# Revisiting the Steering Principal of Tropical Cyclone Motion in a Numerical Experiment 

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#### Abstract

The steering principle of tropical cyclone motion has been applied to tropical cyclone forecast and research for nearly 100 years. Two fundamental questions remain unanswered. One is why the steering flow plays a dominant role in tropical cyclone motion and the other is when tropical cyclone motion deviates considerably from the steering. A high-resolution numerical experiment was conducted with the tropical cyclone in a typical large-scale monsoon trough over the western North Pacific. The simulated tropical cyclone experiences two eyewall replacement processes.

Based on the potential vorticity tendency (PVT) diagnostics, this study demonstrates that the conventional steering, which is calculated over a certain radius from the tropical cyclone center in the horizontal and a deep pressure layer in the vertical, plays a dominant role in tropical cyclone motion since the contributions from other processes are largely cancelled out due to the coherent structure of tropical cyclone circulation. Resulting from the asymmetric dynamics of the tropical cyclone inner core, the trochoidal motion around the mean tropical cyclone track cannot be accounted for by the conventional steering. The instantaneous tropical cyclone motion can considerably derivate from the conventional steering that approximately accounts for the combined effect of the contribution of the advection of the symmetric potential vorticity component by the asymmetric flow and the contribution from the advection of the wavenumber-one potential vorticity component by the symmetric flow.


## 1. Introduction

The environmental steering principle has been applied to tropical cyclone track forecasting for nearly 100 years (Fujiwara and Sekiguchi 1919; Bowie 1922), which states that a tropical cyclone tends to follow the large-scale flow in which it is embedded. Such a steering concept has been extended to include the beta drift (also called secondary steering) that arises mainly from the interaction between tropical cyclone circulation and the planetary vorticity gradient (Holland 1983; Chan 1984; Chan and Williams 1987; Fiorino and Elsberry 1989; Carr and Elsberry 1990; and Wang and Li 1992; Wang and Holland 1996a). The steering flow is usually calculated over a certain radius from the tropical cyclone center in the horizontal and a deep pressure layer in the vertical (Dong and Neumann 1986; Velden and Leslie 1991; Franklin et al. 1996). For convenience, here we call it the conventional steering flow. As a rule of thumb, the conventional steering flow has been extensively used in tropical cyclone track forecasting and understanding of tropical cyclone motion (e.g., Simpson 1948; Riehl and Burgner 1950; Chan and Gray 1982; Fiorino and Elsberry 1989; Neumann 1993; Wu and Emanuel 1995a, b; Wang and Holland 1996b, c; Wu et al. 2011a; Wu et al. 2011b). Given complicated interactions between tropical cyclone circulation and its environment, tropical cyclone motion should be not like a leaf being steered only by the currents in the stream. Therefore, two fundamental issues are still remaining on the steering principle. First, why can the conventional steering plays a dominant role in tropical cyclone motion? Second, when may tropical cyclone motion deviate considerably from the conventional steering?

The potential vorticity tendency (PVT) paradigm for tropical cyclone motion was proposed by Wu and Wang (2000), in which a tropical cyclone tends to move to the region of the PVT maximum. In other words, tropical cyclone motion is completely determined by the azimuthal wavenumber-one component of PVT and all of the factors that contribute to the azimuthal wavenumber-one component of PVT play a potential role in tropical cyclone motion. The contributions of individual factors can be quantified through the PVT diagnosis and the steering effect is one of the factors (Wu and Wang 2000). Wu and Wang (2000, 2001a) evaluated the PVT approach using the output of idealized numerical experiments with a coarse spacing of 25 km and understood the vertical coupling of tropical cyclone circulation under the influence of vertical wind shear. Wu and Wang (2001b) found that convective heating can affect tropical cyclone motion by the heating-induced flow and the positive PVT that is directly generated by convective heating.

The PVT paradigm was further verified by Chan et al. (2002). The observational analysis indicated that the potential vorticity advection process is generally dominant in tropical cyclone motion without much change in direction or speed while the contribution by diabatic heating is usually less important. An interesting finding in the study is that the contribution of diabatic heating becomes important for irregular tropical cyclone motion, suggesting that track oscillations as well as irregular track changes may be explained by changes in the convection pattern. The PVT approach has been used in understanding tropical cyclone motion in the presence of the effects of land surface friction, river deltas, coastal lines, mountains, islands, cloud-radiative
processes and sea surface pressure gradients (e.g., Wong and Chan 2006; Yu et al. 2007; Fovell et al. 2010; Hsu et al. 2013; Wang et al. 2013; Choi et al. 2013).

As we know, the coarse resolution of the numerical experiment in Wu and Wang (2000) was unable to resolve the eyewall structure and tropical cyclone rainbands, which may affect tropical cyclone motion (Holland and Lander 1993; Nolan et al. 2001; Oda et al. 2006; Hong and Chang 2009). Under the PVT paradigm, in this study we use the output from a high-resolution numerical experiment to address the aforementioned two fundamental issues that are important to understanding tropical cyclone motion. The numerical experiment was conducted with the advanced research version of the Weather Research and Forecast (ARW-WRF) model. In particular, an initially symmetric baroclinic vortex is embedded in the low-frequency atmospheric circulation of Typhoon Matsa (2005) to simulate tropical cyclone motion in a realistic large-scale environment. For simplicity, the present study focuses on the numerical experiment without the influences of land surface and topography.

## 2. The output of the numerical experiment

The numerical experiment conducted with the WRF model (version 2.2) in this study contains a coarsest domain centered at $30.0^{\circ} \mathrm{N}, 132.5^{\circ} \mathrm{E}$ and four two-way interactive domains. In order to better simulate the tropical cyclone rainbands and eyewall structure, the horizontal resolutions are $27,9,3,1,1 / 3 \mathrm{~km}$, respectively. The three innermost domains move with the tropical cyclone (Fig. 1). The model consists of 40 vertical levels with a top of 50 hPa . The WRF single-moment 3-class scheme and the Kain-Fritsch cumulus parameterization scheme (Kain and Fritsch 1993) are
used in the outmost domain. The WRF single-moment 6-class scheme (Hong and Lim 2006) and no cumulus parameterization scheme are used in the four inner domains. The other model physics options are the Rapid Radiative Transfer Model (RRTM) longwave radiation scheme (Mlaewe et al. 1997), the Dudhia shortwave radiation scheme (Dudhia 1989), and the Yonsei University scheme for planetary boundary layer parameterization (Noh et al. 2003).

The National Centers for Environmental Prediction (NCEP) Final (FNL) Operational Global Analysis data with resolution of $1.0^{\circ} \times 1.0^{\circ}$ at every 6 h were used for deriving the large-scale background with a 20-day low-pass Lanczos filter (Duchon 1979). The low-frequency fields were taken from those of Typhoon Matsa (2005) from 0000 UTC 5 August to 0000 UTC 9 August 2005. At 0000 UTC 5 August, the typhoon was located to the northeast of Taiwan Island with the maximum surface wind of $45 \mathrm{~m} \mathrm{~s}^{-1}$. During the following three days, Matsa moved northwestward in the monsoon trough and made landfall on mainland China at 1940 UTC 5 August. The sea surface temperature is spatially uniform being $29^{\circ} \mathrm{C}$. The analysis nudging for the wind components above the lower boundary layer is used in the coarsest domain to maintain the large-scale patterns with a nudging coefficient of $1.5 \times 10^{-4} \mathrm{~s}^{-1}$.

A symmetric vortex is initially embedded at $25.4^{\circ} \mathrm{N}, 123.0^{\circ} \mathrm{E}$ (Matsa's center) in the background (Fig. 1). The vortex was spun up for 18 hours on an f-plane without environmental flows to make it relatively consistent with the WRF model dynamics and physics. Considering several hours of the initial spin-up, here we focus only on the 72 -hour period from 6 h to 78 h with the output at one-hour intervals. The
simulated tropical cyclone takes a northwest north track (Fig. 1), generally similar to that of Typhoon Matsa (2005). The evolution of tropical cyclone intensity is shown in Figure 2.

Figure 3 shows the simulated wind and radar reflectivity fields at 700 hPa . The vertical wind shear, which is calculated between 200 hPa and 850 hPa over a radius of 500 km from the tropical cyclone center, is also plotted in the figure. The tropical cyclone center is defined as the geometric center of the circle on which the azimuthal mean tangential wind speed reaches a maximum (Wu et al. 2006). We use a variational method to determine the tropical cyclone center each hour at each level. Different definitions of the tropical cyclone center are also used and it is found that fluctuations in tropical cyclone translation do not depend on the specific definition of the tropical cyclone center. At 24 h (Fig. 3a), the vertical wind shear is more than 10 $\mathrm{m} \mathrm{s}^{-1}$. The eyewall is open to the southwest and strong eyewall convection occurs mainly on the downshear left side (Frank and Ritchie 2001). The rainbands simulated in the inner most domain exhibit apparent cellular structures (Houze 2010), mostly on the eastern side. The eyewall replacement cycle (ERC), which is important for tropical cyclone intensity change (Wu et al. 2012; Huang et al. 2012), is simulated in this numerical experiment. At 48 h (Fig. 3b), the vertical wind shear is weaker and the tropical cyclone undergoes an ERC. At 72 h (Fig. 3c), the outer eyewall just forms while the inner one is breaking during the second ERC. Figure 3 suggests that the simulated tropical cyclone has a structure similar to a typical observed one, especially in the inner core region.

Two eyewall replacement processes, which may affect tropical cyclone motion (Oda et al. 2006; Hong and Chang 2009), can be further shown in Figure 4. The evolution of the azimuthal mean component of the $700-\mathrm{hPa}$ wind in the $9-\mathrm{km}$ domain indicates the eyewall replacement processes around 42 h and 68 h , respectively. During the first eyewall replacement, for example, the wind starts to intensify outside the eyewall around 36 h , in agreement with previous numerical studies ( Wu et al. 2012; Huang et al. 2012). The radius of maximum wind is located at about 40 km after the $6-\mathrm{h}$ spin-up and decreases to about 30 km at 42 h . We also conducted a similar sensitivity experiment without the sub-kilometer domain. The tropical cyclone track in the experiment is generally similar to that in the sub-kilometer simulation, but no eyewall replacement cycle can be observed in the sensitivity experiment.

## 3. Dominant role of the conventional steering

The relationship between PVT and tropical cyclone motion can be written as (Wu and Wang 2000)

$$
\begin{equation*}
\left(\frac{\partial \mathrm{P}_{1}}{\partial \mathrm{t}}\right)_{\mathrm{f}}=\left(\frac{\partial \mathrm{P}_{1}}{\partial \mathrm{t}}\right)_{\mathrm{m}}-\mathrm{C} \cdot \nabla \mathrm{P}_{\mathrm{s}} \tag{1}
\end{equation*}
$$

Where subscripts $m$ and $f$ indicate, respectively, the moving and fixed reference frames and $\boldsymbol{C}$ is the velocity of the reference frame that moves with the tropical cyclone. In other words, $\boldsymbol{C}$ is the velocity of tropical cyclone motion, which can vary in the vertical. $P_{l}$ and $P_{\mathrm{s}}$ are the azimuthal wavenumber-one and symmetric components of potential vorticity with respect to the storm center. It can be seen that the PVT generated in the fixed reference frame (the term on the left hand side) is provided for the development of the wavenumber one component (the first term on the right hand
side) and for tropical cyclone motion (the second term on the right hand side). The first term on the right hand side of Eq. (1) was neglected in Wu and Wang (2000), but we retain it in this study. The term can be calculated with the two-hour change of the wavenumber one component in the frame that moves with the tropical cyclone center.

The PVT generated in the fixed reference frame can be calculated with the PVT equation in $p$-coordinates as

$$
\begin{equation*}
\frac{\partial P}{\partial t}=-\boldsymbol{V} \cdot \nabla P-\omega \frac{\partial P}{\partial p}-g \nabla_{3} \cdot\left(-\frac{Q}{c_{p} \pi} \boldsymbol{q}+\nabla \theta \times \boldsymbol{F}\right), \tag{2}
\end{equation*}
$$

Where $P, \boldsymbol{V}$ and $\boldsymbol{\omega}$ are potential vorticity, horizontal and vertical components of the wind velocity, respectively. Eq. (2) contains horizontal advection (HA), vertical advection (VA), diabatic heating (DH) and friction (FR) terms on the right hand side. $Q, \theta, \boldsymbol{q}$ and $\boldsymbol{F}$ are diabatic heating rate, potential temperature, absolute vorticity and friction, while $g, c_{p}$ and $\pi$ are the gravitational acceleration, the specific heat of dry air at constant pressure and the Exner function. $\nabla_{3}$ and $\nabla$ denote the three and two dimensional gradient operators.

Following Wu and Wang (2000), a least square method is used to estimate the velocity of tropical cyclone motion ( $\boldsymbol{C}$ ) in Eq. (1). The translation velocity is also calculated with the hourly positions of the tropical cyclone center. For convenience, the tropical cyclone motion estimated with the PVT diagnostic approach and with the center position is referred to as the PVT velocity and the tropical cyclone velocity, respectively, in the following discussion. In the PVT approach, we find that the estimated tropical cyclone motion is not much sensitive to the size of the calculation domain. As we know, however, determination of the conventional steering flow for a
given tropical cyclone is not unique and depends on the size of the calculation domain (Wang et al. 1998). Here we select the calculation domain to minimize the difference between the tropical cyclone speed and the conventional steering flow. After a series of tests, we find that such a minimum can be reached when the $270-\mathrm{km}$ radius is used. This is consistent with the analysis of the airborne Doppler radar data in Marks et al. (1992) and Franklin et al. (1996). The analysis indicated that tropical cyclone motion was best correlated with the depth-mean flow averaged over the inner region within $3^{\circ}$ latitudes. Note that the PVT, tropical cyclone and steering velocities are calculated at each level and then the depth-mean ones are averaged over the layer between 850-300 hPa .

Figure 5a shows the time series of the magnitudes of the tropical cyclone velocity (black), the PVT velocity (blue) and the conventional steering (red). Note that the PVT velocity and the conventional steering are instantaneous, whereas the tropical cyclone velocity is calculated based on the two-hour difference of the center position. For consistence, a three-point running mean is applied to the PVT speed and the conventional steering. These magnitudes generally increase as the tropical cyclone takes a northwest north track. The mean speeds calculated from the PVT approach and the center positions are $2.86 \mathrm{~m} \mathrm{~s}^{-1}$ and $2.75 \mathrm{~m} \mathrm{~s}^{-1}$ over the $72-\mathrm{h}$ period. Compared to the tropical cyclone speed, the root-mean-square error (RMSE) of the PVT speed is $0.22 \mathrm{~m} \mathrm{~s}^{-1}$, only accounting for $8 \%$ of the tropical cyclone speed.

Figures 5 b and 5 c further display the zonal and meridional components of the tropical cyclone velocity (black), the PVT velocity (blue) and the conventional
steering (red). While the westward component fluctuates about the mean zonal tropical cyclone (PVT) speed of $-1.0 \mathrm{~m} \mathrm{~s}^{-1}$, the northward component generally increases with time. Figure 5 clearly indicates that the translation velocity of the tropical cyclone can be well estimated with the PVT approach.

The environmental and secondary steering flows are indistinctly referred to the conventional steering flow in this study. The conventional steering shown in Fig. 5 is averaged over the same radius ( 270 km ) and the $850-300 \mathrm{hPa}$ layer, as used in the calculation of the PVT speed. The 72-h mean magnitudes of the tropical cyclone velocity and the conventional steering are $2.86 \mathrm{~m} \mathrm{~s}^{-1}$ and $2.87 \mathrm{~m} \mathrm{~s}^{-1}$, respectively, only with a difference of $6.7^{\circ}$ in the motion direction. We also calculated the root-mean-square-error (RMSE) of the conventional steering averaged over various time periods with the tropical cyclone speed (Fig. 6). The RMSE of the magnitude decreases with the increasing average period, generally less than $9 \%$ of the translation speed of the tropical cyclone. The difference in direction also decreases with the increasing average period within $9-11$ degrees. Considering uncertainties in determining tropical cyclone centers and calculating the steering, we conclude that the conventional steering plays a dominant role in tropical cyclone motion. However, Figure 5 indicates that the instantaneous tropical cyclone motion can considerably derivate from the conventional steering. The conventional steering cannot account for the fluctuations in tropical cyclone motion, which will be further discussed in Section 5.

## 4. Contributions of individual processes

The individual contributions of various terms in the PVT equation to tropical cyclone motion can also be estimated with Eq. (1), as shown by Wu and Wang (2000). In this study, the contribution of the friction (FR) term is calculated as the residual of the PVT equation. Figure 7 shows the individual contributions of the terms in the PVT equation to tropical cyclone motion. While the contribution of the HA term plays a dominant role (Fig. 7c), the figure exhibits considerable fluctuations, suggesting that the contributions of the DH and VA terms tend to cancel each other (Figs. 7a and 7b). Here we discuss the contribution of each term in the PVT equation to understand the dominant role of the conventional steering in tropical cyclone motion.

## a. Horizontal advection

As discussed in Wu and Wang (2001b), the HA term in the PVT equation can be approximately written as: $-\mathbf{V}_{1} \cdot \nabla \mathbf{P}_{\mathrm{s}}-\mathbf{V}_{\mathrm{s}} \cdot \nabla \mathbf{P}_{1}$, where $\mathbf{V}_{\mathrm{s}}$ is the symmetric component of the tangential wind and $\mathbf{V}_{\mathbf{1}}$ is the wavenumber-one component of the asymmetric wind. The first term (HA1) represents the advection of the symmetric potential vorticity component by the asymmetric flow. The second term is the advection of the wavenumber-one potential vorticity component by the symmetric flow (HA2).

The contribution of the HA1 term is literally the steering effect, but it is not the conventional steering that is calculated as the velocity of the mean wind averaged over $300-850 \mathrm{hPa}$ within the radius of 270 km from the tropical cyclone center in this study. Wu and Wang (2001a) pointed out that the steering effect in the HA1 term is
associated also with the gradient of the symmetric potential vorticity component, which make its contribution be confined to the inner region of tropical cyclones.

Figure 8 shows the contributions of the HA1 and HA2 terms, which exhibit considerable fluctuations with time. The contribution of HA and the conventional steering are also plotted. For clarity, the conventional steering is removed from the contribution of HA1 (i.e. HA1'). The 72-hour mean difference between the contribution of HA1 and the conventional steering is $-1.25 \mathrm{~m} \mathrm{~s}^{-1}$ in the zonal component and $1.62 \mathrm{~m} \mathrm{~s}^{-1}$ in the meridional component, suggesting that the contribution of the HA1 term is considerably different from the conventional steering. In fact, the contributions of the HA1 and HA2 terms are highly anticorrelated. The correlations for the zonal and meridional components are -0.82 and -0.80 , respectively. The negative correlations suggest the cancellation between the contributions of the HA1 and HA2 terms. As a result, the combined effect of the HA1 and HA2 terms can actually account for the effect of the conventional steering except the short-time fluctuations, as shown in Fig. 8. It is interesting to note that the contributions of the HA1 and HA2 terms increase in magnitude during the two eyewall replacement processes around 42 h and 68 h , suggesting that the tropical cyclone motion considerably deviates from the steering of the asymmetric flow during eyewall replacement.

The cancellation between the contributions of the HA1 and HA2 terms arises from the interaction between the symmetric and wavenumber-one components of the tropical cyclone circulation. As an example, Figure 9a shows HA1 and the
wavenumber-one components of potential vorticity (contours) and winds at 700 hPa after 18 hours of the integration. The positive (negative) anomalies of potential vorticity are nearly collocated with the cyclonic (anticyclonic) circulation. Since the potential vorticity in the inner core is generally elevated, the advection of the symmetric potential vorticity component by the flows between the cyclonic and anticyclonic circulations leads to the maximum (minimum) HA1 in the exit (entrance) of the flows between the cyclonic and anticyclonic circulation. On the other hand, the advection of the wavenumber-one component of potential vorticity by the symmetric cyclonic flow leads to the maximum HA2 in the entrance and the minimum HA1 in the exit (Fig. 9b). Although the contributions of the HA1 and HA2 terms can fluctuate with a magnitude of about $4 \mathrm{~m} \mathrm{~s}^{-1}$ (Fig. 8), their combined effect shows only small-amplitude fluctuations in the tropical cyclone motion. The short-time fluctuations will be discussed in the next section.

## b. Contributions of diabatic heating and vertical advection

Some individual contributions in Figs. 7a and 7b are statistically correlated. For example, the zonal contribution of the HA term is negatively correlated with that of the DH term with a coefficient of -0.44 , and the meridional contribution of the HA term is negatively correlated with that of the VA terms with a coefficient of -0.54 . The correlation coefficients pass the significance test at the $95 \%$ confidence level. It is suggested that the contributions of individual terms can partially cancel each other due to the coherent structure of the tropical cyclone.

We first discuss the contribution of the VA term. The VA contains two primary terms: the advection of the symmetric component of potential vorticity by the wavenumber-one component of vertical motion (VA1) and wavenumber-one component of potential vorticity by the symmetric component of vertical motion (VA2). Our examination indicates that the contribution of the VA term is dominated by that of VA1. That is, the direction of the contribution of the VA term is determined by the orientation of the wavenumber-one component of vertical motion. Figure 10 shows the wavenumber-one components of the $500-\mathrm{hPa}$ vertical motion, $700-\mathrm{hPa}$ winds relative to tropical cyclone motion, and $500-\mathrm{hPa}$ heating rate after 18 hours of integration. We can see that the upward (downward) motion generally occurs in the entrance (exit) region of the $700-\mathrm{hPa}$ winds. Bender (1997) found that vorticity stretching and compression is closely associated with the vorticity advection due to the relative flow (difference between the wavenumber-one flow and the TC motion), but Riemer (2016) recently argued that Bender's mechanism did not work in his idealized experiment. We find that the contribution of the HA term is indeed significantly correlated with those of the VA and DH terms, suggesting the relationship between the vertical motion (diabatic heating) and the relative flow.

The contribution of diabatic heating results mainly from $-\mathbf{q}_{s} \cdot \nabla_{3} h_{1}$, where $\mathbf{q}_{s}$ is the symmetric component of the absolute vorticity, $\nabla_{3}$ the three-dimensional gradient operator, $h_{1}$ the wavenumber-one component of diabatic heating rate. Since the absolute vorticity is dominated by the vertical component of relative vorticity and diabatic heating rate reaches its maximum in the middle troposphere, it is conceivable
that the contribution of diabatic heating should cancel each other in the low and upper troposphere. Figure 11 shows the contribution of diabatic heating at 700 hPa and 400 hPa . The correlation between 700 hPa and 400 hPa is -0.68 in the zonal direction and -0.67 in the meridional direction.

## 5. Trochoidal motion

As shown in Fig. 5, the tropical cyclone motion exhibits considerable fluctuations. In an instant, the steering can significantly deviate from the tropical cyclone motion. At 60 h , for example, the zonal steering is $-0.55 \mathrm{~m} \mathrm{~s}^{-1}$, about one third of the zonal motion of the tropical cyclone $\left(-1.42 \mathrm{~m} \mathrm{~s}^{-1}\right)$; The meridional steering is $2.71 \mathrm{~m} \mathrm{~s}^{-1}$, slower than the meridional motion of the tropical cyclone ( $3.05 \mathrm{~m} \mathrm{~s}^{-1}$ ). The deviation from the tropical cyclone motion is $13.5^{\circ}$ in the direction and $18 \%$ in the magnitude. It is clear that the instantaneous velocity of tropical cyclone motion can considerably derivate from the conventional steering.

Based on radar data and satellite images, many studies documented the oscillation of a tropical cyclone track with respect to its mean motion vector (e. g., Jordan and Stowell 1955; Lawrence and Mayfield 1977; Muramatsu 1986; Itano et al. 2002; Hong and Chang 2005). The periods of track oscillations range less than an hour to a few days (Holland and Lander 1993). In this study, the small-scale oscillation with amplitudes of that comparable to the eye size and periods of several hours is referred to as the trochoidal motion of the tropical cyclone center. Willoughby (1988) showed that a pair of rotating mass and sink source could lead to trochoidal motion with periods ranging from 2-10 hours. Flatau and Stevens (1993) argued that
wavenumber-one instabilities in the outflow layer of tropical cyclones could cause trochoidal motion. Nolan et al. (2001) found that the small-amplitude trochoidal motion is associated with the instability of the wavenumber-one component of tropical cyclone circulation due to the presence of the low-vorticity eye. The instability in their three-dimensional simulation with a baroclinic vortex quickly led to substantial inner-core vorticity redistribution and mixing, displacing the vortex center that rotates around the vortex core. Our spectral analysis indicates two peaks of the fluctuations of the tropical cyclone motion centered at 5 hours and 9 hours (Figure not shown), suggesting that the trochoidal motion is simulated in our high-resolution numerical simulation.

Figure 12 shows the oscillation of the tropical cyclone track with respect to the 9-hour running mean track for the periods $6-18 \mathrm{~h}$ and $59-70 \mathrm{~h}$. We can see that the displacement from the mean track is usually less than 6 km with a period of several hours in this study. This displacement is less than the size of the tropical cyclone eye. In general, the tropical cyclone center rotates cyclonically relative to the mean track position, in agreement with previous observational and numerical studies (Lawrenece and Mayfield 1977; Muramatsu 1986; Itano et al. 2002; Willoughby 1988; Nolan et al. 2001). In association with the trochoidal motion of the tropical cyclone center, as suggested by Nolan et al. (2001), substantial potential vorticity redistribution and mixing can be observed in the inner core region (Fig. 13). During the period of 13-18 hours, the tropical cyclone eye generally looks like a triangle, but the orientation of
the triangle changes rapidly, suggesting the potential vorticity redistribution and mixing in the eye.

The trochoidal motion is well indicated in the translation speed estimated with the PVT approach. Figure 14a shows the fluctuations of tropical cyclone speed, the PVT speed, and the difference between the tropical cyclone speed and the conventional steering, in which the 9 -hour running mean has been removed. We can see that the fluctuations of tropical cyclone motion are well represented in the PVT speed. Moreover, the consistence between the fluctuations of tropical cyclone motion and those with the conventional steering removed suggests that the small-amplitude oscillation of the tropical cyclone motion cannot be accounted for by the conventional steering. Figure 14 b further compares the time series of tropical cyclone motion relative to the conventional steering with the time series of the contribution of the HA term relative to the conventional steering. The two time series are correlated with a coefficient of 0.60 . We can see that the contribution of the HA term plays an important role in the fluctuations. Since the non-steering effect can well account for the fluctuations (Fig. 14a), Figure 14b suggests that the VA and DH tend to reduce the magnitude of the fluctuations.

## 6. Summary

In this study, we addressed two fundamental questions regarding the steering principle that has been widely applied to tropical cyclone forecast and research for about a century (Fujiwara and Sekiguchi 1919; Bowie 1922). One is why the conventional steering plays a dominant role in tropical cyclone motion and the other
is when tropical cyclone motion deviates considerably from the steering. The PVT diagnosis approach proposed by Wu and Wang (2000) is used with the output from a high-resolution numerical experiment. It is found that the PVT approach can well estimate tropical cyclone motion, including the small-amplitude trochoidal motion relative to the mean tropical cyclone track.

The effect of the conventional steering flow that is averaged over a certain radius from the tropical cyclone center and a deep pressure layer (e.g., 850-300 hPa) actually represents the combined contribution from both of the advection of the symmetric potential vorticity component by the asymmetric flow (HA1) and the advection of the wavenumber-one potential vorticity component by the symmetric flow (HA2), although the contribution of the HA1 term is literally the effect of steering (Wu and Wang 2001a, 2001b). The conventional steering generally plays a dominant role in tropical cyclone motion since the contributions from other processes are largely cancelled out due to the coherent structure of tropical cyclone circulation.

The trochoidal motion of the tropical cyclone center is simulated in the numerical experiment with amplitudes smaller than the eye radius and periods of several hours. The tropical cyclone center rotates cyclonically around the mean track, in agreement with previous observational and numerical studies (Lawrenece and Mayfield 1977; Muramatsu 1986; Itano et al. 2002; Willoughby 1988; Nolan et al. 2001). It is found that the small-amplitude trochoidal motion cannot be accounted for by the effect of the conventional steering although the contribution of the HA term plays an important role in the fluctuations. In agreement with previous studies (Willoughby 1988; Nolan
et al. 2001), we suggest that the small-amplitude trochoidal motion results from the asymmetric dynamics of the tropical cyclone inner core. It is also found that the instantaneous speed of tropical cyclone motion can considerably derivate from the conventional steering, while the latter better represents tropical cyclone motion when averaged over a reasonable time period.

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Figure 1 Model domains of the numerical experiment with the three innermost domains moving with the storm, the initial $850-\mathrm{hPa}$ wind ( $\mathrm{m} \mathrm{s}^{-1}$ ) field (vectors), and the simulated tropical cyclone track (red)


Figure 2 Time series of tropical cyclone intensity: a) sea level minimum pressure (hPa); b) maximum wind speed at $10 \mathrm{~m}_{\left(\mathrm{m} \mathrm{s}^{-1}\right)}$.


Figure 3 Simulated wind (vectors, $\mathrm{m} \mathrm{s}^{-1}$ ), radar reflectivity (shading, dBz ) fields at 700 hPa , and the vertical wind shear (bold arrows in the center) between 200 hPa and 850 hPa after (a) $24-\mathrm{h}$, (b) $48-\mathrm{h}$, and (c) $72-\mathrm{h}$ integration. The x and y axes indicate the distance (km) relative to the storm center. The upper (lower) scale vector at the right lower corner is for the $700-\mathrm{hPa}$ wind (vertical wind shear).


Figure 4 Evolution of the simulated azimuthal mean component $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ of the $700-\mathrm{hPa}$ wind in the $9-\mathrm{km}$ domain. The x -axis and y -axis indicate the distance $(\mathrm{km})$ from the storm center and the integration time (hours).


Figure 5 Time series of tropical cyclone speed (thick black), PVT speed (blue) and conventional steering (red): a) magnitude, b) zonal component, and c) meridional component


Figure 6 Changes of the RMSEs of the speed (blue boxes, \%) and direction (black dots, ${ }^{\circ}$ ) of the conventional steering averaged over various time periods




Figure 7 Contributions of the horizontal advection (HA, black), vertical advection (VA, blue), diabatic heating (DH, red) and friction (FR, purple) terms in the PVT equation to tropical cyclone motion: a) zonal component, b) meridional component, and c) magnitude


Figure 8 Time series of the conventional steering (thick black) and the contributions of the HA (thick purple) and the HA1 (red) and HA2 (blue) terms: a) zonal component, b) meridional component. Note that the conventional steering is deducted from the contribution of the HA1 term.


Figure 9 (a) HA1 (shaded, $10^{-10} \mathrm{~m}^{2} \mathrm{~s}^{-2} \mathrm{~K} \mathrm{~kg}^{-1}$ ) and (b) HA2 (shaded, $10^{-10} \mathrm{~m}^{2} \mathrm{~s}^{-2} \mathrm{~K}$ $\mathrm{kg}^{-1}$ ) with the wavenumber-one and symmetric components of potential vorticity (contours, $10^{-6} \mathrm{~m}^{2} \mathrm{~s}^{-1} \mathrm{~K} \mathrm{~kg}^{-1}$ ) and winds (vectors, $\mathrm{m} \mathrm{s}^{-1}$ ) at 700 hPa after 18 hours of integration. The dashed circle indicates the radius of maximum wind.


Figure 10 The wavenumber-one components of the $500-\mathrm{hPa}$ vertical motion (contours, $\mathrm{m} \mathrm{s}^{-1}$ ), $700-\mathrm{hPa}$ winds relative to the tropical cyclone motion (vectors, $\mathrm{m} \mathrm{s}^{-1}$ ), and $500-\mathrm{hPa}$ heating rate (shaded, $10^{-4} \mathrm{~K} \mathrm{~s}^{-1}$ ) after 18 hours of integration. The dashed circle indicates the radius of maximum wind.


Figure 11 Time series of the contributions of diabatic heating at 700 hPa (blue) and 400 hPa (red) and the contribution of diabatic heating (thick black) averaged over the layer between 300 hPa and 850 hPa


Figure 12 Small-amplitude oscillation of the tropical cyclone track with respect to the 9-hour running mean track: a) 6-18 h and b) 59-69 h . The x and y axes indicate the distance $(\mathrm{km})$ relative to the 9 -hour running mean track.

Figure 13 Distribution of potential vorticity (shaded, $10^{-6} \mathrm{~m}^{2} \mathrm{~s}^{-1} \mathrm{~K} \mathrm{~kg}^{-1}$ ) and magnitude of wind (contour, $\mathrm{m} \mathrm{s}^{-1}$ ) within inner-core region during 13-18 h at 700 hPa . The dashed circle shows the radius of maximum wind with the tropical cyclone center indicating with crosses.


Figure 14 Fluctuations (deviation from the 9 -hour running mean) of (a) the tropical cyclone speed (black solid), the PVT speed (black dashed) and the difference between the tropical cyclone speed and the conventional steering (red solid), and (b) the difference between the tropical cyclone speed and the conventional steering (red solid), and the difference between the contribution of the HA term and the conventional steering (black).

