173	Geological re-absorption
$a_1$	0.2240	0.2607	0.2240	Deep ocean invasion / equilibration
$a_2$	0.2824	0.2909	0.2824	Biospheric uptake / ocean thermocline invasion
$a_3$	0.2763	0.3218	0.2763	Rapid biospheric uptake / ocean mixed-layer invasion
$\tau_0$ (yr)	$1 \times 10^6$	$1 \times 10^6$	$1 \times 10^6$	Geological re-absorption
$\tau_1$ (yr)	394.4	302.8	394.4	Deep ocean invasion/equilibration
$\tau_2$ (yr)	36.54	31.61	36.54	Biospheric uptake / ocean thermocline invasion
$\tau_3$ (yr)	4.304	4.240	4.304	Rapid biospheric uptake / ocean mixed-layer invasion
$c_1$	0.46	0.46	0.46	Thermal adjustment of upper ocean
$c_2$	0.27	0.27	0.27	Thermal equilibration of deep ocean
$d_1$ (yr)	8.4	8.4	8.4	Thermal adjustment of upper ocean
$d_2$ (yr)	409.5	409.5	409.5	Thermal equilibration of deep ocean
$r_0$ (yr)	-	-	35	Pre-industrial iIRF <sub>100</sub>
$r_C$ (yr/GtC)	-	-	0.02	Increase in iIRF <sub>100</sub> with cumulative carbon uptake
$r_T$ (yr/K)	-	-	4.5	Increase in iIRF <sub>100</sub> with warming

**Table 1.** Default parameter values for simple impulse-response climate-carbon-cycle models used in this paper. Note that, for consistency with (Myhre et al., 2013), the ordering of indices is fast-slow for the thermal response and slow-fast for the carbon cycle.

updated thus:

$$\frac{dR_i}{dt} = a_i E - \frac{R_i}{\alpha \tau_i} \quad ; \quad i = 1, 4 \quad (4)$$

where  $E$  are annual  $\text{CO}_2$  emissions, converted to ppm/year. Atmospheric  $\text{CO}_2$  concentrations are given by  $C = C_0 + \sum_i R_i$ , and radiative forcing by:-

$$5 \quad F = \frac{F_{2X}}{\ln(2)} \ln \left( \frac{C}{C_0} \right) + F_{\text{ext}} \quad ,$$

where  $C_0$  is the pre-industrial  $\text{CO}_2$  concentration,  $F_{2X}$  the forcing due to  $\text{CO}_2$  doubling, and  $F_{\text{ext}}$  the non- $\text{CO}_2$  forcing. GMST anomalies are computed thus:-

$$\frac{dT_j}{dt} = \frac{c_j F - T_j}{d_j} \quad ; \quad T = \sum_j T_j \quad ; \quad j = 1, 2$$

with coefficients  $a_i$ ,  $d_j$  and  $\tau_i$  as given in AR5 Chapter 8, tables 8.SM.9 and 8.SM.10 (Myhre et al., 2013). The sole difference between this model and that used for metric calculations in AR5 is that the  $c_j$  are set to give an Equilibrium Climate Sensitivity (ECS)=2.75K and Transient Climate Response (TCR)=1.6K (corresponding to  $c_1 = 0.46$  and  $c_2 = 0.27$  (Millar et al., 2015)), closer to current best-estimate values than the ECS=3.9K and TCR=2.2K implied by the parameter-values given in Myhre et al. (2013); and the introduction of the state-dependent coefficient  $\alpha$ .

To identify a suitable state-dependence, we focus on parameterising variations in the 100-year integrated impulse response function,  $iIRF_{100}$ . ~~Focussing A focus~~ on the integrated impulse response, ~~or~~ (average airborne fraction over a period of time), as opposed to the airborne fraction at a particular point in time, ~~it~~ is more closely related to the impact of emissions on the global energy budget, and also to other metrics such as Global Warming Potential (GWP) (Joos et al., 2013). With other coefficients

5 fixed, ~~this~~  $iIRF_{100}$  is a monotonic (but non-linear) function of  $\alpha$ :

$$iIRF_{100} = \sum_i \alpha a_i \tau_i \left[ 1 - \exp\left(\frac{-100}{\alpha \tau_i}\right) \right]. \quad (5)$$

Following other simplified carbon-cycle models (Meinshausen et al., 2011a; Glotter et al., 2014), we assume  $iIRF_{100}$  is a function of accumulated perturbation carbon stock in the land and ocean (equivalent to the amount of emitted carbon that no longer resides in the atmosphere),  $C_{acc} = \sum_t E - (C - C_0)$ , ~~converted to GtC~~, and of GMST anomaly from pre-industrial conditions,  $T$ . A simple linear relationship appears to give an adequate approximation to the behaviour of ~~more complex models~~ ESMs and EMICs (as will be shown subsequently in section 3):

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$$iIRF_{100} = r_0 + r_C C_{acc} + r_T T. \quad (6)$$

Values of  $r_0=35$  years,  $r_C =0.02$  years/GtC, recalling that 2.12 GtC = 1ppm and  $r_T =4.5$  years/K, with ECS=2.75K and TCR=1.6K, give a numerically-computed  $iIRF_{100}$  of 53 years for a 100 GtC pulse released against a background  $CO_2$  concentration of 389ppm following a historical build-up, consistent with the central estimate of Joos et al. (2013). These parameters also approximately replicate the relationship between warming-driven outgassing of carbon in the bulk of CMIP5 ESMs (see section 3.3). The values of  $r_0$ ,  $r_T$  and  $r_C$  given here are intended to be taken only as approximate best-estimate values that capture important carbon-cycle dynamics in ESMs. The exact values of these parameters could be tuned (along with the other parameters in the model) to best-reproduce the aspect of ESM/EMIC behaviour of interest (e.g. see Figure 4).

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~~For a prognostic model, we~~ We compute  $iIRF_{100}$  at each time-step using  $C_{acc}$  and  $T$  from the previous time-step and equation 6, convert to a  $\alpha$  using equation 5 and apply to the carbon-cycle equations (equation 4). This means the  $iIRF_{100}$  is only exactly reproduced under constant background conditions with infinitesimal perturbations. Values of  $iIRF_{100}$  larger than 100 years correspond to a net carbon source in response to a perturbation, and, as perturbations to the carbon stock in the atmosphere would grow indefinitely, makes the model unstable. In this regime there is no solution for  $\alpha$ , so we set  $iIRF_{100}$  to a

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25 maximum value of 95 years, corresponding with these parameters to  $\alpha=65.4$ . This physically corresponds to a near-absence of carbon sinks in the Earth system following a very large injection, with very slow rates of decay of atmospheric concentrations.

### 3 Results

~~As a test of the behaviour of the FAIR model, we here conduct a set of experiments under idealised  $CO_2$  emission and concentration scenarios that have also been conducted in the literature for more complex ESMs. Throughout this section, we contrast the performance of the FAIR model, described in the previous section, to the AR5-IR model, the MAGICC model (Meinshausen et al., 2011a) and the BEAM model (Glotter et al., 2014), a simple~~ In this section we initially set out the need

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for the FAIR model by showing that state-independent impulse-response model cannot simultaneously reproduce the observed carbon-cycle model that explicitly represents the physical effects of oceanic carbon uptake on ocean carbonate chemistry (Williams and Follows, 2011). We use a version of the BEAM model with no temperature dependence of model parameters, which has been shown to be small in Glotter et al. (2014). response over the historical period and the future carbon-cycle evolution as projected by ESMs under possible future emissions scenarios (section 3.1). We subsequently evaluate the ability of the FAIR model to capture the responses shown by ESMs and EMICs under a range of idealised experiments (sections 3.2 and 3.3), before discussing climate response uncertainty in the FAIR model and describing a strategy to sample climate response uncertainty within the model structure (section 3.4).

Gregory et al. (2009) conducted a set of experiments with two different ESMs under specified  $\text{CO}_2$  concentrations, followed up with a broader range of models by Arora et al. (2013). Concentrations were increased at  $0.5\% \text{yr}^{-1}$ ,  $1\% \text{yr}^{-1}$  and  $2\% \text{yr}^{-1}$  respectively and consistent emissions were derived for different configurations of the ESMs: a “biogeochemically-coupled” experiment, where the carbon-cycle is only allowed to respond to the direct effect of increasing  $\text{CO}_2$  concentrations and not to the resultant warming; a “radiatively-coupled” experiment in which

### 3.1 The necessity for a state-dependent impulse-response model

A key requirement for simple climate-carbon-cycle models is to reproduce the historical period and the present-day state of the climate system is allowed to respond to the radiative forcing of successfully. Compatibility with the present-day climate state can be important for accurately assessing the scale of future mitigation ambition required to achieve specific policy targets (Rogelj et al., 2011). Atmospheric  $\text{CO}_2$  but the carbon-cycle is only allowed to respond to the simulated warming and not to increasing  $\text{CO}_2$ ; and a “fully-coupled” experiment in which the carbon-cycle is allowed to respond to both warming and concentrations increase faster than observed when computed from estimated historical emissions (Le Quéré et al., 2015) with the AR5-IR model (Figure 1a). This leads to a bias of over 30ppm in 2011 concentrations, due to the slower than observed decay of  $\text{CO}_2$  concentrations. Within our simple model framework, we recreate the “biogeochemically-coupled” experiment by setting  $r_T = 0$ . We approximate the “radiatively-coupled” experiment by evaluating the difference between the “fully-coupled” and “biogeochemically-coupled” experiments. Although Gregory et al. (2009) found that the relationship between the experiments was not simply a linear summation at high  $\text{CO}_2$  concentrations, this serves as an adequate approximation for our purposes here, since our objective is from the atmosphere over the historical period. The AR5-IR displays a too-large instantaneous airborne fraction over the entire historical period and is less consistent with the observations than the correct representation of aggregate feedbacks rather than a breakdown into specific contributions.

Figure 6 shows the response of the FAIR model (blue) described in section 2 under the three experiments described above. The responses of Figure 1c). The PI-IR model maintains a lower instantaneous airborne fraction than the AR5-IR (red) and BEAM models (green) are also shown. Figure 6a shows total carbon uptake by ocean and land,  $C_{\text{acc}}$ , as a function of time in the  $1\% \text{yr}^{-1}$  increasing  $\text{CO}_2$  model throughout the historical period, and matches the observed record much better, however neither state-independent impulse-response model matches observations as well as the state-dependent FAIR model. Large amplitude variations in the instantaneous airborne fraction can be seen in the observational record that are likely to be driven in

large part by unforced variability in the Earth-system and as such would not be expected to be reproduced by any of these simple models. More complex carbon-cycle models are required to understand the drivers of these variations and any implications that they have for future carbon-cycle responses. A similar relationship between the models is seen for emissions derived from each model consistent with prescribed observed CO<sub>2</sub> experiment. As in the full-complexity ESMs shown in Arora et al. (2013) and Gregory et al. (2009), the coupling between temperature changes and the carbon cycle in concentrations (Figure 1b), where required emissions are too low relative to observed values over much of the historical period for both the FAIR model acts to suppress carbon uptake, shown by the difference between the thick and thin blue lines, a mechanism that is absent (by construction) in both AR5-IR and biogeochemical version of the BEAM model considered here. The coupling with cumulative carbon uptake in the FAIR model also increases airborne fraction in the later stages of PI-IR models.

Another key test of simple coupled climate-carbon-cycle models is the ability to replicate the response of ESMs to possible scenarios of future emissions. Commonly-used future scenarios are generally defined in terms of concentration pathways (Van Vuuren et al., 2011) and therefore do not have a model-independent set of emissions associated with them. In this paper we drive all three simple impulse-response climate-carbon-cycle models by a single set of emissions for each future scenario that are derived from the MAGICC model (Meinshausen et al., 2011b) in order to allow a comparison of both concentrations and temperatures between simple models. MAGICC has been shown to be a good emulator of the experiment relative to earlier stages (see figure 1), as illustrated by the approximately linear increase in  $C_{acc}$  in the “biogeochemically-coupled” experiment, also consistent with ESM responses. A constant airborne fraction necessarily gives an approximately quadratic increase in  $C_{acc}$  in this experiment, as illustrated by the CMIP5 ensemble and therefore offers a comparison by proxy to the projection of CMIP5 ESMs (Meinshausen et al., 2011a). Whilst the PI-IR model might do a better job of reproducing historical concentrations, under high future emissions scenarios such as RCP8.5 (Riahi et al., 2011), it underestimates end of century concentrations, relative to MAGICC, to an even greater extent than the AR5-IR model (Figure 2a) and concentrations fall from peak even quicker than MAGICC under the high mitigation RCP2.6 scenario (Figure 2b). It is clear that any state-insensitive impulse-response model is therefore unsuitable, unless modified, for calculations of, for example, the social cost of carbon against realistic baseline trajectories or long integrations with historical and projected emissions.

Figure 6b shows  $C_{acc}$  as a function of atmospheric CO<sub>2</sub> concentration: again, the FAIR model captures the concave-downward form of this diagnostic, in contrast to compares well to MAGICC, particularly for the AR5-IR model. Figure 6c shows the impact of GMST increase on cumulative uptake, or the difference between the biogeochemically coupled and fully coupled experiments shown in panel (a), as a function of warming. 1%yr<sup>-1</sup>, 0.5%yr<sup>-1</sup> and 2%yr<sup>-1</sup> experiments all lie along the same line in panel (c). Panel (d) show cumulative airborne fraction increases after an initial decline, similar to the ambitious mitigation scenario. There is some divergence after 2100 in the high emission scenario, but the behaviour of the ESMs. In contrast, the IPCC-AR5 model shows a steady decrease in the cumulative airborne fraction with higher concentrations due to MAGICC (or indeed any other model) under these more extreme forcing scenarios has not been verified. Whilst comparing the performance of one simple model to another is not as rigorous a test of model performance as comparing directly to the behaviour of ESMs, it is encouraging that the FAIR model shows a close correspondence with a well-known and well-used simple model that has been used extensively to emulate the response of ESMs (Rogelj et al., 2012).

### 3.2 Response to pulse emission experiments

The social cost of carbon is conventionally calculated by applying a pulse emission of a specified magnitude of carbon in near to present-day conditions as a perturbation on top of a certain future emission scenario (NAS, 2016). As calculating the social cost of carbon is a key element of cost-benefit analysis of climate change policy in IAMs, simple climate-carbon-cycle models used in IAMs should aim to reproduce the dependencies of the state-invariant rates at which a pulse of carbon is removed from response to the atmosphere. The initial airborne fraction is higher in the AR5-IR model, as the carbon-cycle response parameters used here are those representative of present-day pulse-response experiments of Joos et al. (2013). An integration of this model under historical emissions therefore produces historical concentrations significantly in excess of those observed in the early 21<sup>st</sup> century (see figure 1) perturbation on pulse size and background state that has been highlighted in ESMs and EMICs (Joos et al., 2013; Herrington and Zickfeld, 2014).

The BEAM model (run with parameters as given in Glotter et al. (2014), which are tuned for long time-scales) displays a very low cumulative uptake of carbon from the atmosphere relative to the other simple models considered here, associated with a cumulative airborne fraction initially around 0.9 which does not decline substantially with time. This model therefore also displays much higher concentrations than observed over the historical period when driven with estimates of historical emissions (not shown), although it must be noted again that the model was tuned to long time-scale responses of more complex models and not over the historical period. Although the BEAM model explicitly solves the equations of Joos et al. (2013) documented the ocean carbonate chemistry, it displays a roughly constant cumulative airborne fraction under the exponential concentration increase scenarios considered here, unlike the ESMs which behave more like the FAIR model constructed in this paper (see figure 3, 4, 5 and 6 in Gregory et al. (2009)). In addition, as BEAM focuses solely on oceanic feedback mechanisms and cannot capture saturation of land carbon sinks and the dependence of land carbon sinks on warming, an important part of the ESM feedbacks (Arora et al., 2013). Land carbon uptake can contribute an equal (or even greater) fraction of the total system carbon uptake in ESM models over time periods on the order of a century. However, it must be noted that physical mechanisms of the land carbon-cycle response to warming remain poorly understood compared to oceanic mechanisms and a wide uncertainty exists about their future responses. Due to this lack of potentially important feedback processes associated with the land carbon-cycle, we do not extend the comparison with the BEAM model in any of the subsequent analyses.

Figure 3 shows the response of the simple climate-carbon-cycle models to the emissions pulse experiments of Joos et al. (2013). In these experiments, a set of ESMs are used to derive emissions that are consistent with concentrations rising as observed historically they exceed an ensemble of ESMs and EMICs to pulses of various sizes and against various different background conditions (black lines in Figure 3). In the PD100 experiment (100GtC pulse in approximately present-day background conditions - upper two panels), future emissions are derived that stabilise concentrations at 389ppm (achieved in 2012) and held constant thereafter. Figure 3a shows the emissions and warming consistent with the baseline (no additional pulse emission) experiment for the FAIR and AR5-IR models. A declining but sustained low level of diagnosed emissions are required following stabilisation of atmospheric concentrations in order to maintain them to stabilise atmospheric concentrations at a constant level (Figure 3a). In a second experiment, a 100GtC pulse is added to these calculated emissions in the year that concentrations ex-

ceed 389ppm and the resulting concentration and temperature anomalies are compared ~~with and to the case~~ without the pulse emission to isolate the coupled response to the pulse emission alone –

(~~Figure 3b shows the concentration and warming response to a 100GtC pulse emission at 389ppm background~~). After 100 years the pulse in the concentration anomaly in the fully coupled FAIR model has decayed to 0.46 of its initial value, slightly greater than the multi-model average of the ESM responses of 0.41, but, the  $iIRF_{100}$  of 53 years is consistent with the ESM multi-model mean of 52.4 years (Joos et al., 2013). Excluding temperature feedbacks (~~the “biogeochemically-coupled” version - setting  $r_T = 0$~~ ) on the carbon-cycle increases the decay of the temperature response to the pulse over the century following the pulse emission ~~which reduces the  $iIRF_{100}$  airborne fraction by 11%~~. The “fully-coupled” FAIR model shows temperature initially adjusting rapidly followed by near-constant temperature over the remainder of the century. ~~The “biogeochemically-coupled” version of the FAIR model, in which temperature-induced feedbacks are suppressed, reduces the integrated 100-year airborne fraction by 11%.~~

Figure 3c and 3d also show the response to a 100GtC and a 5000GtC pulse respectively, applied in pre-industrial conditions (~~named PI100 and PI5000 respectively~~). Similarly to the response shown by ESMs ~~discussed in Joos et al. (2013)~~, the 100GtC pre-industrial pulse decays faster than the present-day case, due to reduced saturation of the land and ocean carbon sinks. With these parameters, the FAIR  $iIRF_{100}$  is approximately 30% lower in the pre-industrial case compared to the present day, consistent with corresponding ratio in the ~~most-detailed ESMs Joos et al. (2013) ensemble~~, with its value of 36 years within the 34-47 years range of the ESMs. The magnitude of temperature response is similar in both ~~the PD100 and PI100~~ cases due to the increased radiative efficiency of a pulse of  $CO_2$  at lower background concentrations counteracting the faster decay of carbon out of the atmosphere. The 89% increase of  $iIRF_{100}$  in the 5000GtC pre-industrial pulse relative to the 100GtC pre-industrial, whilst smaller than the approximate doubling observed in the ESMs, shows that the FAIR model can capture the dependence of the pulse-response on pulse size as well as background conditions, whilst the AR5-IR model displays identical pulse response independent of pulse size or background conditions.

A difference between the FAIR model and the ESMs is that restricting temperature-induced feedbacks on the carbon-cycle does not result in a substantial reduction in the  $iIRF_{100}$  for the pre-industrial 100GtC pulse experiment (the “fully-coupled” and “biogeochemically-coupled” experiments lie on top of each other in figure 3c), whereas a 13% reduction in  $iIRF_{100}$  is observed for the ESMs (Joos et al., 2013) (~~not shown~~). It is only for the 5000GtC pre-industrial pulse experiment that we see a reduction in the  $iIRF_{100}$  associated with suppression of the temperature-induced feedbacks on the carbon cycle ~~in FAIR~~.

~~Herrington and Zickfeld (2014)~~ Significant diversity is seen in the range of responses to the PD100 and PI100 experiments across different ESMs/EMICs (grey shading in Figures 3b and 3c). Whilst this diversity is ultimately attributable to a range of differences in carbon-cycle process representations within the models, variations in just a sub-set of the FAIR parameters are sufficient to span the ranges of responses in both the PD100 and PI100 experiments, as well as the ratio between the two responses. Figure 4 shows this by fitting individual model responses in a two-step process. First, the carbon-cycle parameters of the FAIR model are optimised to minimise the combined residual sum of squares of the FAIR fit to the Joos et al. (2013) multi-model mean airborne fraction in the PD100 and PI100 experiments (whilst maintaining the same ratio between the  $r_T$  and  $r_C$  parameters as the default parameters given in section 2 and assuming fixed  $\tau_i$  at their table 1 values). Then, as a second step,

the response for individual models are fitted by minimising the combined PD100 and PI100 residual sum of squares whilst allowing only the  $r_0$ ,  $r_T$  and  $r_C$  parameters to vary from the model parameters found in the first stage, whilst again maintaining the same ratio between the  $r_T$  and  $r_C$  parameters as the default and therefore reducing the effective degrees of freedom of the fit to just two. The timeseries of change in GMST are taken as given by the individual models. While a much better fit could be obtained by adjusting all the parameters of the FAIR model, this subset appears sufficient to successfully capture much of the response to both the PD100 and PI100 experiments for individual models, as well as their range of behaviour (Figure 4). The FAIR model offers a simple framework to emulate the range of ESM responses whilst at the same time maintaining the dependency on background condition and pulse size for the specific model in question.

As an final test of the FAIR model's sensitivity to pulse size, we also consider the response of the FAIR and AR5-IR models under the idealised pulse experiments of Herrington and Zickfeld (2014). Herrington and Zickfeld (2014) conducted several experiments with the UVic Earth System Model of intermediate complexity (Weaver et al., 2001). We here emulate the PULSE experiments of Herrington and Zickfeld (2014) by integrating the FAIR model and AR5-IR models with historical fossil fuel and land-use CO<sub>2</sub> emissions (as derived from historical concentrations using the MAGICC model, (Meinshausen et al., 2011a)) together with estimates of the historical radiative forcing from non-CO<sub>2</sub> factors. Pulse emissions of various sizes were then applied over a two-year period from 2008 in order to restrict total all time cumulative emissions to specified totals (see Herrington and Zickfeld (2014) for details). Non-CO<sub>2</sub> forcings are held constant at 2008 levels after following RCP8.5 (Riahi et al., 2011) trajectories for 2005-2008.

Figure 5 shows the response of the FAIR model, as well as the AR5-IR model, to the experiments described above. Ricke and Caldeira (2008) used a version of the AR5-IR model to find that the maximum warming from a pulse emissions of CO<sub>2</sub> occurs approximately a decade after emission, but as shown here (Figure 5) and as highlighted by Zickfeld and Herrington (2015), not accounting for feedbacks on the carbon-cycle fails to capture the plateau of CO<sub>2</sub>-induced warming over the century following emission. For all pulse sizes (denoted with different linestyles) contrasting the fully coupled FAIR (thick blue) and the AR5-IR (red) models shows that including carbon-cycle feedbacks is essential to prevent a substantial decay in the temperature anomaly over the first 100 years following the pulse emission. Ricke and Caldeira (2014) used a version of the AR5-IR model to find that the maximum warming from a pulse emissions of CO<sub>2</sub> occurs approximately a decade after emission, but as shown here and as highlighted by Zickfeld and Herrington (2015), not accounting for feedbacks on the carbon-cycle fails to capture the plateau of CO<sub>2</sub>-induced warming over the century following emission. At higher pulse sizes, the temperature response in the FAIR model fails to plateau as quickly as at lower pulses, where the balance between carbon-cycle cooling and long-timescale thermal warming takes centuries to reach balance (Figure 3 of Herrington and Zickfeld (2014)).

As a validation of the FAIR model over the historical period, Figure 1 shows the comparisons to historical data from Le Quéré et al. (2015). In figure 1 simulations are commenced from 1850, which is assumed to be a quasi-equilibrium state for

### 3.3 Response to idealised concentration increase experiments

To explore the response to sustained emissions, rather than an emission pulse, we consider the experiments of Gregory et al. (2009) and Arora et al. (2013), in which ESMs are subjected to specified rates of increase in CO<sub>2</sub> concentrations. Concentrations were increased from pre-industrial values at 0.5%yr<sup>-1</sup>, 1%yr<sup>-1</sup> and 2%yr<sup>-1</sup> respectively and consistent emissions were derived for different configurations of the ESMs: a “biogeochemically-coupled” experiment, where the carbon-cycle is only allowed to respond to the direct effect of increasing CO<sub>2</sub> concentrations and not to the resultant warming; a “radiatively-coupled” experiment in which the climate system is allowed to respond to the radiative forcing of CO<sub>2</sub> but the carbon-cycle is only allowed to respond to the simulated warming and not to increasing CO<sub>2</sub>; and a “fully-coupled” experiment in which the carbon cycle, in order to facilitate comparisons with the observed data (which is available from 1850 onwards). Panel (a) shows the concentration response to estimated global historical carbon-cycle is allowed to respond to both warming and CO<sub>2</sub> concentrations (light pastel coloured lines in Figure 6) for the 1%/yr concentration increase scenario. Such idealised scenarios can be highly informative with regard to the physical drivers of carbon-cycle feedbacks under increased emissions. Successfully emulating the approximate balance between warming-induced and biogeochemically-induced contributions to carbon-cycle feedbacks could be important for integrated assessment of solar radiation management scenarios and mitigation scenarios in which the balance of contributions to warming from CO<sub>2</sub> emissions. The FAIR model replicates concentrations over the past several decades well, as opposed to the AR5-IR model, which has abias of over 30ppm in 2011. Similarly, emissions derived from the time-series of historical and non-CO<sub>2</sub> sources changes significantly in the future.

Within the FAIR framework we recreate the “biogeochemically-coupled” experiment by setting  $r_T = 0$ , and approximate the “radiatively-coupled” experiment by evaluating the difference between the “fully-coupled” and “biogeochemically-coupled” experiments. Although Gregory et al. (2009) found that the relationship between the experiments was not simply a linear summation at high CO<sub>2</sub> concentrations, this serves as an adequate approximation for our purposes here, since our objective is the correct representation of aggregate feedbacks rather than a breakdown into specific contributions.

Similarly to the ESMs from Arora et al. (2013), the coupling between temperature changes and the carbon-cycle in the FAIR model acts to suppress carbon uptake, shown by the difference between the thick and thin lines in Figure 6a, a mechanism that is absent (by construction) in the AR5-IR model. The coupling with cumulative carbon uptake in the FAIR model also increases airborne fraction in the later stages of the experiment relative to earlier stages (Figure 1c), as illustrated by the approximately linear increase in  $C_{acc}$  in the “biogeochemically-coupled” experiment, also consistent with ESM responses. A constant airborne fraction necessarily gives an approximately quadratic increase in  $C_{acc}$  in this experiment, as illustrated by the AR5-IR model. Figure 6b shows  $C_{acc}$  as a function of atmospheric CO<sub>2</sub> concentrations (Figure 1b), are lower in concentration: again, the FAIR model captures the concave-downward form of this diagnostic, in contrast to the AR5-IR model.

Whilst oceanic carbon-cycle feedbacks are almost exclusively driven by biogeochemical effects (Glotter et al., 2014), for simple climate-carbon-cycle models to be of use in representing the entire climate system, they need to capture dependencies of the land carbon cycle on warming. Aside from 3 ESMs that display global-mean carbon-cycles insensitive to warming, Figure 6c shows a coherent relationship between temperature increases and the size of the carbon outgassing back to the atmosphere (Arora et al., 2013). The impact of GMST increase on cumulative uptake, or the difference between the biogeochemically coupled and fully coupled experiments shown in Figure 6a, as a function of warming, indicating that values of  $r_T$  close

to 4.5yr/K allow the FAIR model to reproduce this relationship well.  $1\%yr^{-1}$ , due to the slower decay of  $CO_2$  from the atmosphere over the historical period. Whilst the time mean-value of the  $0.5\%yr^{-1}$  and  $2\%yr^{-1}$  experiments all lie along the same line in panel (c), indicating minimal scenario dependence of this effect in FAIR, in contrast to the two ESMs analysed in Gregory et al. (2009).

5 The initial decrease in cumulative airborne fraction (defined as the fraction of emissions remaining in the air after one year) is captured well by the time-integrated instantaneous airborne fraction) followed by a subsequent increase (Figure 6d) displayed by the FAIR model, fluctuations are of much greater magnitude in the observed record, indicating short timescale processes and variability in the carbon cycle that is not captured by these simple models. The AR5-IR (which is tuned to the present-day response) displays a too large airborne fraction over is a feature of the entire historical period and is less consistent with the observations than the FAIR model. Retuning the response of many ESMs under a  $1\%/yr$  increasing  $CO_2$  scenario. In contrast, the IPCC-AR5 model shows a steady decrease in the cumulative airborne fraction with higher concentrations due to the state-invariant rates at which a pulse of carbon is removed from the atmosphere. The initial decrease in cumulative airborne fraction followed by subsequent increase can be understood in terms of the saturation of carbon sinks. If atmospheric anomalies of carbon decay with fixed timescales,  $\tau_i$  (as in the AR5-IR model to pre-industrial conditions would improve its fit to past emissions, but would then give a worse fit to the behaviour of more complex models under impulse-response experiments and future scenarios model case), then instantaneous airborne fraction remains constant in time, which necessarily means that cumulative airborne fraction must decline over time (as emissions from previous years decay further, so the cumulative fraction of the emitted carbon continually decays from the instantaneous airborne fraction). However, if carbon sinks become saturated, the instantaneous airborne fraction would be expected to increase with time (this is represented in the FAIR model by increases to the decay timescales through the parameterised increase in iIRF100). As more recent emissions (which increase monotonically under the  $1\%/yr$  scenario) have a higher instantaneous airborne fraction, the initial decrease in cumulative airborne fraction stops and then begins to increase as this accelerating saturation becomes the dominant effect.

We also check the FAIR model's response to a pair of benchmark scenarios against that of the widely used MAGICC model (Meinshausen et al., 2011a), a simplified box-model of the climate system response to emissions of various greenhouse gases that has commonly been used to assess climate mitigation scenarios (Clarke et al., 2014)

### 3.4 Uncertainty and probabilistic parameter sampling within the FAIR model

Uncertainty is a crucial factor in the integrated assessment of climate policies. Despite significant advances in climate system understanding, non-negligible uncertainties remain in the responses of the coupled climate-carbon-cycle system to emissions of  $CO_2$  (Gillett et al., 2013). Uncertainty in aspects of the climate response to  $CO_2$  remains broad and climate policies have to be constructed and assessed in the light of this continued uncertainty (Millar et al., 2016). Integrated assessment activities require a representation of the physical climate system that can transparently and simply sample physically-consistent modes of climate response uncertainty.

The impulse-response formulation of the physical climate response to radiative forcing used by both the AR5-IR and FAIR models offers a convenient structure for simply sampling plausible ranges of TCR and ECS, as a unique combination of

TCR and ECS (for fixed response time-scales  $d_j$ ) are associated with a unique combination of the model parameters  $c_j$  (see Millar et al. (2015) for details). Panels a) and b) of figure 2 show the  $\text{CO}_2$  concentration from the FAIR model under RCP8.5 and RCP2.6 emissions respectively when forced with emissions diagnosed from the RCP concentration profiles using MAGICC, contrasted with the concentration timeseries from MAGICC (purple), which is by definition equal to the corresponding scenario-defined concentration profiles. Non- $\text{CO}_2$  forcing is the same in all cases. Figure 4 shows how the likely range of TCR and ECS as assessed by IPCC-AR5 (TCR: 1.0-2.5K and ECS: 1.5-4.5K) can be spanned for assessing the climate response to any radiative forcing scenario.

The FAIR model compares well to MAGICC, particularly for the ambitious mitigation scenario (which is less well reproduced by other simplified climate-carbon-cycle models such as BEAM). There is some divergence after 2100 in the high emission scenario, but the behaviour of MAGICC (or indeed any other model) under these more extreme forcing scenarios has not been verified. Whilst comparing the performance of one simple model to another is not as rigorous a test of model performance as comparing directly to the behaviour of ESMs, it is encouraging that the FAIR model shows a close correspondence with a well-known and well-used simple model that has been used extensively to emulate the response of ESMs (Rogelj et al., 2012).

Temperature responses are shown in the lower left panel, and the relationship between temperature and cumulative carbon emissions in the lower right. Crucially, this modified impulse response model. A robust feature of the carbon-cycle response in all ESMs is an increase in the cumulative airborne fraction over time associated with a saturation of carbon sinks (upward curving black lines in Figure 4c imply that a rising fraction of cumulative emissions remain resident in the atmosphere). Unlike the AR5-IR model, which displays a slowly declining cumulative airborne fraction over time due to the state-independence of its response function, coherent perturbations of +/-13% (approximately equivalent to a present-day  $\text{iIRF}_{100}$  change of +/- 7 years) to the  $r_0$ ,  $r_T$  and  $r_C$  parameters (combined with perturbations to  $c_1$  and  $c_2$  consistent with the IPCC-AR5 likely ranges) in the FAIR model all show increasing cumulative airborne fraction over time (blue shading in Figure 4c) and approximately span the range of responses seen in the CMIP5 models under a  $1\% \text{yr}^{-1}$  concentration increase scenario.

Crucially, the FAIR model also captures the straight-line relationship between cumulative carbon emissions and human-induced warming (Figure 4d) that was highlighted in the IPCC 5th Assessment, and is becoming an integral part of climate change policy analysis (Millar et al., 2016). When integrated under a  $1\% \text{yr}^{-1}$  concentration increase scenario, the FAIR model, with parameter settings given in section 2, has a Transient Response to Cumulative Emissions (TCRE) = 1.5K/TtC (see figure 4). A common +/-10% perturbation to the parameters  $r_T$ ,  $r_C$  (thick blue line in Figure 4d). Perturbations to the model parameters as described above (and  $r_0$  (combined with perturbations to  $c_1$  and  $c_2$  consistent with the IPCC-AR5 likely ranges for TCR: 1.0-2.5K and ECS: 1.5-4.5K) identical to Figure 4c) allow the IPCC-AR5 likely TCRE range of 0.8-2.5K/TtC to be spanned, as shown in figure 4 (Figure 4d). In contrast, the AR5-IR model, with a constant airborne fraction, shows a clear concave-downward shape in a plot of realised warming against cumulative carbon emissions, because the decline of the cumulative airborne fraction is unable to compensate (as it does in more complex models) for the logarithmic relationship between  $\text{CO}_2$  concentration and radiative forcing (Millar et al., 2016). The FAIR model also displays some curvature at high cumulative emissions, consistent with Ledue et al. (2015) the behaviour of ESMs (Ledue et al., 2015).

### 3.5 Probabilistic parameter sampling within the FAIR model

Integrated assessment of climate change often requires probabilistic projections of the climate response to CO<sub>2</sub> emissions, partly in order to capture and assess the possibility of extreme, and highly costly, sensitivities within the Earth system (often called “fat-tailed” [events/outcomes](#)) (Weitzman, 2011). Uncertainty in the global climate response to emissions of CO<sub>2</sub> is associated with several factors, which are each considered in turn here.

Uncertainty in the thermal response to radiative forcing typically tends to dominate uncertainty in the response of the global climate system to CO<sub>2</sub> emissions (Gillett et al., 2013). ECS and TCR co-vary in global climate models (Knutti et al., 2005; Millar et al., 2015), with TCR typically considered the more policy-relevant parameter and the parameter better constrained by climate observations to date (Frame et al., 2006; Gillett et al., 2013). Hence varying ECS alone in a probabilistic assessment risks introducing an implicit distribution for TCR that is inconsistent with available observations. Millar et al. (2015) observed that, within the coupled models of the CMIP5 ensemble, TCR and the ratio TCR/ECS (referred to as the Realised Warming Fraction or RWF) are approximately independent. IPCC-AR5 provided formally assessed uncertainty ranges for TCR and ECS (Collins et al., 2013) but not for their ratio. RWFs for the CMIP5 models lie within the range 0.45-0.7, while observationally-constrained estimates typically lie in the upper half of this range (Millar et al., 2015).

As IPCC-AR5 likely (>66% probability) ranges for a physical climate parameter attempt to capture structural uncertainties that might be present in all studies, therefore, IPCC-AR5 likely intervals are generally comparable to the 90% confidence intervals in the underlying studies. IPCC-AR5 gives no assessment of the shape of the distribution associated with structural uncertainty as, by definition, this encompasses “unknown unknowns” that are not included in any model or study available. For quantitative modelling purposes, likely ranges are best interpreted as 5-95 percentiles of input distributions for IPCC-AR5 assessed parameters, provided a similar “structural degradation” is applied to interpret the 5-95 percentiles of output quantities as corresponding only to a likely range, propagating the possibility of structural uncertainty in the assessed parameter through the study. We here assume a bounded (between 0 and 1) Gaussian distribution for RWF and a log-normal distribution for TCR, ~~representative of a positive skew (a long, high response, tail to the probability distribution) (Pueyo, 2012)~~ in many estimates reproducing the positive skewness (fat high tail) of many estimated distributions for this parameter. A log-normal distribution has some theoretical justification for a so-called “scale parameter”, or one in which uncertainty increases with parameter size, which is arguably the case for TCR (Pueyo, 2012). Convolving a bounded Gaussian RWF distribution (with 5-95 percentiles of 0.45-0.75) with a log-normal TCR distribution (with 5-95 percentiles of 1.0-2.5K), gives a corresponding ECS 5-95 percentile range of 1.6-4.5K, in good agreement with the IPCC-AR5 assessed likely range (1.5-4.5K). A sample of 300 ECS and TCR values drawn from these distributions are shown in figure 8a.

Another key uncertainty is the short thermal response timescale,  $d_1$ , an important determinant of the Initial Pulse-adjustment Time (IPT), the initial [e-folding](#) adjustment time of the temperature response to a pulse emission of CO<sub>2</sub> (NAS, 2016). This can be approximated for the FAIR model as  $IPT=d_1(1-a_3)$ . Throughout this paper we have used the IPCC-AR5 default value for  $d_1$  of 8.4 years, but this is longer than indicated by most climate models (Geoffroy et al., 2013). We therefore sample the short thermal response timescale using a Gaussian distribution with a median value of 4 years and a 5-95% probability interval

of 2-8 years. This corresponds to an ~~approximated~~approximate median estimate of 2.8 years with 5-95 percentile range of 1.4-5.6 years for the IPT.

We consider uncertainties in the carbon cycle by sampling  $r_0$ ,  $r_T$  and  $r_C$  with Gaussian distributions of 5-95% probability intervals equal to +/- ~~10%-13%~~(present-day iIRF<sub>100</sub> +/- 7 years) of their default value. Combined with the thermal response uncertainty sampling, the emergent 5-95% range (based on 300 draws from the input parameter distributions) for TCRC (figure 8c) of 1.0-~~2.4~~2.5K/TtC is broadly consistent with the IPCC-AR5 *likely* range (0.8-2.5K/TtC).

Sampling these parameters independently, as described above, produces a range of responses to a 100 GtC pulse emissions in 2020 against the background of the RCP2.6 scenario (figure 8d). However, we consistently observe a rapid warming on the order of a decade followed by an approximate warming plateau (at differing values) that persists for a century or more. Such behaviour is broadly consistent, in all cases, with the range of pulse-response behaviour observed across the ensemble of ESMs in Joos et al. (2013).

## 4 Conclusions

In this paper we have presented a simple Finite Amplitude Impulse Response (FAIR) ~~carbon-cycle-climate~~climate-carbon-cycle model, which adjusts the carbon-cycle impulse-response function based on feedbacks from the warming of the climate and cumulative CO<sub>2</sub> uptake, through a parameterisation of the 100-year integrated impulse-response function, iIRF<sub>100</sub>. This metric provides a potential parallel to those used to assess the thermal response to radiative forcing, namely the Transient Climate Response (TCR) and the Equilibrium Climate Sensitivity (ECS). Although a useful composite metric for the coupled climate-carbon-cycle system exists, the Transient Climate Response to Cumulative Emissions (TCRE), future studies of carbon cycle behaviour could report on ranges of iIRF<sub>100</sub>, and importantly for carbon cycle feedbacks, the evolution of this metric over time under specific emissions scenarios, in order to isolate the changing response of the carbon cycle.

We have shown that including both explicit CO<sub>2</sub> uptake- and temperature- induced feedbacks are essential to capture ESM behaviour. Important dependences of the carbon-cycle response to pulse size, background conditions and the suppression of temperature-induced feedbacks are generally well captured ~~with~~by the FAIR model. As present-day pulse responses are an essential part of calculations of the social cost of carbon (Marten, 2011), the inclusion of climate-carbon-cycle feedbacks in the FAIR model offers an improvement on several simple and transparent climate-carbon-cycle models that have been proposed for policy analysis which either incorporate no feedbacks on the carbon-cycle or do not fully capture the operation of these feedbacks in ESMs.

We believe that the FAIR model could be a useful tool for offering a simple and transparent framework for assessing the implications of CO<sub>2</sub> emissions for climate policy analyses. It offers a structure that both replicates the essential physical mechanisms of the climate system's response to cumulative emissions, whilst at the same time can easily be modified to sample representative climate response uncertainty in either the thermal climate response component, the unperturbed carbon-cycle or the coupled climate-carbon-cycle response to anthropogenic CO<sub>2</sub> emissions. Tuning of parameters within the FAIR framework allows the range of ESM behaviour to be emulated whilst maintaining the physically-understood dependency of pulse-response

on background conditions and pulse size exhibited by a particular ESM. This model structure could thus be adapted to be an effective emulator of CMIP6 ESM responses under a variety of scenarios.

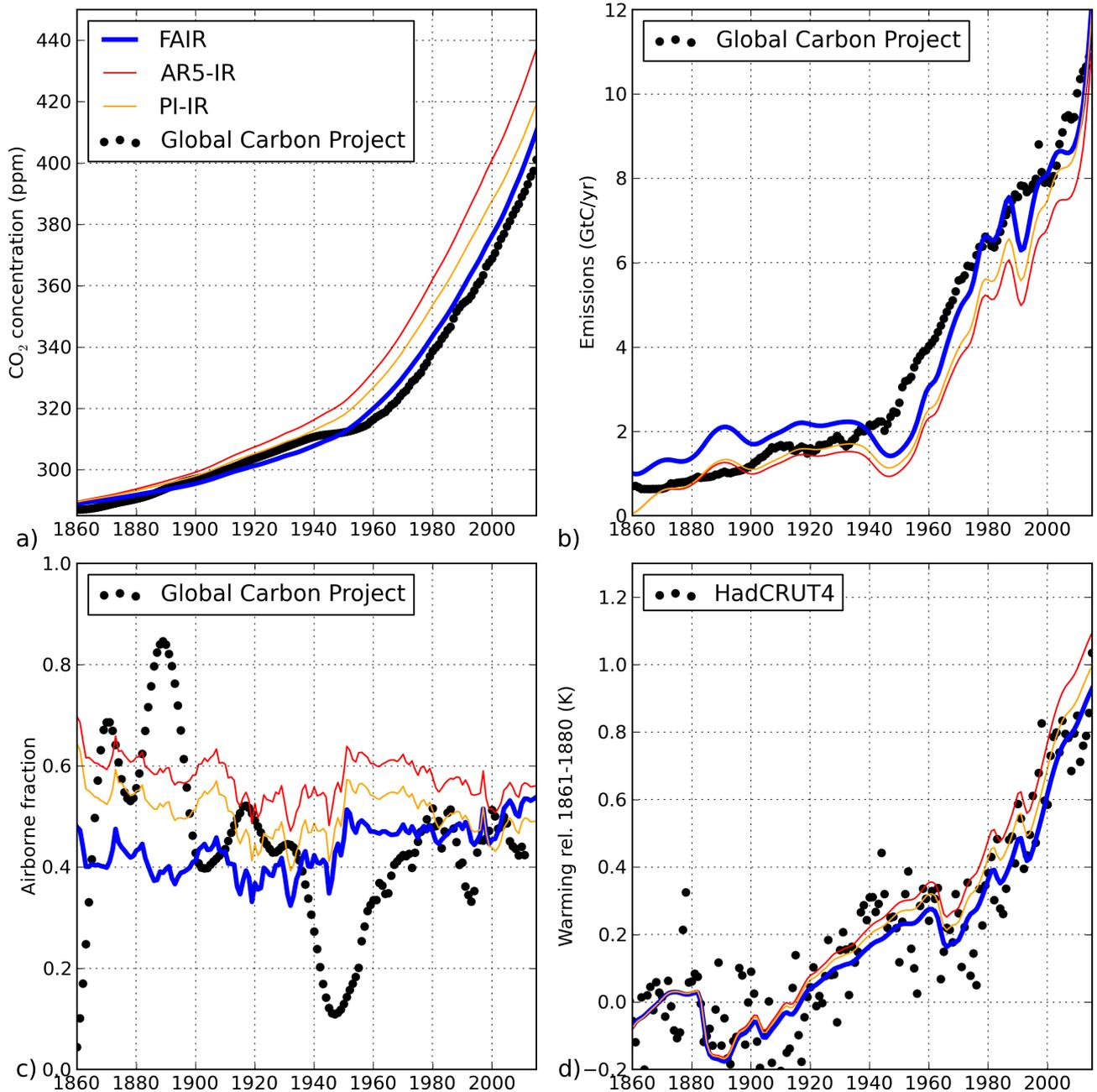
*Author contributions.* RJM, ZRN and MRA developed the FAIR model formulation. PF and MRA identified the need for the feedback term in the AR5-IR model while RJM developed the final formulation. MRA designed the tests and RJM made the figures, except Figure 4 which  
5 was made by ZRN. RJM wrote the first draft of the manuscript and all authors contributed to the editing and revisions of the paper.

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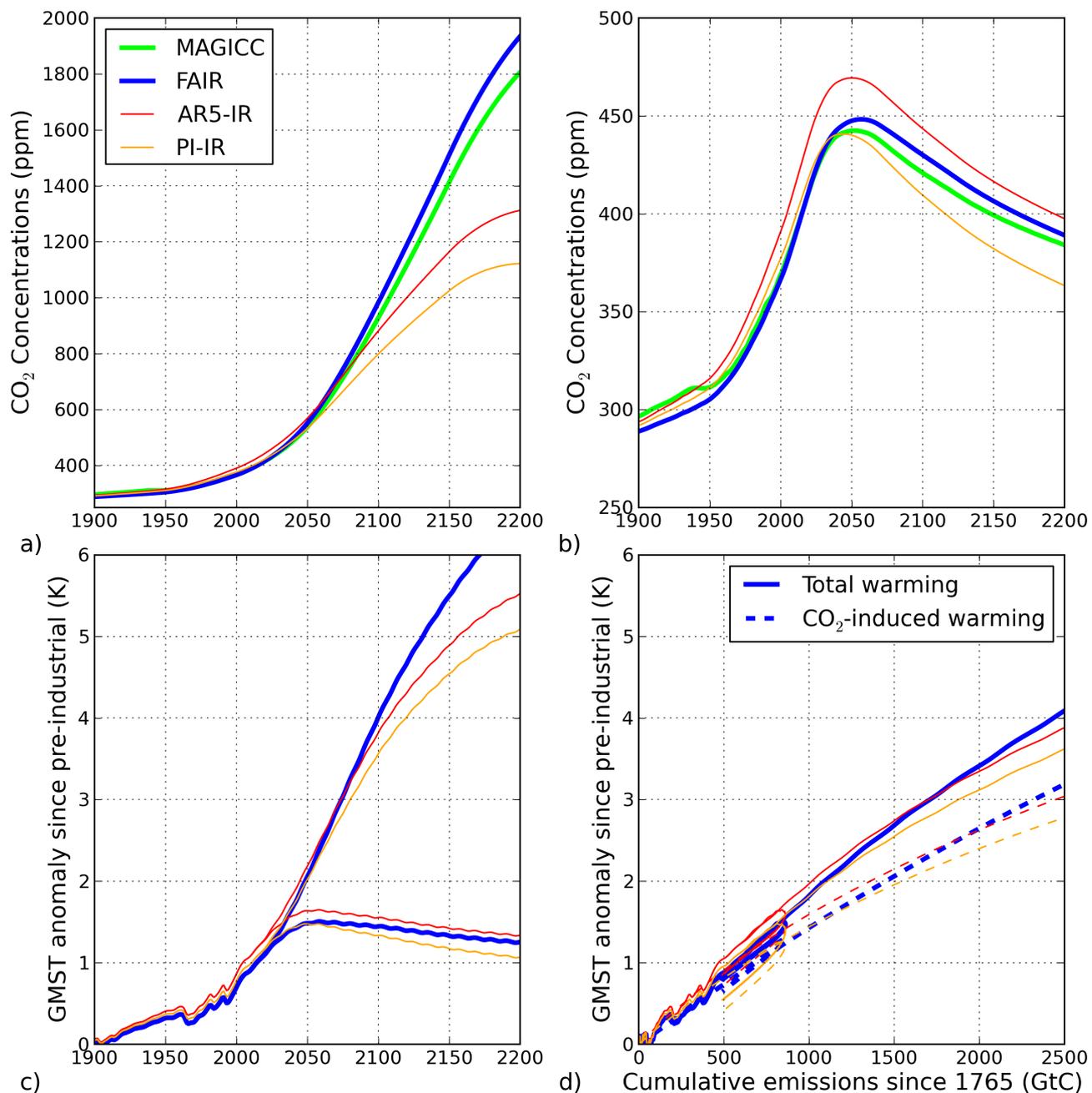
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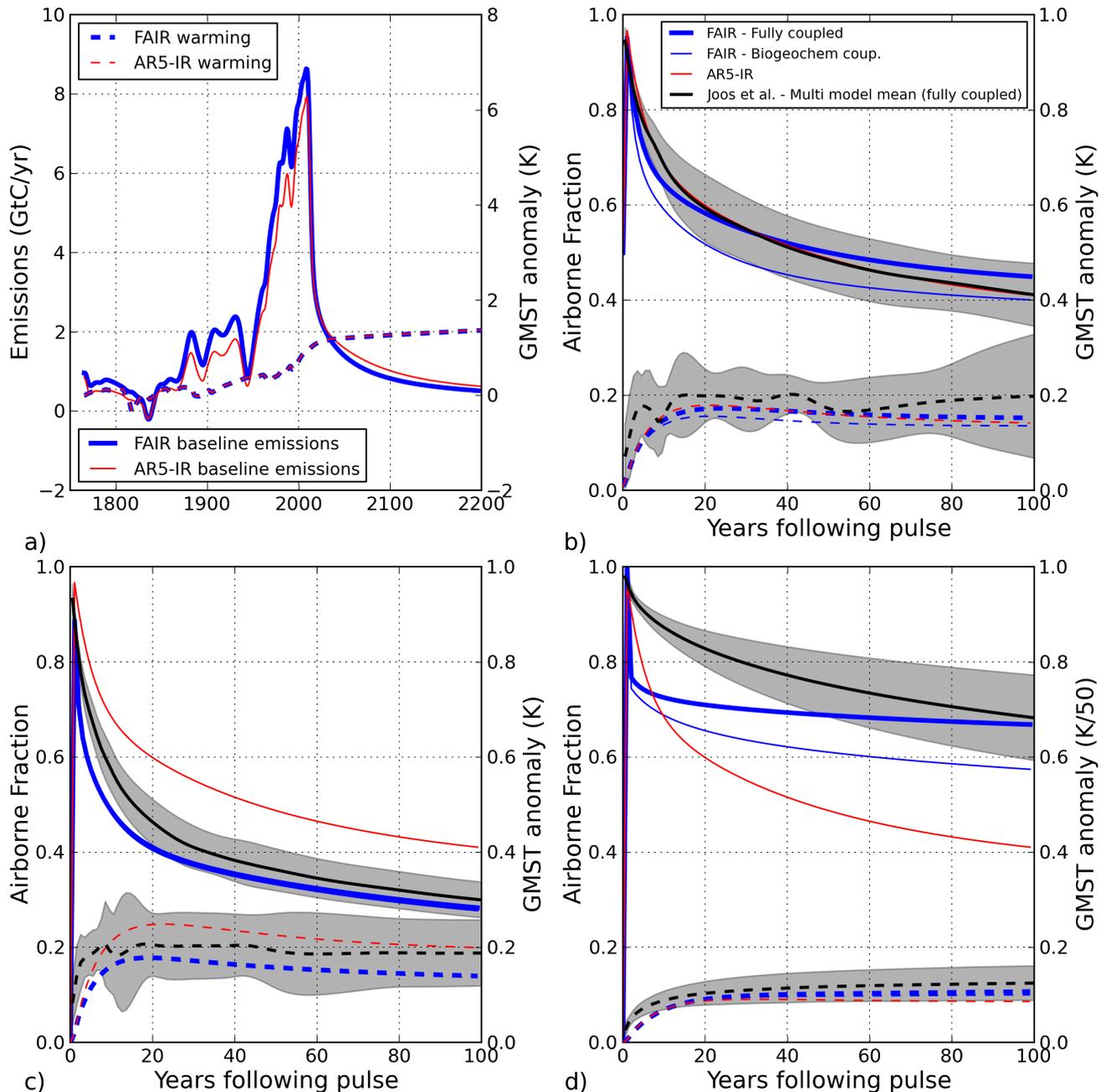
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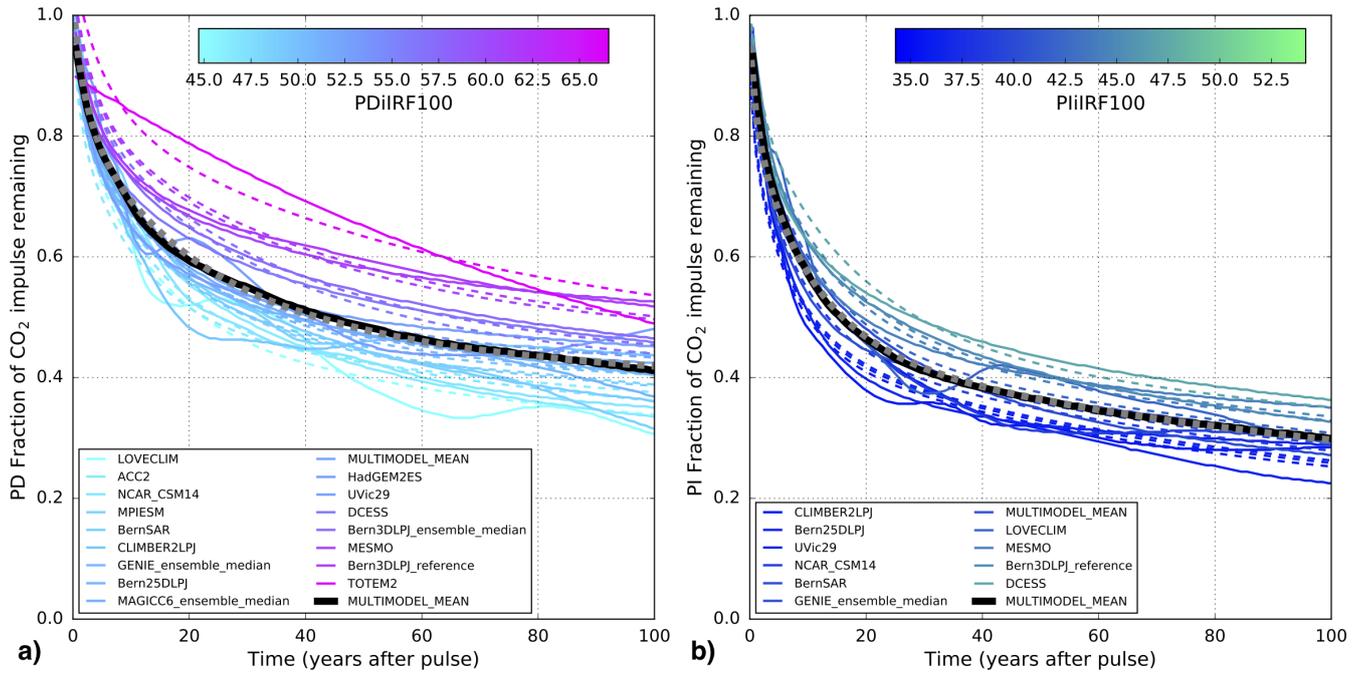
**Figure 1.** Response to idealised concentration increase experiments from Gregory et al. (2009) for historical validation of the FAIR (blue), IPCC-AR5-AR5-IR (red) and BEAM-PI-IR (greenorange) models. Panel a) shows the cumulative total carbon uptake over time in the “fully coupled”  $1\%yr^{-1}$   $CO_2$  concentration increase scenario response when integrated under historical emissions (and historical non- $CO_2$  radiative forcing for the RCP scenarios). Panel b) shows the derived  $CO_2$  emissions consistent with historical concentrations. Panel c) shows the evolution of cumulative total carbon uptake as annual airborne fraction (smoothed with a function of atmospheric concentration 7-year running mean for the observations) in the “biogeochemically-coupled” experiment for  $1\%yr^{-1}$  models when driven by historical emissions (solid as in panel a),  $2\%yr^{-1}$  (dashed) and  $0.5\%yr^{-1}$  (dotted) experiments. Panel d) shows the cumulative uptake as a function of temperature warming anomaly in the “radiatively-coupled” experiment models when driven by historical emissions. Panel Historical observations are shown as black dots in all panels. Panels a), b) and c) all show data from Le Quéré et al. (2015) and panel d) shows the evolution of the cumulative airborne



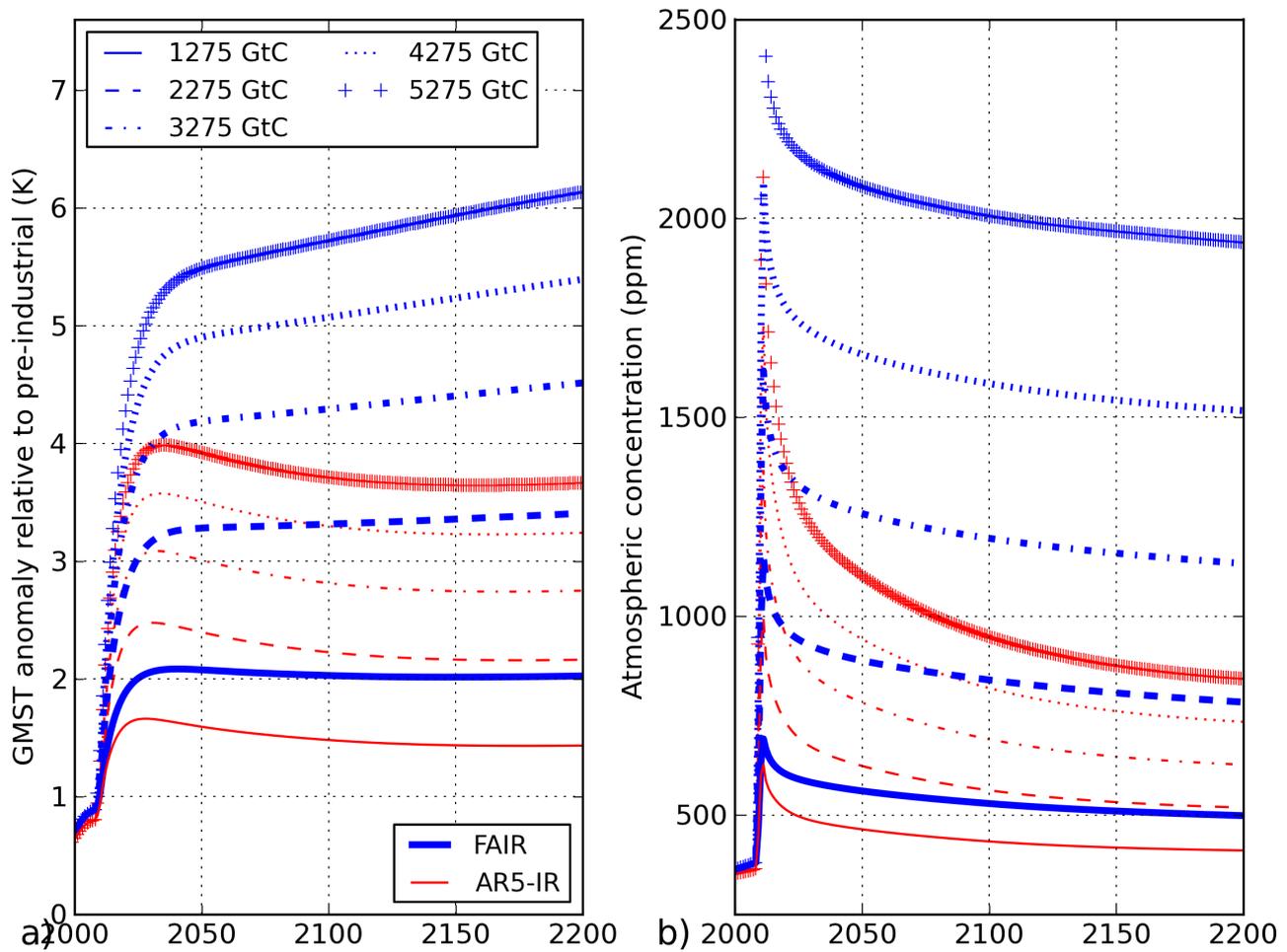
**Figure 2.** Panels a) and b) shows the CO<sub>2</sub> concentrations under RCP8.5 and RCP2.6 respectively for the FAIR (blue), AR5-IR (red), PI-IR (orange) and MAGICCC (green) models. Panel c) shows the temperature response under both RCP2.6 and RCP8.5. Panel d) shows the evolution of total warming (full) and CO<sub>2</sub>-induced warming (dashed) as a function of cumulative carbon emissions.



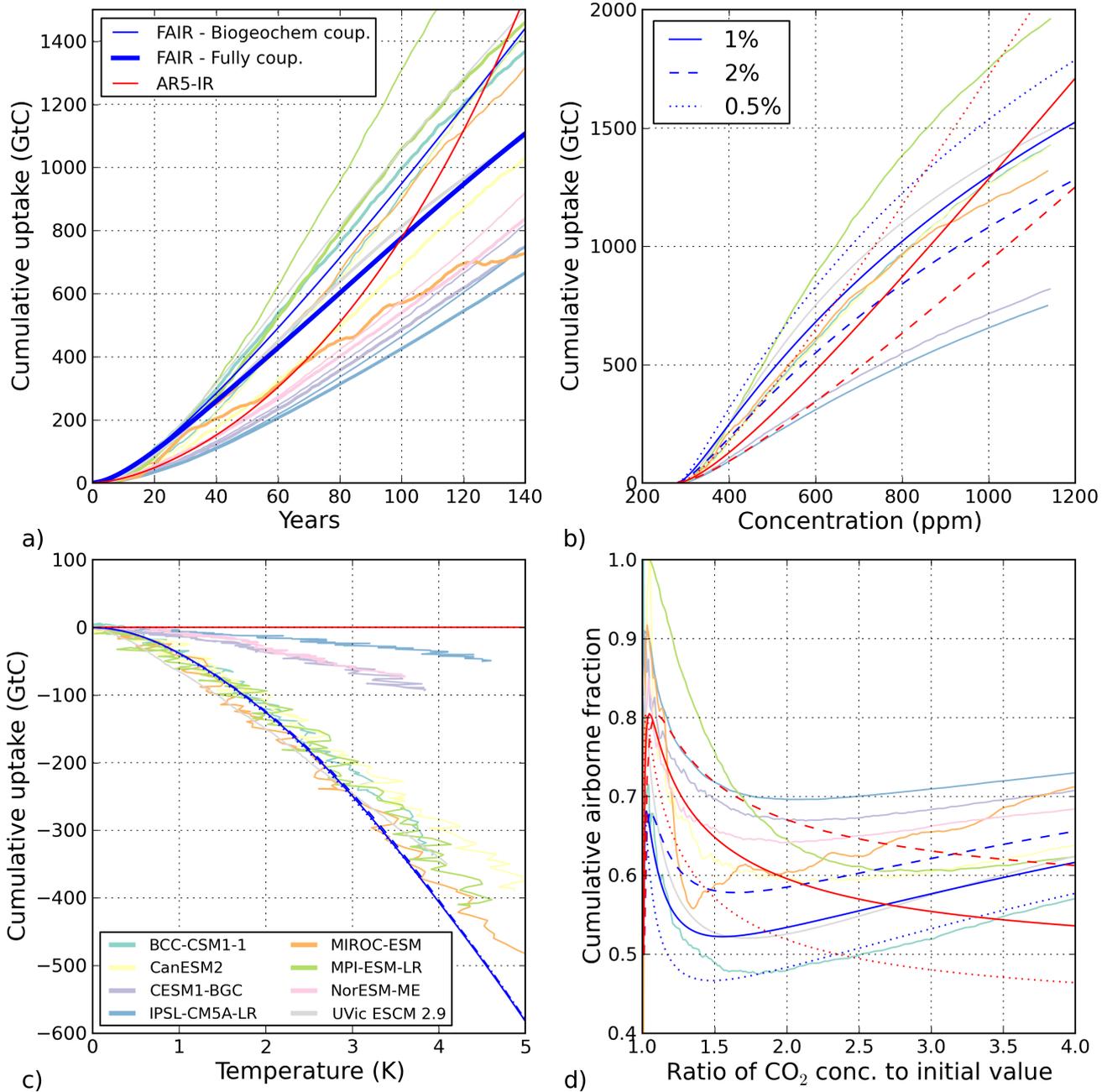
**Figure 3.** Response to pulse emission experiments of Joos et al. (2013). Panel a) shows the “baseline” emissions (left-hand axis, solid) and warming (right-hand axis, dashed) when concentrations are stabilised at 389ppm for the FAIR (blue) and AR5-IR (red) models. Panel b) shows the response to a 100GtC imposed on present-day (389ppm) background conditions ([PD100 experiment](#)). Panel c) shows the response to a 100GtC pulse in pre-industrial conditions ([PI100 experiment](#)). Panel d) shows the response to a 5000GtC pulse in pre-industrial conditions ([PI5000 experiment](#)), with the warming normalised by the increase in pulse size between panels c) and d). [The black lines in panels b\), c\) and d\)](#) shows the Joos et al. (2013) multi-model mean for airborne fraction (solid) and warming (dashed), with the black shading indicating one standard deviation uncertainty across the ensemble.



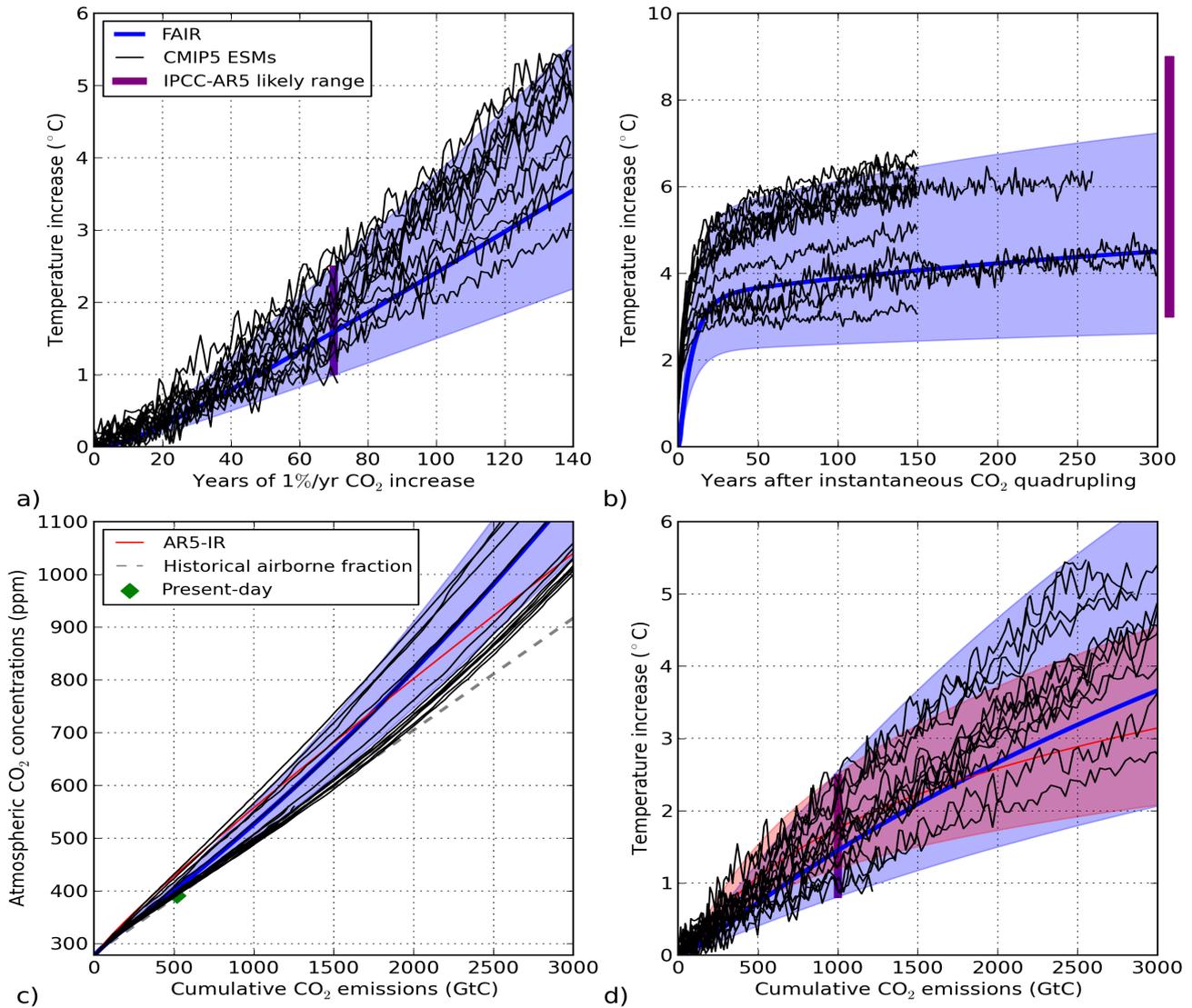
**Figure 4.** The left hand Fitting individual models from Joos et al. (2013) with FAIR. Panel a) shows the remaining airborne fraction for the PD100 experiment and panel b) for those models that additionally completed the PI100 experiment. Solid lines show the original model response coloured by the  $iIRF_{100}$  values. Emulations with FAIR are shown by the same coloured dashed lines. The multi-model mean is shown by a solid black line with the FAIR fit denoted by a dashed grey line.



**Figure 5.** [Panel a\)](#) shows the global mean surface temperature (GMST) response to the pulse experiments of Herrington and Zickfeld (2014). Pulse emissions are applied over a 2-year period from 2008, with differing total cumulative carbon emissions denoted by different line styles. Responses are shown for the FAIR (blue) and AR5-IR (red) models. [The right hand panel Panel b\)](#) shows the corresponding concentration response.

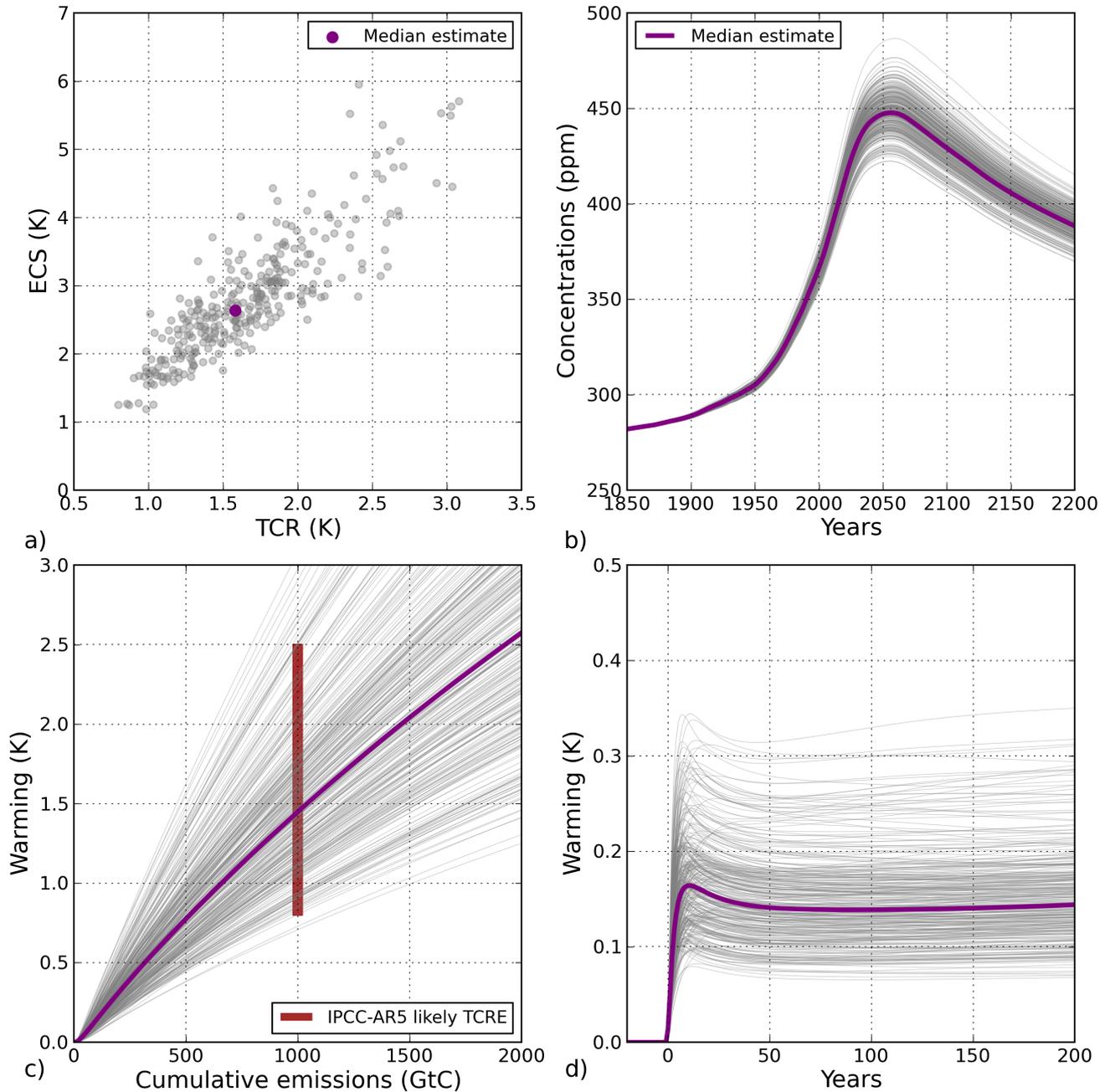


**Figure 6.** Panel a) shows the CO<sub>2</sub> Response to idealised concentration response when integrated under historical emissions (and non-CO<sub>2</sub> radiative forcing) increase experiments from Gregory et al. (2009) for the FAIR (blue) and AR5-IR (red) models. Light pastel colours show the ESMs from Joos et al. (2013) for the 1%/yr concentration increase scenario only. Panel b) shows the derived CO<sub>2</sub> emissions consistent with historical concentrations cumulative total carbon uptake over time in the “fully coupled” 1%yr<sup>-1</sup> concentration increase scenario. Panel c) shows the evolution of annual airborne fraction (smoothed with cumulative total carbon uptake as a 7-year running mean for the observations) function of atmospheric concentration in the models when driven by historical emissions “biogeochemically coupled” experiment for 1%yr<sup>-1</sup> (solid), 2%yr<sup>-1</sup> (dashed) and 0.5%yr<sup>-1</sup> (dotted) experiments. Panel d) shows the warming anomaly cumulative uptake as a function of temperature in the models when driven by historical emissions “radiatively coupled” experiment. Historical observations are shown as black dots in all panels. Panels a), b) and c) all show data from Le Ouéré et al. (2015) and panel d) shows



TCRE uncertainty in the FAIR model 1%/yr scenario. Straight upward curving lines indicate a constant TCRE and an increase cumulative airborne fraction. Dashed brown lines (The plumes in panels c) and d) show the IPCC-AR5 likely 0.8-2.5K/TtC assessed range for TCRE. The blue plume shows the response to 1%/yr<sup>-1</sup> increase in CO<sub>2</sub> concentrations for the IPCC-AR5 likely TCR and ECS ranges in the FAIR model, with an additional +/-10% perturbation to the  $r_0$ ,  $r_T$  and  $r_C$  parameters for the high/low end the likely ranges respectively in the FAIR model. The red plume dashed grey line indicates a constant cumulative airborne fraction that is consistent with the present-day state of the climate system (green diamond). Panel d) shows warming as a function of cumulative emissions in the AR5-IR model response 1%/yr scenario. Straight lines indicate a constant TCRE. The purple bar shows the IPCC-AR5 likely 0.8-2.5K/TtC assessed range for TCRE.

TCRE uncertainty in the FAIR model 1%/yr scenario. Straight upward curving lines indicate a constant TCRE and an increase cumulative airborne fraction. Dashed brown lines (The plumes in panels c) and d) show the IPCC-AR5 likely 0.8-2.5K/TtC assessed range for TCRE. The blue plume shows the response to 1%/yr<sup>-1</sup> increase in CO<sub>2</sub> concentrations for the IPCC-AR5 likely TCR and ECS ranges in the FAIR model, with an additional +/-10% perturbation to the  $r_0$ ,  $r_T$  and  $r_C$  parameters for the high/low end the likely ranges respectively in the FAIR model. The red plume dashed grey line indicates a constant cumulative airborne fraction that is consistent with the present-day state of the climate system (green diamond). Panel d) shows warming as a function of cumulative emissions in the AR5-IR model response 1%/yr scenario. Straight lines indicate a constant TCRE.



**Figure 8.** Probabilistic sampling in the FAIR model. Grey lines show 300 random draws from the input parameter distributions, as described in the text. Panel (a) shows the joint distribution of TCR and ECS. Panel (b), the concentration response under MAGICC-derived RCP2.6 emissions. Panel (c), warming as a function of cumulative emissions in the  $1\%yr^{-1}$  concentration increase experiment. The [dashed-brown lines bar in panel c](#) represent the IPCC-AR5 likely TCRE range. Panel (d), the warming response to a 100GtC pulse emitted in 2020 on top of the MAGICC-derived RCP2.6 emissions. The purple line/dot represents the median estimate in all panels.