Network design for quantifying urban CO_2 emissions: Assessing trade-offs between precision and network density

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Contents

1 WRF-STILT

We use meteorological fields from the Weather Research and Forecasting (WRF v3.5, [1]) model, a mesoscale meteorological model, to drive the Stochastic Time-Inverted Lagrangian Transport (STILT [2]) model, a Lagrangian particle dispersion model. The coupling between the WRF and STILT models (WRF-STILT) was developed by Nehrkorn *et al.*[3]. Meteorological fields were generated at four gridded horizontal resolutions (27, 9, 3, and 1 km) in a one-way nested arrangement centered around California's Bay Area (see Fig. 1). All WRF domains had 50 vertical levels (see caption of Fig. 1). Initial and lateral boundary conditions were provided by the North American Regional Reanalysis[4]. Overlapping 30-hour forecasts were initialized every 24 hours, at 00 UTC, and the first 6 hours of each forecast were discarded to allow for model spinup. Grid nudging was used in the outer-most domain. WRF simulations used the MYJ planetary boundary layer scheme and the 5-layer SLAB land surface model[1].

Figure 1: **WRF domains.** Plot window covers the outermost domain (d01), d02 covers the majority of California, d03 covers a section of Northern California, and d04 covers California's Bay Area. Vertical levels: 28, 97, 190, 309, 460, 652, 887, 1110, 1301, 1497, 1697, 1994, 2391, 2791, 3195, 3602, 4013, 4427, 4845, 5266, 5691, 6121, 6554, 6991, 7433, 7879, 8330, 8785, 9246, 9711, 10182, 10657, 11139, 11626, 12119, 12618, 13124, 13636, 14155, 14681, 15215, 15757, 16304, 16853, 17401, 17950, 18498, 19046, 19594, and 20141 m.

The STILT model advects an ensemble of 500 particles 3-days backward in time from the spatio-temporal receptor points using the meteorological fields from WRF. Fig. 2 shows some example particle trajectories. These trajectories can be used to construct measurement footprints (see main text).

2 Prior error covariance matrix

Following Meirink *et al.*[5], Singh *et al.*[6], and Yadav & Michalak[7], we express our prior error covariance matrix ($\mathbf{B}$; $m \times m$) as a Kronecker product of a temporal covariance matrix ($\mathbf{D}$; $n_t \times n_t$) and a spatial covariance matrix ($\mathbf{E}$; $n_x n_y \times n_x ny$), in our application, $m = 2,133,120$, $n_t = 240$, $n_x = 88$, and $n_y = 101$. This allows us to write $\mathbf{B}$ as:

$$\mathbf{B} = \mathbf{D} \otimes \mathbf{E} = \begin{pmatrix} d_{(1,1)}\mathbf{E} & \cdots & d_{(1,n_t)}\mathbf{E} \\ \vdots & \ddots & \vdots \\ d_{(n_t,1)}\mathbf{E} & \cdots & d_{(n_t,n_t)}\mathbf{E} \end{pmatrix} \tag{1}$$

where $\otimes$ is the Kronecker product. Our implementation is adapted from Yadav & Michalak[7].

The temporal and spatial covariance matrices can be expressed in terms of correlation matrices and diagonal variance matrices:

$$\mathbf{\Sigma} = \mathbf{V}^{1/2}\mathbf{M}\mathbf{V}^{1/2} \tag{2}$$

where $\mathbf{\Sigma}$ is an $p \times p$ covariance matrix, $\mathbf{M}$ is an $p \times p$ correlation matrix, and $\mathbf{V}$ is an $p \times p$ diagonal matrix of variances:

$$\mathbf{V} = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_p^2 \end{pmatrix} \tag{3}$$

Thus, the temporal covariance matrix is $\mathbf{D} = \mathbf{V}_t^{1/2}\mathbf{M}_t\mathbf{V}_t^{1/2}$ and the spatial covariance matrix is $\mathbf{E} = \mathbf{V}_s^{1/2}\mathbf{M}_s\mathbf{V}_s^{1/2}$.

We construct $\mathbf{V}_t$, $\mathbf{V}_s$, $\mathbf{M}_t$, and $\mathbf{M}_s$ from the $BEACO_2N$ emission inventory described in the main text. $\mathbf{X}$ is an $n_x \times n_y \times n_t$ third-order tensor of CO_2 emissions from the $BEACO_2N$ emission inventory. $\mathbf{V}_t$ and $\mathbf{V}_s$ are constructed as:

$$\mathbf{V}_t = f_\sigma \cdot \begin{pmatrix} \mathrm{var}\left(\mathbf{X}_{(:,:,1)}\right) & 0 & \cdots & 0 \\ 0 & \mathrm{var}\left(\mathbf{X}_{(:,:,2)}\right) & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathrm{var}\left(\mathbf{X}_{(:,:,n_t)}\right) \end{pmatrix} \tag{4}$$

$$\mathbf{V}_s = f_\sigma \cdot \begin{pmatrix} \mathrm{var}\left(\mathbf{X}_{(1,1,:)}\right) & 0 & \cdots & 0 \\ 0 & \mathrm{var}\left(\mathbf{X}_{(1,2,:)}\right) & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathrm{var}\left(\mathbf{X}_{(n_x,n_y,:)}\right) \end{pmatrix} \tag{5}$$

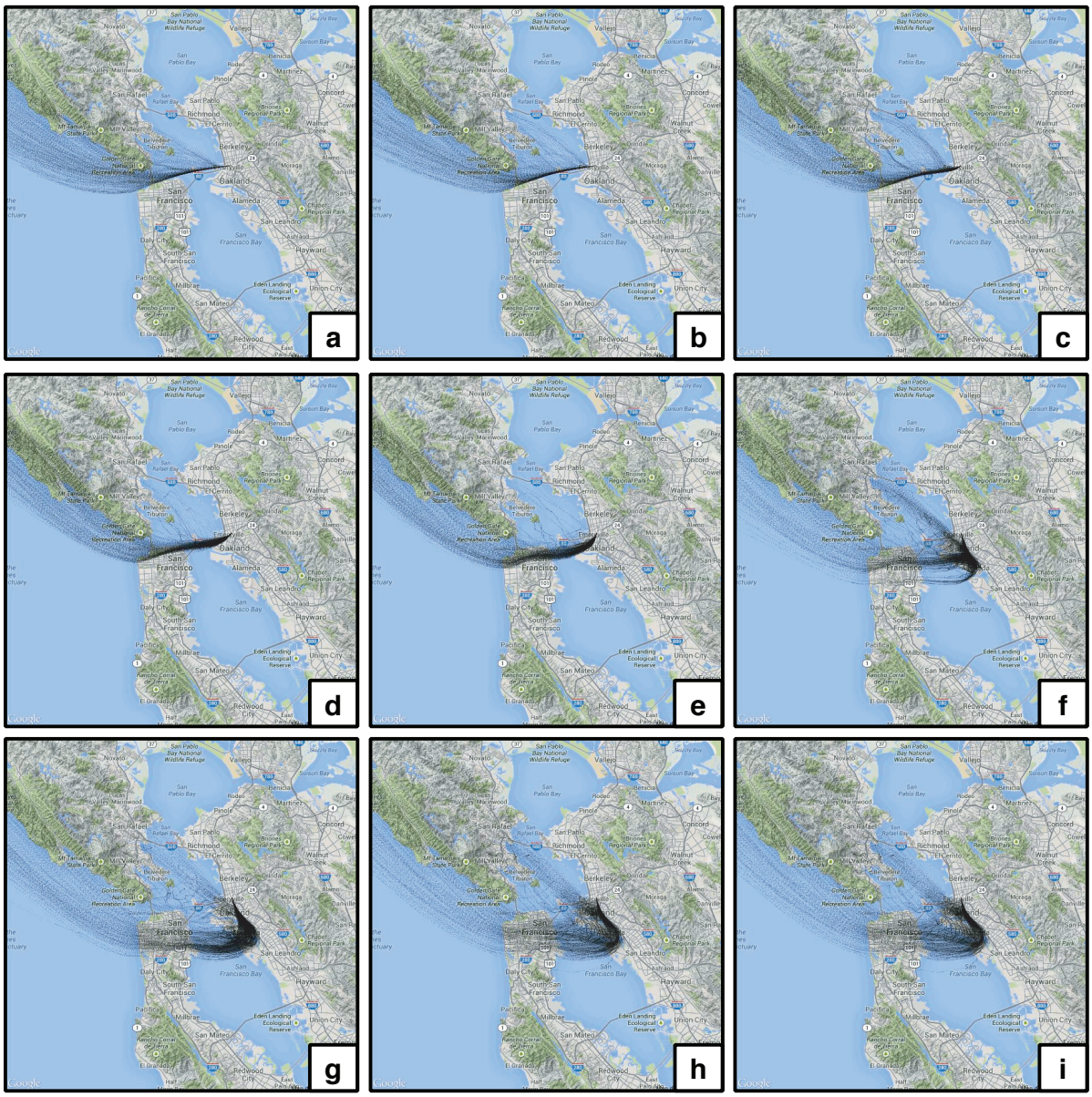

Figure 2: **STILT back trajectories.** Example back trajectories computed using WRF-STILT beginning from a BEACO$_2$N node. Panels show different particle release times.

where f_σ is an uncertainty scaling factor. Here we have chosen $f_\sigma = 1$, corresponding to a 100% uncertainty. $\mathbf{M}_t$ and $\mathbf{M}_s$ are constructed as:

$$\mathbf{M}_t = \begin{pmatrix} \text{corr}\left(\mathbf{X}_{(:,:,1)}, \mathbf{X}_{(:,:,1)}\right) & \cdots & \text{corr}\left(\mathbf{X}_{(:,:,1)}, \mathbf{X}_{(:,:,n_t)}\right) \\ \vdots & \ddots & \vdots \\ \text{corr}\left(\mathbf{X}_{(:,:,n_t)}, \mathbf{X}_{(:,:,1)}\right) & \cdots & \text{corr}\left(\mathbf{X}_{(:,:,n_t)}, \mathbf{X}_{(:,:,n_t)}\right) \end{pmatrix} \circ \exp\left(-\frac{\mathbf{Z}_t}{\tau_t}\right) \tag{6}$$

$$\mathbf{M}_s = \begin{pmatrix} \text{corr}\left(\mathbf{X}_{(1,1,:)}, \mathbf{X}_{(1,1,:)}\right) & \cdots & \text{corr}\left(\mathbf{X}_{(1,1,:)}, \mathbf{X}_{(n_x,n_y,:)}\right) \\ \vdots & \ddots & \vdots \\ \text{corr}\left(\mathbf{X}_{(n_x,n_y,:)}, \mathbf{X}_{(1,1,:)}\right) & \cdots & \text{corr}\left(\mathbf{X}_{(n_x,n_y,:)}, \mathbf{X}_{(n_x,n_y,:)}\right) \end{pmatrix} \circ \exp\left(-\frac{\mathbf{Z}_s}{\tau_s}\right) \tag{7}$$

where $\circ$ is the Hadamard product, $\mathbf{Z}_t$ ($n_t \times n_t$) and $\mathbf{Z}_s$ ($n_x n_y \times n_x n_y$) represent the separation lags/distances in between locations in time and space, respectively, and τ_t and τ_s are the temporal and spatial decay parameters, respectively. Here we have chosen $\tau_t = 3$ hr and $\tau_s = 5$ km.

The resulting correlation structure can be seen in Fig. 3. We can see that the temporal correlation matrix is diagonal with an exponential decay (Fig. 3c). The spatial structure shown in Fig. 3d and 3e is more complicated. The banded structure in panels Fig. 3d and 3e is from reshaping the state vector from matrices to a vector.

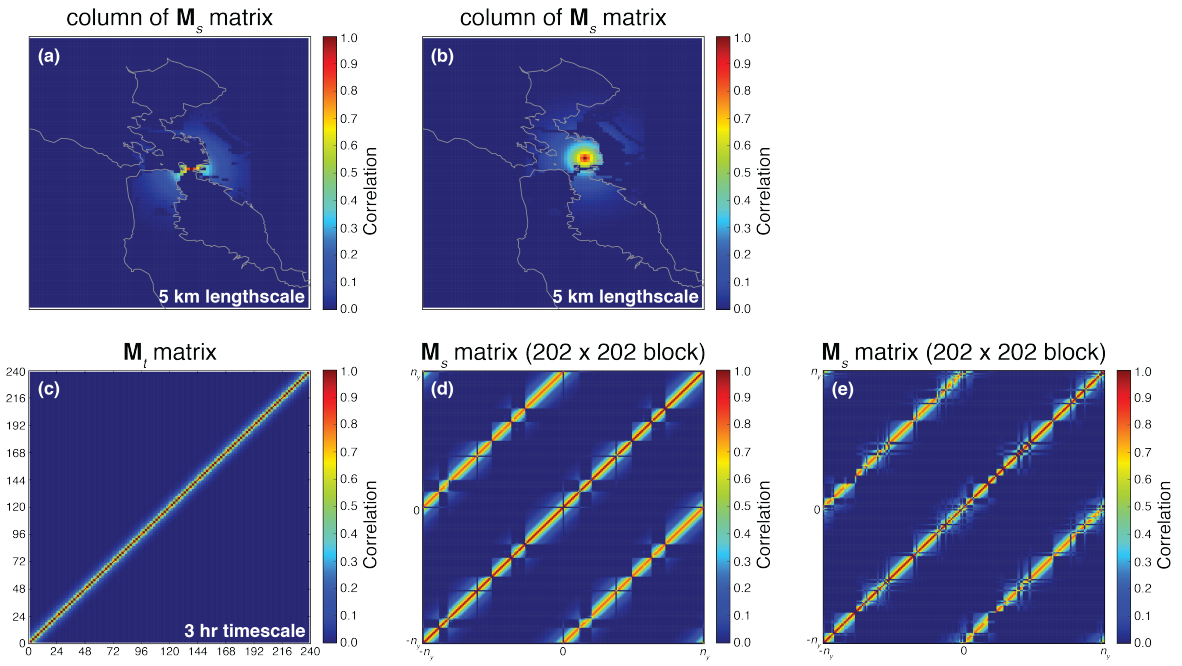

Figure 3: **Prior correlation matrix structure.** **(a)** A column of the spatial correlation matrix for a grid cell on a road. **(b)** A column of the spatial correlation matrix for a grid cell in the bay. **(c)** Temporal correlation matrix. **(d)** A 202×202 block of the spatial correlation matrix centered on the grid cell from panel (a). **(e)** A 202×202 block of the spatial correlation matrix showing some of the more complicated structure. n_y in panel (d) and (e) is the number of grid cells in the longitudinal direction ($n_y = 101$).

Here we have used knowledge of the true emissions, $\mathbf{X}$, in constructing $\mathbf{M}_t$ and $\mathbf{M}_s$. At first glance this would seem to be an overly optimistic specification of the prior covariance

structure. However, in practice, this is equivalent to specifying a correlation that exponentially decays over a specified land-type (e.g., roads). This is because grid cells from similar land-types have a similar diurnal cycle and will be strongly correlated with each other and have negligible correlations with other land-types. This can be seen in Fig 3a and 3b. This is similar to the "hybrid" spatial error correlation used in Basu *et al.*[8].

3 Implementation of the error metrics

For computing the error metric we use three third-order tensors (all dimension $n_x \times n_y \times n_t$): the prior emissions ($\mathbf{W}$), the true emissions ($\mathbf{X}$), and the posterior emissions ($\mathbf{Y}$). And evaluate them using:

$$\eta = 1 - \frac{||\mathbf{y} - \mathbf{x}||_2}{||\mathbf{w} - \mathbf{x}||_2} \tag{8}$$

where $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{w}$ are explained for each source type below.

Figure 4: **Source types examined.** Same as right column from main text Fig. 3. The area source, line source, and point source have emission rates of 147 ± 55 tC hr^{-1}, 45 ± 20 tC hr^{-1}, and 9 ± 4 tC hr^{-1} over one week, respectively.

3.1 Area Source

We use the area source mask ($\mathcal{M}_{\text{AS}}$) shown in the left panel of Fig. 4. We sum emissions from within the mask at each timestep to create $n_t \times 1$ vectors of emissions from the area source. $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{w}$ are constructed as:

$$\mathbf{x} = \begin{pmatrix} \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{X}_{(i,j,1)} \\ \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{X}_{(i,j,2)} \\ \vdots \\ \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{X}_{(i,j,n_t)} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{Y}_{(i,j,1)} \\ \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{Y}_{(i,j,2)} \\ \vdots \\ \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{Y}_{(i,j,n_t)} \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{W}_{(i,j,1)} \\ \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{W}_{(i,j,2)} \\ \vdots \\ \sum_{i,j\in\mathcal{M}_{\text{AS}}} \mathbf{W}_{(i,j,n_t)} \end{pmatrix} \tag{9}$$

Posterior emissions are then evaluated using Eq. 8.

3.2 Line Source

We use the line source mask ($\mathcal{M}_{\mathrm{LS}}$) shown in the middle panel of Fig. 4. We sum emissions from within the mask at each timestep to create $n_t \times 1$ vectors of emissions from the line source. $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{w}$ are constructed as:

$$\mathbf{x} = \begin{pmatrix} \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{X}_{(i,j,1)} \\ \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{X}_{(i,j,2)} \\ \vdots \\ \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{X}_{(i,j,n_t)} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{Y}_{(i,j,1)} \\ \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{Y}_{(i,j,2)} \\ \vdots \\ \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{Y}_{(i,j,n_t)} \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{W}_{(i,j,1)} \\ \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{W}_{(i,j,2)} \\ \vdots \\ \sum_{i,j\in\mathcal{M}_{\mathrm{LS}}} \mathbf{W}_{(i,j,n_t)} \end{pmatrix} \tag{10}$$

Posterior emissions are then evaluated using Eq. 8.

3.3 Point Source

We use the locations of the four point sources ($\left[i^{\{1\}},\ldots,i^{\{4\}}\right]$ and $\left[j^{\{1\}},\ldots,j^{\{4\}}\right]$) shown in the right panel of Fig. 4. We extract emissions from the four point sources at each timestep to create $4n_t \times 1$ vectors of emissions from the point sources. $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{w}$ are constructed as:

$$\mathbf{x} = \begin{pmatrix} \mathbf{X}_{(i^{\{1\}},j^{\{1\}},1)} \\ \mathbf{X}_{(i^{\{2\}},j^{\{2\}},1)} \\ \mathbf{X}_{(i^{\{3\}},j^{\{3\}},1)} \\ \mathbf{X}_{(i^{\{4\}},j^{\{4\}},1)} \\ \vdots \\ \mathbf{X}_{(i^{\{1\}},j^{\{1\}},n_t)} \\ \mathbf{X}_{(i^{\{2\}},j^{\{2\}},n_t)} \\ \mathbf{X}_{(i^{\{3\}},j^{\{3\}},n_t)} \\ \mathbf{X}_{(i^{\{4\}},j^{\{4\}},n_t)} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} \mathbf{Y}_{(i^{\{1\}},j^{\{1\}},1)} \\ \mathbf{Y}_{(i^{\{2\}},j^{\{2\}},1)} \\ \mathbf{Y}_{(i^{\{3\}},j^{\{3\}},1)} \\ \mathbf{Y}_{(i^{\{4\}},j^{\{4\}},1)} \\ \vdots \\ \mathbf{Y}_{(i^{\{1\}},j^{\{1\}},n_t)} \\ \mathbf{Y}_{(i^{\{2\}},j^{\{2\}},n_t)} \\ \mathbf{Y}_{(i^{\{3\}},j^{\{3\}},n_t)} \\ \mathbf{Y}_{(i^{\{4\}},j^{\{4\}},n_t)} \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} \mathbf{W}_{(i^{\{1\}},j^{\{1\}},1)} \\ \mathbf{W}_{(i^{\{2\}},j^{\{2\}},1)} \\ \mathbf{W}_{(i^{\{3\}},j^{\{3\}},1)} \\ \mathbf{W}_{(i^{\{4\}},j^{\{4\}},1)} \\ \vdots \\ \mathbf{W}_{(i^{\{1\}},j^{\{1\}},n_t)} \\ \mathbf{W}_{(i^{\{2\}},j^{\{2\}},n_t)} \\ \mathbf{W}_{(i^{\{3\}},j^{\{3\}},n_t)} \\ \mathbf{W}_{(i^{\{4\}},j^{\{4\}},n_t)} \end{pmatrix} \tag{11}$$

Posterior emissions are then evaluated using Eq. 8.

4 Relating the error metric to the flux error

We can relate our error metric (η) to the flux error as:

$$\varepsilon = \underbrace{(1-\eta)}_{\text{unexplained error}} \cdot \underbrace{\left(\frac{1}{m}\sum_{i=1}^{m}\mathbf{x}_i^a\right)}_{\text{mean true emissions}} \cdot \underbrace{\left(\frac{1}{m}\sum_{i=1}^{m}\left|\frac{\mathbf{x}_i^b-\mathbf{x}_i^a}{\mathbf{x}_i^a}\right|\right)}_{\substack{\text{mean relative difference} \\ \text{between prior and true emissions}}} \tag{12}$$

where $\mathbf{x}^a$ is the true emissions, $\mathbf{x}^b$ ($m \times 1$ vector) is the prior emissions, and $m = 2,133,120$.

5 Sensitivity tests

We tested the sensitivity to domain-size, systematic biases, and observational frequency.

5.1 Sensitivity to domain size

The inversion was found to be fairly insensitive to domain size. This was determined by comparing the base case inversion to an inversion using a reduced domain (gray box in Fig. 5). Fig. 6 shows the error for the reduced domain and the difference between the base case. We find roughly 1% less error reduction when using the reduced domain, compared to the base case.

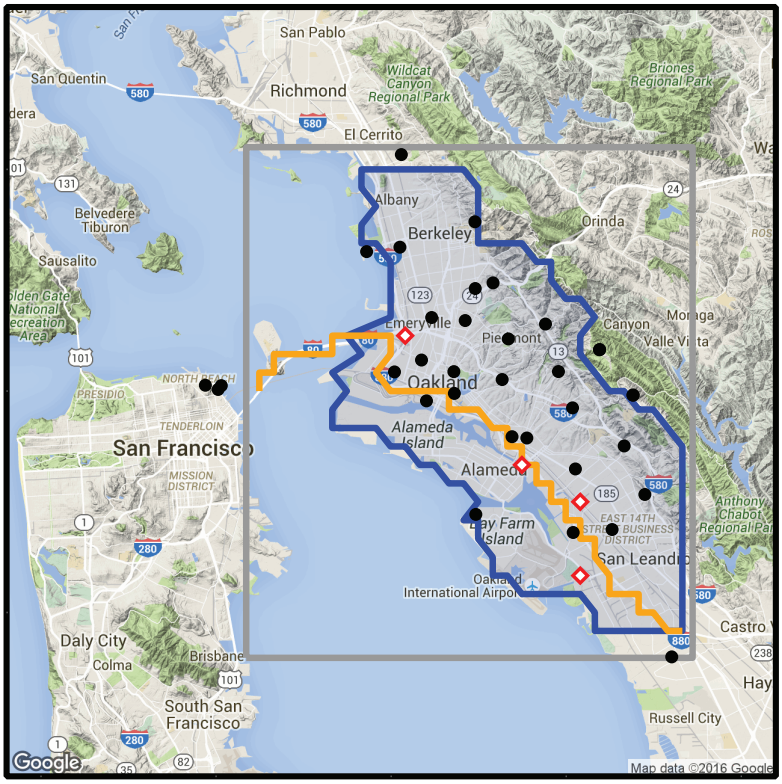

Figure 5: **Map of sites with reduced domain.** Same as top panel from Fig. 2 in the main text except the reduced domain is also shown as a gray box.

5.2 Sensitivity to systematic biases

We performed an ensemble of inversions where each measurement site had a systematic bias (ϵ_b) added to it. The bias for each site (ϵ_b) was drawn from a zero-mean gaussian with a standard deviation $\sigma_b = 1$ ppm: $\epsilon_b \sim \mathcal{N}\left(0, \sigma_b^2\right)$.

Fig. 7 shows the error for an inversion where we have introduced a systematic bias ($\epsilon_b \sim \mathcal{N}\left(0, \sigma_b^2\right)$) at each site.

5.3 Sensitivity to observational frequency

The inversion was found to be sensitive to the observational frequency. This was determined by comparing the base case inversion to an inversion using only daytime observations when we might expect a well developed boundary layer (10am to 5pm local time). Fig. 8 shows the error for the daytime-only inversions and the difference between the base case.

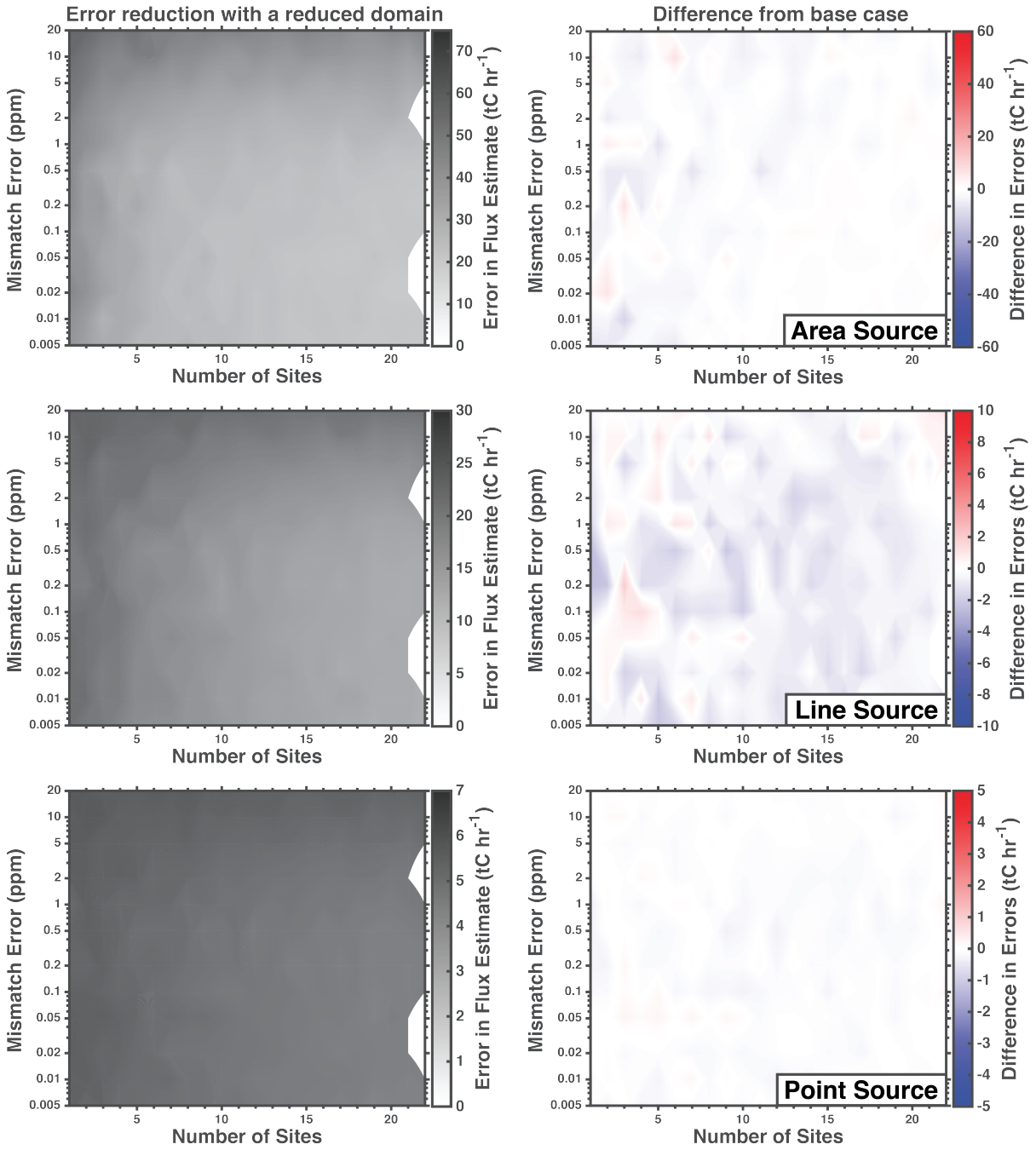

Figure 6: **Error reduction with a reduced domain.** Left column is the same as main text Fig. 3 except the inversions use the reduced domain shown in Fig. 5 and each point only uses 5 ensemble members. Right column is the difference between the left column and the base case.

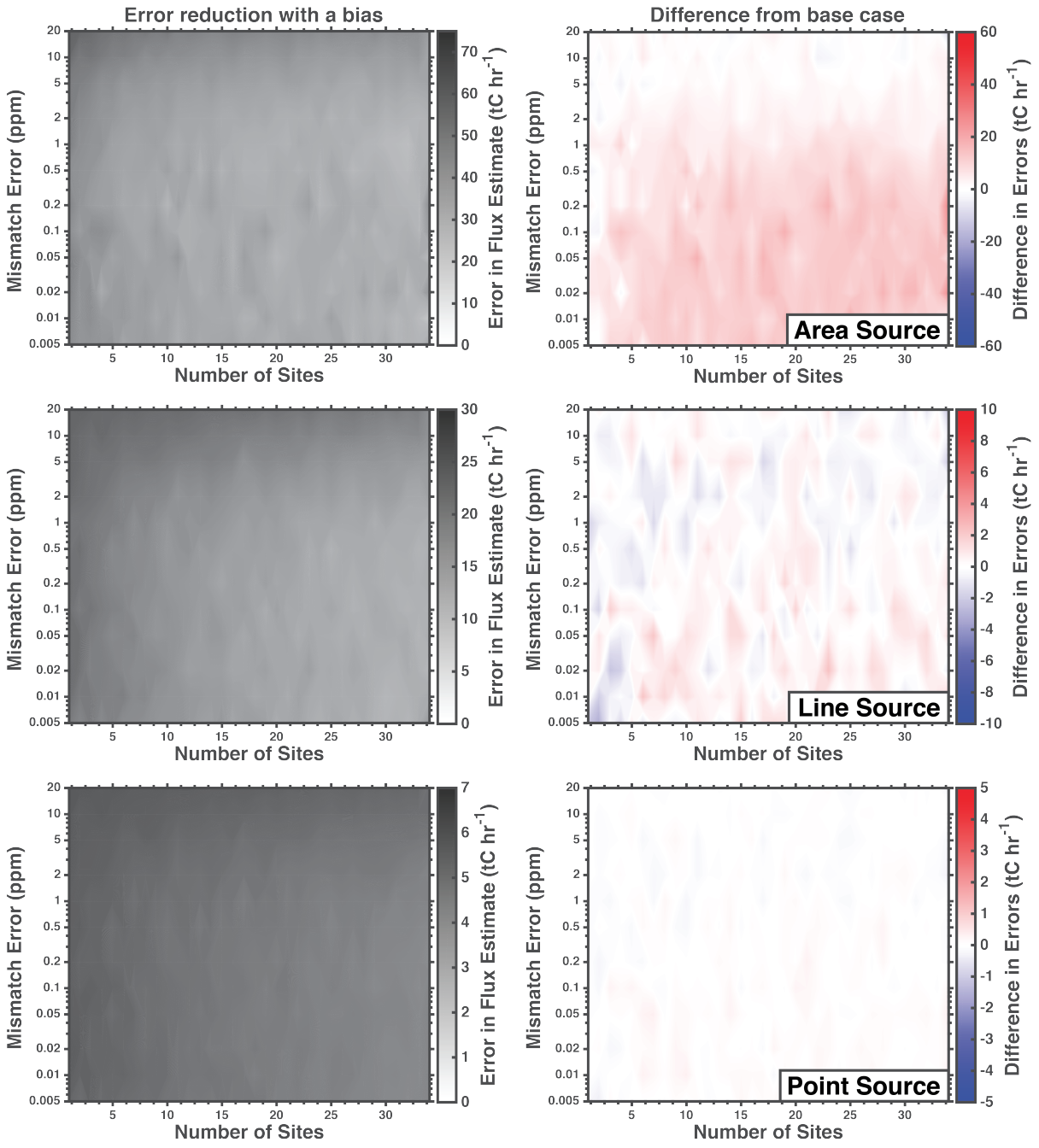

Figure 7: **Error reduction with a systematic bias.** Same as Fig. 6 but for a systematic bias ($\epsilon_b \sim \mathcal{N}\left(0, \sigma_b^2\right)$) at each site.

This is partly due to the poor representation of the diurnal cycle in the prior emissions. The inversion is unable to correct for the overestimated nighttime emissions in the prior without nighttime observations.

6 Model selection criterion

The statistical models presented in the main text were chosen based on an analysis of 127 different models using Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and F-tests. See Appendix A for a list of all 127 models. The selected models all have p-values less than 0.001 in the F-tests. Table 1 shows the model selection criterion for the reduced domain size and Appendix A lists of all the model selection criterion.

Table 1: Model selection criterion and regression coefficients for the base case.

Source Type	AIC	BIC	Model parameters [Regression Coefficients]
Area Source	382	406	$\beta_0 + \beta_2\sigma_o + \beta_3 \ln(n_s) + \beta_4 \ln(\sigma_o) + \beta_5\sqrt{n_s} + \beta_6\sqrt{\sigma_o}$ $[44.16, 0.070, 15.76, 1.23, -3.61, -14.09]$
Line Source	478	502	$\beta_1 n_s + \beta_2\sigma_o + \beta_3 \ln(n_s) + \beta_4 \ln(\sigma_o) + \beta_5\sqrt{n_s} + \beta_6\sqrt{\sigma_o}$ $[-1.44, 0.58, -7.57, 1.18, 23.49, -12.37]$
Point Source	513	533	$\beta_0 + \beta_2\sigma_o + \beta_4 \ln(\sigma_o) + \beta_5\sqrt{n_s} + \beta_6\sqrt{\sigma_o}$ $[6.53, 0.77, 0.81, 4.77, -8.95]$

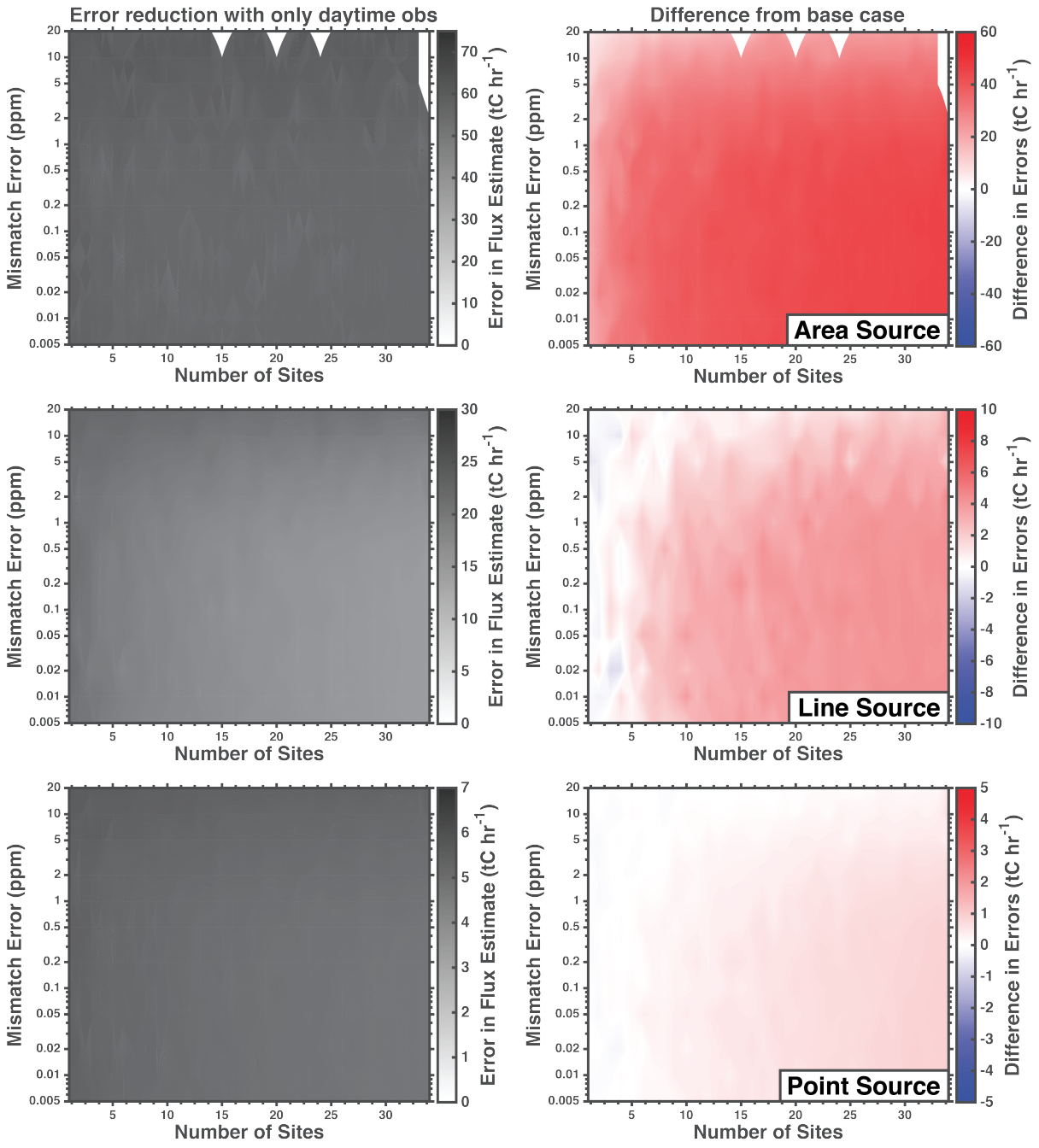

Figure 8: **Error reduction using daytime-only observations.** Same as Fig. 6 but using only daytime observations (10am to 5pm local time).

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A Tables of Model Selection Criterion

Listing 1 : List of all the model combinations

```
Model   1 = constant,nSites,obsErr,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(
    obsErr)
Model   2 = constant,nSites,obsErr,ln(nSites),ln(obsErr),sqrt(nSites)
Model   3 = constant,nSites,obsErr,ln(nSites),ln(obsErr),sqrt(obsErr)
Model   4 = constant,nSites,obsErr,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model   5 = constant,nSites,obsErr,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model   6 = constant,nSites,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model   7 = constant,obsErr,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model   8 = nSites,obsErr,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model   9 = constant,nSites,obsErr,ln(nSites),ln(obsErr)
Model  10 = constant,nSites,obsErr,ln(nSites),sqrt(nSites)
Model  11 = constant,nSites,obsErr,ln(obsErr),sqrt(nSites)
Model  12 = constant,nSites,ln(nSites),ln(obsErr),sqrt(nSites)
Model  13 = constant,obsErr,ln(nSites),ln(obsErr),sqrt(nSites)
Model  14 = nSites,obsErr,ln(nSites),ln(obsErr),sqrt(nSites)
Model  15 = constant,nSites,obsErr,ln(nSites),sqrt(obsErr)
Model  16 = constant,nSites,obsErr,ln(obsErr),sqrt(obsErr)
Model  17 = constant,nSites,ln(nSites),ln(obsErr),sqrt(obsErr)
Model  18 = constant,obsErr,ln(nSites),ln(obsErr),sqrt(obsErr)
Model  19 = nSites,obsErr,ln(nSites),ln(obsErr),sqrt(obsErr)
Model  20 = constant,nSites,obsErr,sqrt(nSites),sqrt(obsErr)
Model  21 = constant,nSites,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  22 = constant,obsErr,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  23 = nSites,obsErr,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  24 = constant,nSites,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  25 = constant,obsErr,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  26 = nSites,obsErr,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  27 = constant,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  28 = nSites,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  29 = obsErr,ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  30 = constant,nSites,obsErr,ln(nSites)
Model  31 = constant,nSites,obsErr,ln(obsErr)
Model  32 = constant,nSites,obsErr,sqrt(nSites)
Model  33 = constant,nSites,obsErr,sqrt(obsErr)
Model  34 = constant,nSites,ln(nSites),ln(obsErr)
Model  35 = constant,nSites,ln(nSites),sqrt(nSites)
Model  36 = constant,nSites,ln(nSites),sqrt(obsErr)
Model  37 = constant,nSites,ln(obsErr),sqrt(nSites)
Model  38 = constant,nSites,ln(obsErr),sqrt(obsErr)
Model  39 = constant,nSites,sqrt(nSites),sqrt(obsErr)
Model  40 = constant,obsErr,ln(nSites),ln(obsErr)
Model  41 = constant,obsErr,ln(nSites),sqrt(nSites)
Model  42 = constant,obsErr,ln(nSites),sqrt(obsErr)
Model  43 = constant,obsErr,ln(obsErr),sqrt(nSites)
Model  44 = constant,obsErr,ln(obsErr),sqrt(obsErr)
Model  45 = constant,obsErr,sqrt(nSites),sqrt(obsErr)
Model  46 = constant,ln(nSites),ln(obsErr),sqrt(nSites)
Model  47 = constant,ln(nSites),ln(obsErr),sqrt(obsErr)
Model  48 = constant,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  49 = constant,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  50 = nSites,obsErr,ln(nSites),ln(obsErr)
Model  51 = nSites,obsErr,ln(nSites),sqrt(nSites)
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Model  52 = nSites,obsErr,ln(nSites),sqrt(obsErr)
Model  53 = nSites,obsErr,ln(obsErr),sqrt(nSites)
Model  54 = nSites,obsErr,ln(obsErr),sqrt(obsErr)
Model  55 = nSites,obsErr,sqrt(nSites),sqrt(obsErr)
Model  56 = nSites,ln(nSites),ln(obsErr),sqrt(nSites)
Model  57 = nSites,ln(nSites),ln(obsErr),sqrt(obsErr)
Model  58 = nSites,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  59 = nSites,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  60 = obsErr,ln(nSites),ln(obsErr),sqrt(nSites)
Model  61 = obsErr,ln(nSites),ln(obsErr),sqrt(obsErr)
Model  62 = obsErr,ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  63 = obsErr,ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  64 = ln(nSites),ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model  65 = constant,nSites,obsErr
Model  66 = constant,nSites,ln(nSites)
Model  67 = constant,nSites,ln(obsErr)
Model  68 = constant,nSites,sqrt(nSites)
Model  69 = constant,nSites,sqrt(obsErr)
Model  70 = constant,obsErr,ln(nSites)
Model  71 = constant,obsErr,ln(obsErr)
Model  72 = constant,obsErr,sqrt(nSites)
Model  73 = constant,obsErr,sqrt(obsErr)
Model  74 = constant,ln(nSites),ln(obsErr)
Model  75 = constant,ln(nSites),sqrt(nSites)
Model  76 = constant,ln(nSites),sqrt(obsErr)
Model  77 = constant,ln(obsErr),sqrt(nSites)
Model  78 = constant,ln(obsErr),sqrt(obsErr)
Model  79 = constant,sqrt(nSites),sqrt(obsErr)
Model  80 = nSites,obsErr,ln(nSites)
Model  81 = nSites,obsErr,ln(obsErr)
Model  82 = nSites,obsErr,sqrt(nSites)
Model  83 = nSites,obsErr,sqrt(obsErr)
Model  84 = nSites,ln(nSites),ln(obsErr)
Model  85 = nSites,ln(nSites),sqrt(nSites)
Model  86 = nSites,ln(nSites),sqrt(obsErr)
Model  87 = nSites,ln(obsErr),sqrt(nSites)
Model  88 = nSites,ln(obsErr),sqrt(obsErr)
Model  89 = nSites,sqrt(nSites),sqrt(obsErr)
Model  90 = obsErr,ln(nSites),ln(obsErr)
Model  91 = obsErr,ln(nSites),sqrt(nSites)
Model  92 = obsErr,ln(nSites),sqrt(obsErr)
Model  93 = obsErr,ln(obsErr),sqrt(nSites)
Model  94 = obsErr,ln(obsErr),sqrt(obsErr)
Model  95 = obsErr,sqrt(nSites),sqrt(obsErr)
Model  96 = ln(nSites),ln(obsErr),sqrt(nSites)
Model  97 = ln(nSites),ln(obsErr),sqrt(obsErr)
Model  98 = ln(nSites),sqrt(nSites),sqrt(obsErr)
Model  99 = ln(obsErr),sqrt(nSites),sqrt(obsErr)
Model 100 = constant,nSites
Model 101 = constant,obsErr
Model 102 = constant,ln(nSites)
Model 103 = constant,ln(obsErr)
Model 104 = constant,sqrt(nSites)
Model 105 = constant,sqrt(obsErr)
Model 106 = nSites,obsErr
Model 107 = nSites,ln(nSites)
Model 108 = nSites,ln(obsErr)
```

```
Model 109 = nSites,sqrt(nSites)
Model 110 = nSites,sqrt(obsErr)
Model 111 = obsErr,ln(nSites)
Model 112 = obsErr,ln(obsErr)
Model 113 = obsErr,sqrt(nSites)
Model 114 = obsErr,sqrt(obsErr)
Model 115 = ln(nSites),ln(obsErr)
Model 116 = ln(nSites),sqrt(nSites)
Model 117 = ln(nSites),sqrt(obsErr)
Model 118 = ln(obsErr),sqrt(nSites)
Model 119 = ln(obsErr),sqrt(obsErr)
Model 120 = sqrt(nSites),sqrt(obsErr)
Model 121 = constant
Model 122 = nSites
Model 123 = obsErr
Model 124 = ln(nSites)
Model 125 = ln(obsErr)
Model 126 = sqrt(nSites)
Model 127 = sqrt(obsErr)
```

Listing 2 : Statistical models for the "Area Source" (base case)

```
*** ERROR REDUCTION MODELS ***
Model 001:  AIC =  383.9276 & BIC =  412.0065  (7 Terms)
Model 002:  AIC =  514.7889 & BIC =  538.8565  (6 Terms)
Model 003:  AIC =  383.7930 & BIC =  407.8606  (6 Terms)
Model 004:  AIC =  451.4532 & BIC =  475.5208  (6 Terms)
Model 005:  AIC =  401.4659 & BIC =  425.5335  (6 Terms)
Model 006:  AIC =  400.9189 & BIC =  424.9865  (6 Terms)
Model 007:  AIC =  382.2329 & BIC =  406.3005  (6 Terms)
Model 008:  AIC =  493.3063 & BIC =  517.3739  (6 Terms)
Model 009:  AIC =  512.9857 & BIC =  533.0420  (5 Terms)
Model 010:  AIC =  543.3056 & BIC =  563.3619  (5 Terms)
Model 011:  AIC =  516.3437 & BIC =  536.4000  (5 Terms)
Model 012:  AIC =  653.1694 & BIC =  673.2258  (5 Terms)
Model 013:  AIC =  512.8003 & BIC =  532.8566  (5 Terms)
Model 014:  AIC =  536.6576 & BIC =  556.7139  (5 Terms)
Model 015:  AIC =  449.8015 & BIC =  469.8579  (5 Terms)
Model 016:  AIC =  512.0753 & BIC =  532.1316  (5 Terms)
Model 017:  AIC =  400.1892 & BIC =  420.2455  (5 Terms)
Model 018:  AIC =  414.4421 & BIC =  434.4984  (5 Terms)
Model 019:  AIC =  575.3743 & BIC =  595.4307  (5 Terms)
Model 020:  AIC =  456.9411 & BIC =  476.9974  (5 Terms)
Model 021:  AIC =  463.3456 & BIC =  483.4019  (5 Terms)
Model 022:  AIC =  449.4542 & BIC =  469.5105  (5 Terms)
Model 023:  AIC =  502.8071 & BIC =  522.8634  (5 Terms)
Model 024:  AIC =  413.9400 & BIC =  433.9964  (5 Terms)
Model 025:  AIC =  477.7444 & BIC =  497.8007  (5 Terms)
Model 026:  AIC =  518.5051 & BIC =  538.5615  (5 Terms)
Model 027:  AIC =  399.1028 & BIC =  419.1591  (5 Terms)
Model 028:  AIC =  492.3375 & BIC =  512.3938  (5 Terms)
Model 029:  AIC =  609.3252 & BIC =  629.3815  (5 Terms)
Model 030:  AIC =  541.7619 & BIC =  557.8069  (4 Terms)
Model 031:  AIC =  557.9078 & BIC =  573.9529  (4 Terms)
Model 032:  AIC =  544.5256 & BIC =  560.5707  (4 Terms)
Model 033:  AIC =  530.8044 & BIC =  546.8495  (4 Terms)
```

```
Model 034:  AIC =  651.2413 & BIC =  667.2864  (4 Terms)
Model 035:  AIC =  702.6344 & BIC =  718.6795  (4 Terms)
Model 036:  AIC =  461.5619 & BIC =  477.6070  (4 Terms)
Model 037:  AIC =  652.3123 & BIC =  668.3574  (4 Terms)
Model 038:  AIC =  512.9356 & BIC =  528.9807  (4 Terms)
Model 039:  AIC =  467.6673 & BIC =  483.7124  (4 Terms)
Model 040:  AIC =  517.6744 & BIC =  533.7194  (4 Terms)
Model 041:  AIC =  541.5262 & BIC =  557.5713  (4 Terms)
Model 042:  AIC =  465.0317 & BIC =  481.0767  (4 Terms)
Model 043:  AIC =  540.2055 & BIC =  556.2506  (4 Terms)
Model 044:  AIC =  620.0587 & BIC =  636.1038  (4 Terms)
Model 045:  AIC =  504.8572 & BIC =  520.9023  (4 Terms)
Model 046:  AIC =  651.1698 & BIC =  667.2149  (4 Terms)
Model 047:  AIC =  423.5506 & BIC =  439.5957  (4 Terms)
Model 048:  AIC =  461.3564 & BIC =  477.4015  (4 Terms)
Model 049:  AIC =  480.3013 & BIC =  496.3464  (4 Terms)
Model 050:  AIC =  579.0750 & BIC =  595.1201  (4 Terms)
Model 051:  AIC =  563.5195 & BIC =  579.5646  (4 Terms)
Model 052:  AIC =  573.5099 & BIC =  589.5549  (4 Terms)
Model 053:  AIC =  543.5969 & BIC =  559.6420  (4 Terms)
Model 054:  AIC =  773.1389 & BIC =  789.1839  (4 Terms)
Model 055:  AIC =  520.0060 & BIC =  536.0511  (4 Terms)
Model 056:  AIC =  653.3265 & BIC =  669.3716  (4 Terms)
Model 057:  AIC =  575.0046 & BIC =  591.0497  (4 Terms)
Model 058:  AIC =  507.2983 & BIC =  523.3434  (4 Terms)
Model 059:  AIC =  516.5122 & BIC =  532.5573  (4 Terms)
Model 060:  AIC =  607.6063 & BIC =  623.6514  (4 Terms)
Model 061:  AIC =  671.7812 & BIC =  687.8262  (4 Terms)
Model 062:  AIC =  610.4250 & BIC =  626.4701  (4 Terms)
Model 063:  AIC =  719.4930 & BIC =  735.5381  (4 Terms)
Model 064:  AIC =  612.5545 & BIC =  628.5996  (4 Terms)
Model 065:  AIC =  571.4328 & BIC =  583.4666  (3 Terms)
Model 066:  AIC =  701.1087 & BIC =  713.1425  (3 Terms)
Model 067:  AIC =  666.7837 & BIC =  678.8175  (3 Terms)
Model 068:  AIC =  702.2077 & BIC =  714.2415  (3 Terms)
Model 069:  AIC =  537.8523 & BIC =  549.8861  (3 Terms)
Model 070:  AIC =  542.2674 & BIC =  554.3012  (3 Terms)
Model 071:  AIC =  636.6359 & BIC =  648.6697  (3 Terms)
Model 072:  AIC =  557.6561 & BIC =  569.6899  (3 Terms)
Model 073:  AIC =  623.5771 & BIC =  635.6109  (3 Terms)
Model 074:  AIC =  653.0281 & BIC =  665.0619  (3 Terms)
Model 075:  AIC =  700.9487 & BIC =  712.9825  (3 Terms)
Model 076:  AIC =  477.6762 & BIC =  489.7100  (3 Terms)
Model 077:  AIC =  660.6026 & BIC =  672.6364  (3 Terms)
Model 078:  AIC =  619.4667 & BIC =  631.5005  (3 Terms)
Model 079:  AIC =  514.0512 & BIC =  526.0850  (3 Terms)
Model 080:  AIC =  600.1499 & BIC =  612.1837  (3 Terms)
Model 081:  AIC =  849.7446 & BIC =  861.7784  (3 Terms)
Model 082:  AIC =  570.1771 & BIC =  582.2109  (3 Terms)
Model 083:  AIC =  856.8198 & BIC =  868.8536  (3 Terms)
Model 084:  AIC =  656.1247 & BIC =  668.1585  (3 Terms)
Model 085:  AIC =  703.3695 & BIC =  715.4033  (3 Terms)
Model 086:  AIC =  573.8958 & BIC =  585.9296  (3 Terms)
Model 087:  AIC =  651.3495 & BIC =  663.3833  (3 Terms)
Model 088:  AIC =  835.8096 & BIC =  847.8434  (3 Terms)
Model 089:  AIC =  522.5167 & BIC =  534.5505  (3 Terms)
Model 090:  AIC =  680.5031 & BIC =  692.5369  (3 Terms)
```

```
Model 091:  AIC =  624.3632 & BIC =  636.3970  (3 Terms)
Model 092:  AIC =  688.4205 & BIC =  700.4543  (3 Terms)
Model 093:  AIC =  747.4873 & BIC =  759.5211  (3 Terms)
Model 094:  AIC =  821.8353 & BIC =  833.8691  (3 Terms)
Model 095:  AIC =  754.1316 & BIC =  766.1654  (3 Terms)
Model 096:  AIC =  663.4622 & BIC =  675.4960  (3 Terms)
Model 097:  AIC =  691.6121 & BIC =  703.6459  (3 Terms)
Model 098:  AIC =  610.5569 & BIC =  622.5907  (3 Terms)
Model 099:  AIC =  753.2032 & BIC =  765.2370  (3 Terms)
Model 100:  AIC =  708.5702 & BIC =  716.5927  (2 Terms)
Model 101:  AIC =  646.6925 & BIC =  654.7150  (2 Terms)
Model 102:  AIC =  700.0558 & BIC =  708.0783  (2 Terms)
Model 103:  AIC =  697.5921 & BIC =  705.6147  (2 Terms)
Model 104:  AIC =  704.5057 & BIC =  712.5282  (2 Terms)
Model 105:  AIC =  622.8196 & BIC =  630.8422  (2 Terms)
Model 106:  AIC =  862.2705 & BIC =  870.2930  (2 Terms)
Model 107:  AIC =  704.1059 & BIC =  712.1285  (2 Terms)
Model 108:  AIC =  848.2829 & BIC =  856.3054  (2 Terms)
Model 109:  AIC =  701.3768 & BIC =  709.3994  (2 Terms)
Model 110:  AIC =  863.8412 & BIC =  871.8637  (2 Terms)
Model 111:  AIC =  686.9972 & BIC =  695.0198  (2 Terms)
Model 112:  AIC =  976.0873 & BIC =  984.1098  (2 Terms)
Model 113:  AIC =  752.7273 & BIC =  760.7498  (2 Terms)
Model 114:  AIC = 1054.1848 & BIC = 1062.2073  (2 Terms)
Model 115:  AIC =  700.4043 & BIC =  708.4268  (2 Terms)
Model 116:  AIC =  707.9458 & BIC =  715.9683  (2 Terms)
Model 117:  AIC =  691.4227 & BIC =  699.4452  (2 Terms)
Model 118:  AIC =  751.4122 & BIC =  759.4347  (2 Terms)
Model 119:  AIC =  915.2317 & BIC =  923.2542  (2 Terms)
Model 120:  AIC =  758.0984 & BIC =  766.1210  (2 Terms)
Model 121:  AIC =  738.2662 & BIC =  742.2775  (1 Terms)
Model 122:  AIC =  861.8462 & BIC =  865.8574  (1 Terms)
Model 123:  AIC = 1109.4646 & BIC = 1113.4759  (1 Terms)
Model 124:  AIC =  727.5389 & BIC =  731.5502  (1 Terms)
Model 125:  AIC =  996.5779 & BIC = 1000.5891  (1 Terms)
Model 126:  AIC =  768.4575 & BIC =  772.4688  (1 Terms)
Model 127:  AIC = 1090.7113 & BIC = 1094.7225  (1 Terms)

*** BEST ERROR REDUCTION MODELS ***
Model 007:  AIC =  382.2329 & BIC =  406.3005 (6 Terms)
```

Listing 3 : Statistical models for the "Line Source" (base case)

```
*** ERROR REDUCTION MODELS ***
Model 001:  AIC =  480.2421 & BIC =  508.3210  (7 Terms)
Model 002:  AIC =  560.6119 & BIC =  584.6795  (6 Terms)
Model 003:  AIC =  496.0918 & BIC =  520.1594  (6 Terms)
Model 004:  AIC =  515.7065 & BIC =  539.7741  (6 Terms)
Model 005:  AIC =  482.1347 & BIC =  506.2023  (6 Terms)
Model 006:  AIC =  486.0717 & BIC =  510.1393  (6 Terms)
Model 007:  AIC =  492.5250 & BIC =  516.5926  (6 Terms)
Model 008:  AIC =  478.4108 & BIC =  502.4784  (6 Terms)
Model 009:  AIC =  565.9010 & BIC =  585.9574  (5 Terms)
Model 010:  AIC =  583.8352 & BIC =  603.8916  (5 Terms)
Model 011:  AIC =  560.0436 & BIC =  580.0999  (5 Terms)
Model 012:  AIC =  718.4606 & BIC =  738.5170  (5 Terms)
```

```
Model 013:  AIC =  564.1466 & BIC =  584.2030  (5 Terms)
Model 014:  AIC =  560.8280 & BIC =  580.8844  (5 Terms)
Model 015:  AIC =  526.3957 & BIC =  546.4520  (5 Terms)
Model 016:  AIC =  581.9360 & BIC =  601.9924  (5 Terms)
Model 017:  AIC =  500.2958 & BIC =  520.3522  (5 Terms)
Model 018:  AIC =  506.0062 & BIC =  526.0626  (5 Terms)
Model 019:  AIC =  548.9225 & BIC =  568.9788  (5 Terms)
Model 020:  AIC =  516.4728 & BIC =  536.5292  (5 Terms)
Model 021:  AIC =  525.9726 & BIC =  546.0289  (5 Terms)
Model 022:  AIC =  523.8456 & BIC =  543.9019  (5 Terms)
Model 023:  AIC =  515.7351 & BIC =  535.7915  (5 Terms)
Model 024:  AIC =  487.5749 & BIC =  507.6313  (5 Terms)
Model 025:  AIC =  517.7705 & BIC =  537.8268  (5 Terms)
Model 026:  AIC =  485.6776 & BIC =  505.7339  (5 Terms)
Model 027:  AIC =  496.8794 & BIC =  516.9357  (5 Terms)
Model 028:  AIC =  484.6623 & BIC =  504.7186  (5 Terms)
Model 029:  AIC =  551.9587 & BIC =  572.0151  (5 Terms)
Model 030:  AIC =  586.3957 & BIC =  602.4408  (4 Terms)
Model 031:  AIC =  613.8148 & BIC =  629.8599  (4 Terms)
Model 032:  AIC =  582.5621 & BIC =  598.6072  (4 Terms)
Model 033:  AIC =  594.4882 & BIC =  610.5333  (4 Terms)
Model 034:  AIC =  718.0235 & BIC =  734.0686  (4 Terms)
Model 035:  AIC =  777.6872 & BIC =  793.7323  (4 Terms)
Model 036:  AIC =  536.0808 & BIC =  552.1258  (4 Terms)
Model 037:  AIC =  716.6368 & BIC =  732.6819  (4 Terms)
Model 038:  AIC =  581.5029 & BIC =  597.5480  (4 Terms)
Model 039:  AIC =  526.6675 & BIC =  542.7126  (4 Terms)
Model 040:  AIC =  571.1750 & BIC =  587.2201  (4 Terms)
Model 041:  AIC =  584.9829 & BIC =  601.0280  (4 Terms)
Model 042:  AIC =  532.9448 & BIC =  548.9899  (4 Terms)
Model 043:  AIC =  574.4002 & BIC =  590.4453  (4 Terms)
Model 044:  AIC =  763.1899 & BIC =  779.2349  (4 Terms)
Model 045:  AIC =  541.9903 & BIC =  558.0354  (4 Terms)
Model 046:  AIC =  717.7983 & BIC =  733.8434  (4 Terms)
Model 047:  AIC =  510.4146 & BIC =  526.4597  (4 Terms)
Model 048:  AIC =  533.9758 & BIC =  550.0208  (4 Terms)
Model 049:  AIC =  519.6370 & BIC =  535.6821  (4 Terms)
Model 050:  AIC =  572.3827 & BIC =  588.4278  (4 Terms)
Model 051:  AIC =  582.5060 & BIC =  598.5510  (4 Terms)
Model 052:  AIC =  550.1118 & BIC =  566.1569  (4 Terms)
Model 053:  AIC =  558.8609 & BIC =  574.9060  (4 Terms)
Model 054:  AIC =  737.7429 & BIC =  753.7879  (4 Terms)
Model 055:  AIC =  514.4740 & BIC =  530.5191  (4 Terms)
Model 056:  AIC =  718.5513 & BIC =  734.5963  (4 Terms)
Model 057:  AIC =  547.3428 & BIC =  563.3878  (4 Terms)
Model 058:  AIC =  526.0873 & BIC =  542.1324  (4 Terms)
Model 059:  AIC =  488.2187 & BIC =  504.2638  (4 Terms)
Model 060:  AIC =  569.6551 & BIC =  585.7002  (4 Terms)
Model 061:  AIC =  550.0599 & BIC =  566.1050  (4 Terms)
Model 062:  AIC =  550.9205 & BIC =  566.9656  (4 Terms)
Model 063:  AIC =  612.5479 & BIC =  628.5930  (4 Terms)
Model 064:  AIC =  552.3027 & BIC =  568.3478  (4 Terms)
Model 065:  AIC =  623.4304 & BIC =  635.4643  (3 Terms)
Model 066:  AIC =  775.9476 & BIC =  787.9814  (3 Terms)
Model 067:  AIC =  734.7450 & BIC =  746.7788  (3 Terms)
Model 068:  AIC =  775.6968 & BIC =  787.7306  (3 Terms)
Model 069:  AIC =  601.4104 & BIC =  613.4442  (3 Terms)
```

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Model 070:  AIC =  592.4083 & BIC =  604.4421  (3 Terms)
Model 071:  AIC =  770.3294 & BIC =  782.3632  (3 Terms)
Model 072:  AIC =  591.0657 & BIC =  603.0995  (3 Terms)
Model 073:  AIC =  763.9131 & BIC =  775.9469  (3 Terms)
Model 074:  AIC =  716.5170 & BIC =  728.5508  (3 Terms)
Model 075:  AIC =  775.8328 & BIC =  787.8666  (3 Terms)
Model 076:  AIC =  540.0868 & BIC =  552.1206  (3 Terms)
Model 077:  AIC =  721.2484 & BIC =  733.2822  (3 Terms)
Model 078:  AIC =  761.9604 & BIC =  773.9942  (3 Terms)
Model 079:  AIC =  552.3054 & BIC =  564.3392  (3 Terms)
Model 080:  AIC =  595.2768 & BIC =  607.3106  (3 Terms)
Model 081:  AIC =  782.0989 & BIC =  794.1327  (3 Terms)
Model 082:  AIC =  580.5883 & BIC =  592.6221  (3 Terms)
Model 083:  AIC =  796.3904 & BIC =  808.4242  (3 Terms)
Model 084:  AIC =  716.6563 & BIC =  728.6901  (3 Terms)
Model 085:  AIC =  776.0825 & BIC =  788.1163  (3 Terms)
Model 086:  AIC =  557.3592 & BIC =  569.3930  (3 Terms)
Model 087:  AIC =  719.3387 & BIC =  731.3725  (3 Terms)
Model 088:  AIC =  791.7155 & BIC =  803.7493  (3 Terms)
Model 089:  AIC =  524.6716 & BIC =  536.7054  (3 Terms)
Model 090:  AIC =  571.0262 & BIC =  583.0600  (3 Terms)
Model 091:  AIC =  592.8115 & BIC =  604.8453  (3 Terms)
Model 092:  AIC =  549.3126 & BIC =  561.3464  (3 Terms)
Model 093:  AIC =  610.5513 & BIC =  622.5851  (3 Terms)
Model 094:  AIC =  881.0454 & BIC =  893.0792  (3 Terms)
Model 095:  AIC =  616.9590 & BIC =  628.9928  (3 Terms)
Model 096:  AIC =  717.3411 & BIC =  729.3749  (3 Terms)
Model 097:  AIC =  550.3268 & BIC =  562.3606  (3 Terms)
Model 098:  AIC =  559.6060 & BIC =  571.6398  (3 Terms)
Model 099:  AIC =  628.6530 & BIC =  640.6869  (3 Terms)
Model 100:  AIC =  784.6650 & BIC =  792.6875  (2 Terms)
Model 101:  AIC =  774.6424 & BIC =  782.6650  (2 Terms)
Model 102:  AIC =  775.0181 & BIC =  783.0407  (2 Terms)
Model 103:  AIC =  815.7513 & BIC =  823.7738  (2 Terms)
Model 104:  AIC =  776.3462 & BIC =  784.3688  (2 Terms)
Model 105:  AIC =  762.5123 & BIC =  770.5348  (2 Terms)
Model 106:  AIC =  795.8500 & BIC =  803.8725  (2 Terms)
Model 107:  AIC =  774.1385 & BIC =  782.1610  (2 Terms)
Model 108:  AIC =  790.5731 & BIC =  798.5956  (2 Terms)
Model 109:  AIC =  775.7453 & BIC =  783.7678  (2 Terms)
Model 110:  AIC =  804.6386 & BIC =  812.6611  (2 Terms)
Model 111:  AIC =  593.6120 & BIC =  601.6345  (2 Terms)
Model 112:  AIC = 1034.3253 & BIC = 1042.3478  (2 Terms)
Model 113:  AIC =  627.7191 & BIC =  635.7416  (2 Terms)
Model 114:  AIC = 1119.4241 & BIC = 1127.4466  (2 Terms)
Model 115:  AIC =  718.2423 & BIC =  726.2648  (2 Terms)
Model 116:  AIC =  774.7821 & BIC =  782.8046  (2 Terms)
Model 117:  AIC =  557.6296 & BIC =  565.6521  (2 Terms)
Model 118:  AIC =  719.5425 & BIC =  727.5650  (2 Terms)
Model 119:  AIC =  980.0107 & BIC =  988.0332  (2 Terms)
Model 120:  AIC =  626.9783 & BIC =  635.0008  (2 Terms)
Model 121:  AIC =  855.1810 & BIC =  859.1923  (1 Terms)
Model 122:  AIC =  825.2913 & BIC =  829.3025  (1 Terms)
Model 123:  AIC = 1179.9119 & BIC = 1183.9231  (1 Terms)
Model 124:  AIC =  776.1363 & BIC =  780.1476  (1 Terms)
Model 125:  AIC = 1056.5279 & BIC = 1060.5392  (1 Terms)
Model 126:  AIC =  774.3804 & BIC =  778.3917  (1 Terms)
```

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Model 127:  AIC = 1164.4041 & BIC = 1168.4154  (1 Terms)

*** BEST ERROR REDUCTION MODELS ***
Model 008:  AIC =  478.4108 & BIC =  502.4784 (6 Terms)
```

Listing 4 : Statistical models for the "Point Source" (base case)

```
*** ERROR REDUCTION MODELS ***
Model 001:  AIC =  516.5162 & BIC =  544.5951  (7 Terms)
Model 002:  AIC =  551.8963 & BIC =  575.9639  (6 Terms)
Model 003:  AIC =  515.9237 & BIC =  539.9913  (6 Terms)
Model 004:  AIC =  522.9692 & BIC =  547.0368  (6 Terms)
Model 005:  AIC =  514.7844 & BIC =  538.8520  (6 Terms)
Model 006:  AIC =  527.0333 & BIC =  551.1009  (6 Terms)
Model 007:  AIC =  514.5781 & BIC =  538.6457  (6 Terms)
Model 008:  AIC =  515.8843 & BIC =  539.9519  (6 Terms)
Model 009:  AIC =  550.5805 & BIC =  570.6368  (5 Terms)
Model 010:  AIC =  584.7050 & BIC =  604.7613  (5 Terms)
Model 011:  AIC =  549.9804 & BIC =  570.0367  (5 Terms)
Model 012:  AIC =  616.8468 & BIC =  636.9031  (5 Terms)
Model 013:  AIC =  549.9063 & BIC =  569.9626  (5 Terms)
Model 014:  AIC =  549.9839 & BIC =  570.0402  (5 Terms)
Model 015:  AIC =  522.1990 & BIC =  542.2554  (5 Terms)
Model 016:  AIC =  527.8178 & BIC =  547.8741  (5 Terms)
Model 017:  AIC =  526.0988 & BIC =  546.1551  (5 Terms)
Model 018:  AIC =  556.6633 & BIC =  576.7196  (5 Terms)
Model 019:  AIC =  550.0185 & BIC =  570.0749  (5 Terms)
Model 020:  AIC =  521.1899 & BIC =  541.2462  (5 Terms)
Model 021:  AIC =  525.0389 & BIC =  545.0952  (5 Terms)
Model 022:  AIC =  521.0206 & BIC =  541.0769  (5 Terms)
Model 023:  AIC =  521.1768 & BIC =  541.2331  (5 Terms)
Model 024:  AIC =  525.2030 & BIC =  545.2594  (5 Terms)
Model 025:  AIC =  513.5629 & BIC =  533.6192  (5 Terms)
Model 026:  AIC =  525.2375 & BIC =  545.2939  (5 Terms)
Model 027:  AIC =  525.0636 & BIC =  545.1199  (5 Terms)
Model 028:  AIC =  525.2001 & BIC =  545.2565  (5 Terms)
Model 029:  AIC =  531.4730 & BIC =  551.5293  (5 Terms)
Model 030:  AIC =  582.9480 & BIC =  598.9931  (4 Terms)
Model 031:  AIC =  557.8958 & BIC =  573.9409  (4 Terms)
Model 032:  AIC =  582.7095 & BIC =  598.7546  (4 Terms)
Model 033:  AIC =  533.0149 & BIC =  549.0600  (4 Terms)
Model 034:  AIC =  615.1222 & BIC =  631.1672  (4 Terms)
Model 035:  AIC =  710.2246 & BIC =  726.2697  (4 Terms)
Model 036:  AIC =  524.1019 & BIC =  540.1470  (4 Terms)
Model 037:  AIC =  614.8677 & BIC =  630.9127  (4 Terms)
Model 038:  AIC =  536.2373 & BIC =  552.2823  (4 Terms)
Model 039:  AIC =  523.2077 & BIC =  539.2528  (4 Terms)
Model 040:  AIC =  577.0988 & BIC =  593.1439  (4 Terms)
Model 041:  AIC =  582.7133 & BIC =  598.7583  (4 Terms)
Model 042:  AIC =  559.1296 & BIC =  575.1746  (4 Terms)
Model 043:  AIC =  548.3828 & BIC =  564.4279  (4 Terms)
Model 044:  AIC =  699.9415 & BIC =  715.9866  (4 Terms)
Model 045:  AIC =  519.8256 & BIC =  535.8707  (4 Terms)
Model 046:  AIC =  614.8487 & BIC =  630.8938  (4 Terms)
Model 047:  AIC =  561.3878 & BIC =  577.4328  (4 Terms)
Model 048:  AIC =  523.0687 & BIC =  539.1138  (4 Terms)
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Model 049:  AIC =  523.8225 & BIC =  539.8676  (4 Terms)
Model 050:  AIC =  552.9248 & BIC =  568.9698  (4 Terms)
Model 051:  AIC =  582.7480 & BIC =  598.7930  (4 Terms)
Model 052:  AIC =  552.6939 & BIC =  568.7389  (4 Terms)
Model 053:  AIC =  547.9858 & BIC =  564.0308  (4 Terms)
Model 054:  AIC =  593.0397 & BIC =  609.0847  (4 Terms)
Model 055:  AIC =  524.3754 & BIC =  540.4205  (4 Terms)
Model 056:  AIC =  615.2662 & BIC =  631.3113  (4 Terms)
Model 057:  AIC =  548.6869 & BIC =  564.7320  (4 Terms)
Model 058:  AIC =  523.2165 & BIC =  539.2616  (4 Terms)
Model 059:  AIC =  527.0500 & BIC =  543.0951  (4 Terms)
Model 060:  AIC =  548.9378 & BIC =  564.9829  (4 Terms)
Model 061:  AIC =  563.1517 & BIC =  579.1968  (4 Terms)
Model 062:  AIC =  530.3237 & BIC =  546.3687  (4 Terms)
Model 063:  AIC =  529.4731 & BIC =  545.5181  (4 Terms)
Model 064:  AIC =  529.7217 & BIC =  545.7667  (4 Terms)
Model 065:  AIC =  586.9887 & BIC =  599.0225  (3 Terms)
Model 066:  AIC =  708.2312 & BIC =  720.2650  (3 Terms)
Model 067:  AIC =  617.9610 & BIC =  629.9948  (3 Terms)
Model 068:  AIC =  708.3244 & BIC =  720.3582  (3 Terms)
Model 069:  AIC =  534.2373 & BIC =  546.2711  (3 Terms)
Model 070:  AIC =  600.6050 & BIC =  612.6388  (3 Terms)
Model 071:  AIC =  701.6617 & BIC =  713.6955  (3 Terms)
Model 072:  AIC =  581.0168 & BIC =  593.0506  (3 Terms)
Model 073:  AIC =  698.5437 & BIC =  710.5775  (3 Terms)
Model 074:  AIC =  626.9277 & BIC =  638.9615  (3 Terms)
Model 075:  AIC =  708.3516 & BIC =  720.3854  (3 Terms)
Model 076:  AIC =  559.3919 & BIC =  571.4257  (3 Terms)
Model 077:  AIC =  612.9811 & BIC =  625.0149  (3 Terms)
Model 078:  AIC =  698.6054 & BIC =  710.6392  (3 Terms)
Model 079:  AIC =  521.8286 & BIC =  533.8624  (3 Terms)
Model 080:  AIC =  589.0602 & BIC =  601.0940  (3 Terms)
Model 081:  AIC =  605.1685 & BIC =  617.2023  (3 Terms)
Model 082:  AIC =  581.2101 & BIC =  593.2439  (3 Terms)
Model 083:  AIC =  646.1754 & BIC =  658.2092  (3 Terms)
Model 084:  AIC =  613.4773 & BIC =  625.5111  (3 Terms)
Model 085:  AIC =  708.2249 & BIC =  720.2587  (3 Terms)
Model 086:  AIC =  551.4370 & BIC =  563.4708  (3 Terms)
Model 087:  AIC =  614.7086 & BIC =  626.7424  (3 Terms)
Model 088:  AIC =  616.3087 & BIC =  628.3425  (3 Terms)
Model 089:  AIC =  525.4738 & BIC =  537.5076  (3 Terms)
Model 090:  AIC =  577.0755 & BIC =  589.1093  (3 Terms)
Model 091:  AIC =  581.1069 & BIC =  593.1407  (3 Terms)
Model 092:  AIC =  561.2581 & BIC =  573.2919  (3 Terms)
Model 093:  AIC =  548.0945 & BIC =  560.1283  (3 Terms)
Model 094:  AIC =  736.4775 & BIC =  748.5113  (3 Terms)
Model 095:  AIC =  528.3875 & BIC =  540.4213  (3 Terms)
Model 096:  AIC =  623.2711 & BIC =  635.3049  (3 Terms)
Model 097:  AIC =  561.6428 & BIC =  573.6766  (3 Terms)
Model 098:  AIC =  530.0829 & BIC =  542.1167  (3 Terms)
Model 099:  AIC =  527.7309 & BIC =  539.7647  (3 Terms)
Model 100:  AIC =  708.1997 & BIC =  716.2222  (2 Terms)
Model 101:  AIC =  705.5771 & BIC =  713.5996  (2 Terms)
Model 102:  AIC =  711.1265 & BIC =  719.1491  (2 Terms)
Model 103:  AIC =  718.0927 & BIC =  726.1152  (2 Terms)
Model 104:  AIC =  706.3602 & BIC =  714.3827  (2 Terms)
Model 105:  AIC =  696.6394 & BIC =  704.6620  (2 Terms)
```

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Model 106:  AIC =  646.1805 & BIC =  654.2030  (2 Terms)
Model 107:  AIC =  706.3013 & BIC =  714.3238  (2 Terms)
Model 108:  AIC =  622.7828 & BIC =  630.8054  (2 Terms)
Model 109:  AIC =  706.8500 & BIC =  714.8726  (2 Terms)
Model 110:  AIC =  646.2357 & BIC =  654.2582  (2 Terms)
Model 111:  AIC =  598.6085 & BIC =  606.6310  (2 Terms)
Model 112:  AIC =  829.4327 & BIC =  837.4552  (2 Terms)
Model 113:  AIC =  579.2348 & BIC =  587.2573  (2 Terms)
Model 114:  AIC =  860.6207 & BIC =  868.6433  (2 Terms)
Model 115:  AIC =  636.4907 & BIC =  644.5133  (2 Terms)
Model 116:  AIC =  709.6320 & BIC =  717.6546  (2 Terms)
Model 117:  AIC =  560.5322 & BIC =  568.5547  (2 Terms)
Model 118:  AIC =  623.5570 & BIC =  631.5795  (2 Terms)
Model 119:  AIC =  791.1095 & BIC =  799.1321  (2 Terms)
Model 120:  AIC =  528.1109 & BIC =  536.1335  (2 Terms)
Model 121:  AIC =  760.9363 & BIC =  764.9476  (1 Terms)
Model 122:  AIC =  709.9145 & BIC =  713.9258  (1 Terms)
Model 123:  AIC =  898.2527 & BIC =  902.2639  (1 Terms)
Model 124:  AIC =  713.2731 & BIC =  717.2844  (1 Terms)
Model 125:  AIC =  861.0349 & BIC =  865.0461  (1 Terms)
Model 126:  AIC =  708.2118 & BIC =  712.2231  (1 Terms)
Model 127:  AIC =  890.1085 & BIC =  894.1198  (1 Terms)

*** BEST ERROR REDUCTION MODELS ***
Model 025:  AIC =  513.5629 & BIC =  533.6192 (5 Terms)
```