Reply to Reviewer 2

Thank you very much for your valuable comments. We have revised the manuscript and the point-to-point responses to your comments are as follows:

1. Caption Figure 3: It has to be mentioned that several time points (24h, 48h and 72h) are shown and that the bold vector in the center indicates the vertical wind shear between 200hPa and 850hPA.

The figure caption has been rewritten in the revised manuscript.

2. *Page 34-35: The picture on page 35 must be replaced by the picture on page 34.* Sorry for the mistake. The picture has been replaced.

3. Caption Figure 13: Please indicate to which quantity the black isolines refer. Added.

Reply to Reviewer 3

Thank you very much for your valuable comments. In the revised manuscript, we have added more description about the potential vorticity tendency method. The point-to-point responses to your comments are as follows:

I believe that the TC community's answer to question 1) is that a TC can be approximated by a point vortex that is advected by the environmental flow (e.g. Emanuel, 2005, "Divine Wind"). This approximation implies that a) PV tendencies are dominated by horizontal advection and b) that the TC's (horizontal) PV structure and changes thereof do not play a leading-order role in TC motion. Why would this well-known picture be in question? Deviations of this leading-order picture have been demonstrated in the context of track deflection near Taiwan, tilted TC vortices, and trochoidal motion (see references in the manuscript), i.e. these studies give answers to question 2) above. For this manuscript to be suitable for publication, the authors need to clarify to the reader how their PV tendency analysis improves the current state of knowledge.

- 1) Dominant role of the steering flow: As mentioned in the introduction, the real tropical cyclone is not a point vortex. Given complicated interactions between tropical cyclone circulation and its environment, tropical cyclone motion should be not like a leaf being steered only by the currents in the stream. In theory (Wu and Wang 2000), tropical cyclone motion is completely determined by the azimuthal wavenumber-one component of potential vorticity tendency (PVT) and all of the factors that contribute to the azimuthal wavenumber-one component of PVT play a potential role in tropical cyclone motion. The steering effect is only one of the factors. In addition, although the dominant role of the conventional steering flow was widely accepted, the reason was not well investigated in previous studies.
- 2) **Deviation from the steering:** So far, most studies interpreted tropical cyclone motion in terms of the steering flow (so-called conventional steering flow in this manuscript), including the track deflection near Taiwan, the

motion of tilted vortices, and trochoidal motion. These studies did not show why the tropical cyclone motion can be fully represented by the conventional steering flow. In fact, we demonstrate that the horizontal PV advection (HA) contains two main processes: the advection of the symmetric PV component by the asymmetric flow (HA1) and the advection of the wavenumber-one PV component by the symmetric flow (HA2). The contribution of the HA1 term (literally the steering) can considerably differ from the tropical cyclone motion. The conventional steering is best represented by the combined effect of the HA1 and HA2 terms.

1. A secondary eyewall cycle is described in some detail early in the manuscript but is not referred to later in relation to TC motion.

Thank you for the suggestion. In the revised manuscript, we added a few words for the influence of the eyewall replacement. The contributions of the HA1 and HA2 terms increase in magnitude during the two eyewall replacement processes around 42 h and 68 h, suggesting that the tropical cyclone motion considerably deviates from the steering of the asymmetric flow during eyewall replacement.

2. The terminology of steering, conventional steering, secondary steering, etc. is confusing.

The environmental and secondary steering flows are indistinctly referred to the conventional steering flow in this study. The conventional steering flow, which has been widely used in previous studies, is obtained by averaging the winds within the radius of 270 km from the tropical cyclone center over a layer of 300-850 hPa. The contribution of the HA1 term is literally the steering effect, but it is not the conventional steering. We use the contribution of the HA1 term to replace the steering effect in the revised manuscript. We have made their differences clearer in the revision.

3. A rationale for dividing and approximating the horizontal advection into HA1 and

HA2 - it looks akin to a linearization - needs to be given and the calculation of the TC motion C in Eq. 1 needs clarification.

You are right. The HA term can be written as $-V \cdot \nabla P$. Considering that each variable is comprised of symmetric and asymmetric components with respect to the tropical cyclone center, the HA term can be rewritten as $-V_a \cdot \nabla P_s - V_s \cdot \nabla P_a - V_a \cdot \nabla P_a - V_s \cdot \nabla P_s$. The last term has no contribution to tropical cyclone motion. Since the wavenumber-1 component plays the most dominant role in asymmetric components, the HA term can be approximately rewritten as $-V_1 \cdot \nabla P_s - V_s \cdot \nabla P_1$. As shown in the attached figure, the contribution of the HA term can be well represented by those of $-V_1 \cdot \nabla P_s - V_s \cdot \nabla P_1$.



Figure 1 Time series of the (a) zonal and (b) meridional contributions of the HA term (black) and the sum (red) of the HA1 $(-V_1 \cdot \nabla P_s)$ and HA2 $(-V_s \cdot \nabla P_1)$ terms

As described in the manuscript, the tropical cyclone center at each level is defined as the geometric center of the circle on which the azimuthal mean tangential wind speed reaches a maximum (Wu et al. 2006). The translation speed is calculated at each level and then averaged over the layer between 850-300 hPa.

We examined the TC center defined in different ways, including minimum SLP center, minimum geopotential height center and the pressure centroid method, as discussed in Nguyen et al (2014). The results are shown in the attached figure (Fig. 2). We can see that the resulting speeds and fluctuations are generally similar. In particular, the results from the pressure centroid method is very close to those from the method in our study. We conclude that the trochoidal motion discussed in this study does not depend on the definition of the tropical cyclone center.



Figure 2 The TC speed calculated by different TC center definition (Black: the method used in this study; Red: the minimum SLP center; Purple: the minimum geopotential center on each level; Blue: the pressure centroid center defined by Nguyen et. al. 2014): (a) translation speed (m s⁻¹), and (b) fluctuation of translation speed (m s⁻¹)

4. Importantly, the authors consider finite differences over 2 hours to determine TC motion, which sets the timescale of the resolved deviations from steering. Why 2 h?

The model output is at 1-h intervals and the TC translation speed (t h) is calculated by the central difference with the TC centers at t+1 h and t-1 h. Since the

PVT velocity and the conventional steering are instantaneous, for consistence, a three-point running mean is applied to the PVT speed and the conventional steering.

5. I cannot follow the authors' distinction between layer-wise steering and the attempt to find a vertical average that best represents the steering flow for the TC as different processes. Velden and Leslie (1991) and Galarneau and Davis (2013), e.g., consider a PV(or pressure)-weighted vertical average as the best way to define a steering flow, making explicit the idea that steering is governed by horizontal PV advection on individual levels.

Sorry for the confusion. The PVT, tropical cyclone and steering velocities are calculated at each level and then the depth-mean ones are averaged over the layer between 850-300 hPa. We have revised the text. We agree that steering is governed by horizontal PV advection (HA1+HA2) at individual levels, but we find that the tropical cyclone motion can considerably deviate from the steering at individual levels.

6. Fig. 5 seems incorrect as the individual motion components seem not to add up to the motion speed.

As shown Fig. 5, the meridional component of TC speed is much larger than the zonal component and the speed magnitude is calculated as $C = \sqrt{c_x^2 + c_y^2}$. We have carefully examined the figure. For instance, at the 24th hour, the zonal component is -1.28 m s⁻¹ and the meridional component is 3.00 m s⁻¹, thus the magnitude is 3.26 m s⁻¹.

7. What is angular RMS in the caption of Fig. 6?

The angular RMS should be the root-mean-square-error (RMSE), which is the angular difference of the direction between TC motion and conventional steering. The

figure caption has been revised.

8. Compensation between HA1 and HA2 (pg 13/14): Is this basically saying that the wavenumber 0 and 1 PV structure of the TC is stationary (in the storm-relative frame of reference)? If yes, this observation is inconsistent with reference to the argument of vorticity stretching/ compression (Bender, 1997) on pg 15. Recently (Riemer, 2016), a similar compensation between advection terms has been described, consistent with your observation. Riemer (2016) argues that Bender's mechanism is not at play when compensation occurs between symmetric and asymmetric vorticity advection.

Sorry for the confusion in Fig. 8. The asymmetric potential vorticity advection is not totally compensated by the symmetric potential vorticity advection. In this figure, **the conventional steering is deducted from the contribution of the HA1 term**.

Thank you very much for providing the latest reference. It is a very interesting study, in which Riemer (2016) proposed a new mechanism for the formation of the stationary complex. his idealized experiments, band In there is no convergence/vertical motion associated with this kinematic boundary between the vortical TC flow and the storm-relative environmental flow. This is different from the results in Bender (1997). In Fig. 8, we can see that the upward (downward) motion generally occurs in the entrance (exit) region of the 700-hPa winds. Moreover, the contribution of the HA term is indeed significantly correlated with those of the VA and DH terms, suggesting the relationship between the vertical motion (diabatic heating) and the relative flow.

The mechanism for the formation of stationary cloud bands is beyond the scope of this study. In the revised manuscript, we mentioned both of Bender (1997) and Riemer (2016) for reader's reference.

9. Fig. 8: It is unclear to me what is shown in the individual panels. There are 4 panels but only labels a) and b). In addition, terms HA1 and HA2 need to have the same units.

Sorry for the mistake. We replaced this figure, but forgot removing the old one.

We have corrected the unit in the figure caption.

10. The correlations found between individual terms of the PV tendency equation are rather low. Are they statistically significant? If not, I would argue against a physical interpretation of the potentially spurious correlations.

1) The coefficients of zonal and meridional contribution of HA and VA terms are -0.26 and -0.54, respectively, in which the former one at the 95% confidence level and the other at the 99% confidence level.

2) The coefficients of zonal and meridional contribution of VA and DH terms are -0.29 and -0.48, respectively, both at the 99% confidence level.

3) The coefficients of zonal and meridional contribution of HA and VA terms are -0.44 and -0.02, respectively, only the former one at the 99% confidence level.

Overall, most of the negative correlations between zonal and meridional contribution of individual terms are statistically significant. We mention the statistic significance test in the revised manuscript.

11. Section 5 describes the occurrence of trochoidal motion in the authors' experiments and analysis the motion using the PV tendency framework. It is unclear to me, however, what the novel insight is that would be gained in this section.

The trochoidal movement was often explained by the changes in asymmetric flows in previous studies (e.g. Nolan et al. 2001; Hong and Chang 2005). It is found in this study that the fluctuation of TC motion including the trochoidal track cannot be interpreted by the conventional steering. Based on our PVT diagnosis, it is argued that the trochoidal motion is associated with the inner-core structure modification which can be estimated by the PVT method.

12. line 348: Do we miss a "sink"?It has been corrected.

13. line 356: Unclear what sort of spectral analysis was conducted.

The spectral analysis is conducted through NCL functions *specx_anal*. It performs the temporal-to-frequency transformation via the fast Fourier transform.

14. Fig. 12 misses units at the axes.

It has been added in the figure caption.