

Authors Response to the reviews of

*Cloud Photogrammetry with Dense Stereo for
Fisheye Cameras*

by

Christoph Beekmans et al.

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Dear Editors and Reviewers,

thank you very much for your valuable comments on our paper. We addressed all the issues raised by the reviewers and modified/extended the paper as requested. Below, you can find the answers to the questions raised by the referees.

We also want to inform you that we included Mr. Martin Lennefer into the list of co-authors due to his intensive involvement in our work regarding his technical expertise of the camera system and his contributions to the design and implementation of our field campaigns.

Thank you for your efforts,

Christoph Beekmans, Johannes Schneider, Thomas Läbe, Martin Lennefer, Cyrill Stachniss and Clemens Simmer

Point-to-Point response to: anonymous referee #1 from June 9th, 2016

Referee Comment:

„In many parts of the manuscript, rectification is referred as the method that allows dense stereo matching. I find this misleading, because rectification is merely a transformation to translate the epipoles to infinity so that the epipolar lines are parallel in both images, hence matching algorithm is less time consuming and more straightforward to design.“

Authors Response:

We agree with the referee that this can be misleading. Our intention was to emphasize that epipolar rectification is usually a prerequisite to use efficient out of the box dense stereo matching techniques, like the Semi-Global-Matching employed in this paper. The plane-sweep algorithm works also on unrectified images.

To be more precise we added changes at several places.

- 1) In the abstract we point out, that the rectification “allows the use of efficient out-of-the-box dense matching algorithms designed for classical pinhole-type cameras ...” (Page 1, Line 3)
- 2) We added at Page 2, Line 21, a clarification that we are aiming to use dense stereo methods designed for perspective cameras:
"The main contribution of this paper is an approach to combine the large field of view of a fisheye camera with an efficient out-of-a-box dense stereo matching algorithm[...]"
- 3) We added a statement at Page 2, Line 22 in order to clarify that epipolar rectification is not a principal requirement for a dense stereo reconstruction:
"Although epipolar rectification is not required for a dense reconstruction in principle, many dense stereo algorithms require rectified images because computation is greatly simplified."

Referee Comment:

„Page 3, Line 3, Romps and Oktem also studied convective clouds in the following two references Romps and Oktem, Stereo photogrammetry reveals substantial drag on cloud thermals, GRL, 2015 Oktem and Romps, Observing atmospheric clouds through stereo reconstruction, Proceedings of SPIE – The International Society for Optical Engineering, March 2015“

Authors Response:

Thank you for the feedback. We have fully addressed this issue and have incorporated the suggested reference by integrating them into the related work section (Page 3, Line 14):

" To the best of our knowledge, only Hu et al. (2010) and Romps and Öktem (2015) used stereo vision to reconstruct a convective cloud."

Romps, D. M. and Öktem, R.: Stereo photogrammetry reveals substantial drag on cloud thermals, Geophysical Research Letters, 42, 5051–5057, 2015.

Referee Comment:

„In Section 3, the parameters such as theta and phi angles are only displayed in figures but are not introduced in the text nor in the captions. There are many parameters used in the equations, it maybe a good idea to list and define them in a separate table or introduce/explain them in the text.“

Authors Response:

We agree with the referee. In the text we corrected 'r(theta)' to 'theta' (Page 4, Line 21 and 22). The angle phi and the projection function r(theta) are now introduced on Page 4, Line 20-22:

" Each symmetric projection function $r(\theta)$ defines the distance between x' and the principal point x_c as a function of the zenith angle θ between the incoming projection ray and the optical axis as depicted in Figure 2 (a). Accordingly, the

coordinates of x on the image plane are a function of the azimuth angle φ and $r(\theta)$ and are given by[...]"

Referee Comment:

„In Section 3.3, it is claimed that rectification allows to use the complete image content of a fisheye image. It is not clear to me why the whole content of rectified image can be used but the whole content of the non-rectified image cannot. Besides, the distortion (stretching) introduced by the rectification is likely to severely limit the use of data beyond a certain theta.“

Authors Response:

We agree with the referee and we have corrected this issue. The image content of a non-rectified image can also be used, e.g. for feature-based matching. However, the matching algorithm we use requires epipolar rectified images, which allow to use out-of-the-box dense and efficient stereo methods as we mention on Page 2, Line 19-20. As we have fisheye images, we have to use a special rectification model and cannot use the classical perspective rectification using a homography (Page 6, Line 30):

" In the frame of pinhole-type cameras, epipolar image rectification refers to the computation and application of a homography which maps epipolar lines (projections of epipolar planes on the image plane) to image rows. In the omnidirectional camera model however, epipolar lines become epipolar curves due to the non-linear projection and thus cannot be mapped by a homography because of its line-preserving character. Therefore, we employ the rectification scheme following Abraham and Förstner (2005) which is sketched in Figure 4"

The used epipolar rectification allows to keep the complete image content, but also introduces some strong distortions at the image borders (Page 7, Line 4):

"However, epipolar rectification leads to lower accuracies at the margins as the image is stretched in these areas, cf. to Schneider et al. (2016)."

Referee Comment:

„Section 4, Line 14, “Dense stereo is advantageous when dealing with complex geometries but also effectively delivers reasonable results for image regions with low contrast”. I believe that this statement needs revision to clarify the point being made. I understand the clouds are considered as complex geometries but it is not clear to me how dense stereo is advantageous for these cases.

Authors Response:

We addressed this issue and revised this statement by giving more explanation in this paragraph (Page 10, Line 26-30):

"Dense stereo can be advantageous when dealing with complex and dynamic scenes that have limited texture, because it effectively delivers reasonable results for image regions with low-contrast. It propagates information from high-contrast regions into the low-contrasts regions assuming similar depth at nearby pixels with similar intensity. In such regions local methods may deliver few or no information leading to a sparse point cloud, which makes further analysis like segmentation or classification difficult."

Technical Corrections**Referee Technical Comment #1:**

Abstract, Line 8, “..of the a cloud ...”, “the” should be omitted.

Authors Response:

Agreed.

Referee Technical Comment #2:

Abstract, Line 10, there appears to be tense mismatch.

Authors Response:

Agreed: "We implemented and evaluated" has been changed to present tense.

Referee Technical Comment #3:

Introduction, Page 1, Line 22, "...cloud observation.." should be "observations"

Authors Response:

Agreed.

Referee Technical Comment #4:

Introduction, Page 2, Line 9, "...which both need to be", correct as "...both of which need to be..."

Authors Response:

Agreed.

Referee Technical Comment #5:

Page 3, Line 27, "..orientation consist of...", correct as "..orientation consists of..."

Authors Response:

Agreed.

Referee Technical Comment #6:

Page 4, equations with phi uses lowercase phi in one equation, and uppercase phi in

another, do they refer to different parameters?

Authors Response:

Thank you for pointing to this error, which has been corrected.

Referee Technical Comment #7:

Page 6, the last equation which is below Line 30, Is this the intended notation for e.g. $\text{atan2}(z_v, y_v)$ ”? I believe that the notation should be revised to clearly state the formula.

Authors Response:

Yes. The full notation has been inserted in the paper.

Referee Technical Comment #8:

Page 9, the first equation on the top, what is $\sin(\delta)$ ”?

Authors Response:

Thank you for identifying this is error. 'delta' has been changed to 'theta'.

Referee Technical Comment #9:

Page 10, Line 2, “..on the both..”, I recommend “...on both of the...”

Authors Response:

“..on the both..” changed to "...on both..."

Referee Technical Comment #10:

Page 10, Line 10, better separate the last statement into two sentences to make it more readable.

Authors Response:

Changed to:

"Especially the latter poses a problem in cloud photogrammetry. Hence, depending on the cloud situation stereo reconstruction has limitations."

Referee Technical Comment #11:

Page 11, Line 32, “...mounted in a box with ...”, is “with” redundant in this statement?,

Authors Response:

We removed the "with".

Referee Technical Comment #12:

Page 12, Line 3, “..(IDS, 2013).”, remove “.”

Authors Response:

Agreed.

Referee Technical Comment #13:

Page 12, Line 8, “..an reduced..”, correct as “..a reduced..”

Authors Response:

Agreed.

Referee Technical Comment #14:

Page 12, Line 17, “Further, the solid angle..”, correct as “Furthermore, ..”

Authors Response:

Agreed.

Referee Technical Comment #15:

Page 13, Line 9, correct “citepblender”

Authors Response:

Corrected: “(Blender Foundation, 2016)”

Referee Technical Comment #16:

Page 13, Line 26, is “..0.15 seconds...” supposed to be “... 15 seconds ...”

Authors Response:

This refers to the averaging time of a single range resolved scan (a single elevation value / beam), which takes 0.15 seconds. The whole scan (elevation angle from 15 degrees to 165 degrees) takes almost 1 minute. We added a sentence to clarify this (Page 15, Line 8-10):

“For our comparisons we average observations over an integration time of 0.15 seconds for each range resolved beam. A complete cross-section scan then takes approximately one minute for all elevation angles, that range between 15 degrees and 165 degrees.”

Point-to-Point response to: anonymous referee #2 from August 5th, 2016

We thank the referee for the important and valuable comments on the paper. We used them to correct the paper for the revised version. Our response to the referees comments are as follows:

Referee Comment:

"Being not an expert in the field of cloud observation, I would be interested to read a bit more about the significance of the work: what does this technique offer, compared to the existing ones? Is it a better resolution/accuracy, a larger area, reduced cost, more completeness? One could also describe it in terms of requirements: we want to see ..., but existing techniques only give ... and therefore we develop a new system expecting to get ... and here we evaluate it"

Authors Response:

The problem is that the mentioned instruments simply do not have the spatial and/or temporal resolution for near-instantaneous scans and thus representations of e.g. boundary layer clouds which are highly dynamic and comparatively small are quite limited. In case of a cloud radar they also lack sensitivity to cover all parts of a cloud.

In order to better motivate our contribution, we reformulated the respective paragraph in the Introduction (Page 2, Line 1-5):

"Current ground-based cloud observations are made primarily with cloud radars, lidars, lidar-ceilometers and infrared and microwave radiometers, all of which usually only sense clouds along a pencil beam; they record the 3D cloud evolution at time resolutions during which clouds already change significantly. For instance, a cross-section scan of a cloud radar takes up to one minute with a beam width of about 0.6 degrees; moreover its sensitivity does not not allow

to detect the cloud boundaries. A lidar-ceilometer observes the cloud base height with high temporal resolution, but only as zenith point-measurement."

and Page 1, Line 27:

" A more complete and consistent cloud shape can be used in radiative transfer applications where cloud geometry is modeled explicitly. Cloud evolution studies can benefit from the larger geometric data basis regarding segmentation and classification of individual clouds, tracking and visualization, making further analysis more effective."

Regarding the novelty and contribution of our approach in the field of cloud photogrammetry, we state in Page 2, Lines 17-24:

"The main contribution of this paper is an approach to combine the large field of view of a fisheye camera with an efficient out-of-a-box dense stereo matching algorithm in order to obtain consistent and detailed cloud geometries above the area around the cameras.[...]. In contrast to regular feature-based methods used in previous studies on cloud photogrammetry, dense methods seek a correspondence for every pixel in the stereo images, leading to a dense 3D point cloud. At the same time dense stereo methods often impose spatial consistency constraints, which allows us to obtain more reliable correspondences in low-contrast image regions, which are typical for clouds, than sparse feature-based methods"

We also state on Page 3, Line 16, that up to now, fisheye cameras have only been used to derive cloud base height, but not for a recovery of complete 3D cloud geometries of e.g. convective boundary layer clouds:

"Experiments involving sky imagers focused on the derivation of the cloud base height."

Referee Comment:

"There are several challenges being addressed, concerning the „difficult“ geometry of fisheye lenses, the size of the setup (with a baseline of 300m), the use of automatic (dense) matching with „fuzzy“ objects (clouds). All is well

explained, but it is not always completely clear how it relates to the state of the art and where is the novelty – I suppose it is in the application of dense matching to clouds, but in that case the results could be analyzed a bit more exactly there.(Furthermore, I liked the used of stars in the exterior orientation)."

Authors Response:

We updated some passages in section 2 and 5 in order to put our methods and decisions better in the context of the cited methods.

The general contribution and novelty of this paper is mentioned in the introduction, which has also undergone some changes (Page 2, Lines 19-31). We now also state on Page 3, Line 27-31:

" In contrast to previous studies, we used a dense stereo method to recover a dense 3D cloud geometry (Figure 1). Dense stereo methods obtain much more geometric information than feature-based methods, especially in image regions with low contrast, which is a general problem in cloud photogrammetry. This additional geometric information can prove beneficial in cloud evolution and radiation closure studies where the cloud geometry is modeled explicitly."

A state-of-the-art regarding the stereo matching depends primarily on the reconstruction application (clouds in our case) and the number of cameras involved. Multi-view reconstruction for example offers many more constraints on the scene geometry than two cameras and makes visibility and radiometric consistency assumptions more feasible. In this field, feature-based methods (at least as a starting point) are the best choice because each feature / point can be refined towards a consistent maximum-likelihood estimate.

A two-camera stereo application has only the simple epipolar constraint and hence - assuming a relatively small baseline compared to the whole 3D scene - further constraints do not have the same significance than in multi-view applications with a variety of viewports.

Another aspect is, that clouds themselves are fuzzy objects and thus do not have clearly defined 3D boundaries that can be refined the same way as solid object boundaries can be. These implications cause a shift of our focus from geometric accuracy towards geometric completeness, although we do not think that there is a significant trade-off (compare statement about feature-based methods and dense stereo methods in section 4 or in the introduction).

Referee Comment:

"Only in the evaluation section it becomes apparent what one had in mind concerning the size of the area to be measured: results are shown up to 4 km away from the cameras at two different heights (Fig 11) . "

Authors Response:

We updated the abstract and the introduction as well as the caption of Figure 1 to solve this deficit:

"We present a novel approach for dense 3D cloud reconstruction using two hemispheric sky imagers with fisheye lenses in a stereo setup above 10×10 km²."

and

"Two cameras with a spatial displacement and simultaneous time of exposure provide the necessary information for a 3D reconstruction within an area of about 10×10 km² around the cameras."

Referee Comment:

"The accuracy gets rather poor at larger distances, which may be due to the baseline of only 300m (but a larger one might affect matching performance). Some more discussion about this would be welcome."

Authors Response:

We revised section 5.2 extensively and discussed the implications of a larger / shorter baseline or different distances of the object on the reconstruction

uncertainty and the matching performance. For example, on Page 13, Line 16, we state now:

"However, larger baselines (or disparities) usually affect the stereo matching because increasing parts of the object might not be visible by both cameras. Additionally, the object will have significantly different geometric appearance in each camera."

We also want to refer to our statement in section 5.1 where we also discuss this issue at Page 12, Line 13-16:

"Compared to previous studies that mention a baseline between 500 m (Allmen and Kegelmeyer (1996)) and 900 m (Öktem et al. (2014)), this is a rather short distance and results in higher geometric uncertainty of the estimated 3D points on the clouds, but reduces occlusions and enhances the ratio of mutually visible cloud regions in both images"

Referee Comment:

"By the way, what is the area in other Figures , like 15 and 17?"

Authors Response:

Since the baseline is constant the main area of reconstruction does not change.

We added the distance information to the figure captions, e.g. for Fig. 16:

"3D reconstruction of cumulus mediocris at 24 July 2014 approximately 3 km away [...]"

Referee Comment:

"The paper mentions using a third camera, but that would not be covered by the current setup of resampling the images to epipolar geometry (would it?). This implication should be mentioned."

Authors Response:

We added a reference to Heinrichs and Rodehorst (2006), who gives a solution for perspective cameras. An adaption to omnidirectional cameras has, to the best of our knowledge, not been addressed yet, but seems to be possible, but we didn't investigate it. The revised text is in Page 13, Line 30:

"A successful integration of a third camera into the dense stereo matching scheme including epipolar rectification is explained for perspective cameras in Heinrichs and Rodehorst (2006). An adaption to omnidirectional cameras has, to the best of our knowledge, not been addressed yet, but seems to be possible."

References:

[Heinrichs 2006]

Matthias Heinrichs and Volker Rodehorst. "Trinocular rectification for various camera setups." *Symp. of ISPRS Commission III-Photogrammetric Computer Vision PCV*. Vol. 6. 2006.

List of all relevant changes (Pages and Lines w.r.t. the marked-up text):

Description	Page, Line
Updated Abstract to clarify of our approach	1
Updated Introduction for a better explanation of current cloud remote sensing techniques	2,7
Updated Introduction to outline the novelty of our approach and its use	2,27
Added a sentence that epipolar rectification is not mandatory for dense stereo	2,32
Added reference in the Related Work section (Romps and Öktem (2015))	3,29
Added reference in the Related Work section (Nguyen and Kleissl (2014))	4,7
Added introduction of working variables phi and theta	5,9
Replaced 'principal distance' by 'camera constant' for consistency	5,14
Replaced 'projection center' by 'principal point'; could be misleading	5,15
Added variable declaration of A1, A2 and A3 of the polynomial coefficients (Brown)	5,24
Added a "distortion-corrected" to point out that x is not the same as \tilde{x}	6,1
Corrected an error in the upper formula (spherical coordinates)	6,3
Corrected the formula which was still using the distortion terms (triangle u/v)	6,5
Added a general classification of epipolar rectification to section 3.3	7,23
Added the full notation of "atan2()" to the formula	8,7
Introduced the variable x''_V	8,8
Corrected the triangulation formula: ψ_r should be ψ'_r (Figure 5)	9,7
Section 3.5.1 and 3.5.2 are now one level higher: 3.5 and 3.6	9,10
Corrected an error in the equation for the coordinates of stars ($\sin(\delta)$ to $\sin(\theta)$)	10,19
Updated expression of the cost function to be minimized	10,27
Updated expression of the cost function to be minimized	11,12
Updated the paragraph regarding dense stereo and complex cloud geometries	11,31
Added context regarding the baseline of our stereo setup	13,16
Rearranged the order of discussion in section 5.2	14-15

Added further discussion of depth accuracy related to measurement errors and baseline	14,1
Inserted a more detailed description of our choice for the stereo setup	14,21
Added a sentence to clarify the mode of operation of the cloud radar	16,29
Added a remark regarding the outlook	19,5
Added a caption remark regarding the typical range of our reconstructions (Figure 1)	22
Added a symbol for the projection / camera center in Figure 2 ('C')	22
Figure 2 now shows the <i>Optical Axis</i> of a camera	22
Added a new figure (Figure 12) to illustrate the geometric uncertainty at two distances	27
Added the distance of the visualized cloud in the caption of Figure 16	31
Added the distance of the visualized cloud in the caption of Figure 18	32

Cloud Photogrammetry with Dense Stereo for Fisheye Cameras

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Abstract. ~~In this paper, we present our~~ We present a novel approach for dense 3D cloud reconstruction above an area of $10 \times 10 \text{ km}^2$ using two hemispheric sky imagers with fisheye lenses in a stereo setup. We examine an epipolar rectification model designed for fisheye cameras, which allows the use of efficient out-of-the-box dense matching algorithms designed for classical pinhole-type cameras to search for correspondence information at every pixel. The resulting dense point cloud
5 allows to recover a detailed and more complete cloud morphology, compared to previous approaches that employed sparse feature-based stereo or assumed geometric constraints on the cloud field. Our approach is very efficient and can be fully automated. From the obtained 3D shapes, cloud dynamics, size, motion, type and spacing can be derived and used e.g. for radiation closure under cloudy conditions.

Fisheye lenses follow a different projection function than classical pinhole-type cameras ~~,which and~~ provide a large field of
10 view with a single image, ~~but also renders~~. However, the computation of dense 3D information ~~more complicated, such that we cannot rely entirely on is more complicated and~~ standard implementations for dense 3D stereo reconstruction cannot be easily applied.

~~In this work, we examine the epipolar rectification model, which allows the use of dense matching algorithms designed for classical perspective cameras to search for disparity information at every pixel.~~ Together with an appropriate camera calibration,
15 which includes internal camera geometry and global position and orientation of the stereo camera pair, we ~~can use the disparity information use the correspondence information from the stereo matching~~ for dense 3D stereo reconstruction of ~~the a cloud and thus estimate its shape. From the obtained 3D shapes, cloud dynamics, size, motion, type and spacing can be derived and used for radiation closure under cloudy conditions.~~ clouds located around the cameras.

We ~~implemented and evaluated~~ implement and evaluate the proposed approach using real world data and present two case
20 studies. In the first case, we validate the quality and accuracy of the method by comparing the stereo reconstruction of a stratocumulus layer with ~~the~~ reflectivity observations measured by a cloud radar and the cloud base height estimated from a Lidar-ceilometer. The second case analyzes a rapid cumulus ~~convection~~ evolution in the presence of strong wind shear.

1 Introduction

Ground-based photogrammetry has ~~the great a large~~ potential to complement ~~current cloud observations~~ cloud observations
25 from classical remote sensing via radiometers, radars and lidar due to its high spatial and temporal resolution. So-called

hemispheric sky imagers ~~record images with high temporal and spatial resolution and~~ provide a complete hemispheric view of the cloudy sky at arbitrary time intervals. Up to now, such imagers are predominantly used only for the derivation of cloud cover or cloud type classification. The derivation of additional information related to cloud size and extension including their temporal development, especially of convective boundary layer clouds, would provide valuable information for radiation closure studies under cloudy conditions and can be used for validation of LES-scale cloud simulations, e.g. ~~like from~~ the new ICON model (Zängl et al. (2015)).

Current ground-based cloud ~~observation are made~~ observations are made primarily with cloud radars, lidars, lidar-ceilometers and infrared and microwave radiometers, all of which usually only sense clouds along a pencil beam, ~~but are not able to view the whole cloud over time intervals of seconds~~; they record the 3D cloud evolution at time resolutions during which clouds ~~do not already~~ change significantly. ~~Recent work~~ For instance, a cross-section scan of a cloud radar takes up to one minute with a beam width of about 0.6° ; moreover its sensitivity does not allow to detect the cloud boundaries. A lidar-ceilometer observes the cloud base height with high temporal resolution, but only as zenith point-measurement. Recent works show that stereo photogrammetry of clouds may help to close this gap to some extent due to the capability of cameras to capture the visible parts of clouds instantaneously with a high spatial and temporal resolution. The resulting 3D cloud geometries can then be combined with the observations from other instruments to provide valuable information for cloud reconstruction.

In this paper we investigate the potential ~~of deriving to derive~~ the 3D ~~structure morphology~~ of clouds with two hemispheric sky imagers, cf. to Figure 1. Fisheye cameras provide a large field of view, have robust mechanics and are very cost effective. Two cameras with a spatial displacement and simultaneous time of exposure provide the necessary information to perform for a 3D reconstruction within an area of about $10 \times 10 \text{ km}^2$ around the cameras. Such stereo techniques are a well studied field in photogrammetry and computer vision and early approaches of cloud photogrammetry date back to the late 19th century (Koppe, 1896).

3D stereo reconstruction is based on triangulation. Knowing the two camera orientations and the direction vector (baseline) between the cameras, each pair of corresponding image points can be back-projected into ray directions which intersect in the mapped 3D point. This requires accurately known parameters for the interior orientation, e. g. focal length and lens distortion, and also accurate knowledge of the exterior orientation, namely the position and orientation of the cameras in space, ~~which both both of which~~ need to be determined by a calibration procedure.

The main contribution of this paper is ~~the combination of a dense stereo correspondence technique and a large scale stereo setup of hemispheric cameras~~ an approach to combine the large field of view of a fisheye camera with an efficient out-of-a-box dense stereo matching algorithm in order to ~~derive dense 3D cloud geometries, especially for convective clouds.~~ obtain consistent and detailed cloud geometries above the area around the cameras. We achieve this by employing an epipolar rectification technique on the recorded images that is designed for fisheye cameras and is required to apply the dense stereo correspondence algorithm used in this study. Although epipolar rectification is not required for a dense reconstruction in principle, many dense stereo algorithms require rectified images because computation is greatly simplified. In contrast to regular feature-based methods used in previous studies on cloud photogrammetry, dense methods seek a correspondence for every pixel in the stereo images, ~~while at leading to a dense 3D point cloud. At the same time imposing dense stereo methods~~

often impose spatial consistency constraints, which is an important aspect in our application due to the lack of contrast in the appearance of clouds.

As a prerequisite, we employ an epipolar rectification technique on the recorded images, that is designed for fisheye stereo cameras and is needed to apply the dense stereo correspondence algorithm. The obtained correspondence information can then be used for triangulation and allows us to obtain more reliable correspondences in low-contrast image regions, which are typical for clouds, than sparse feature-based methods. A more complete and consistent cloud shape can be used in radiative transfer applications where cloud geometry is modeled explicitly. Cloud evolution studies can benefit from the larger geometric data basis regarding segmentation and classification of individual clouds, tracking and visualization, making further analysis more effective. Once the system is calibrated our approach runs fully automated and provides dense 3D reconstruction geometries over large parts of the hemisphere observed by the fisheye cameras.

The paper is organized as follows. In Sec. 2, we discuss previous studies and their contributions to the field of cloud photogrammetry. In Sec. 3, we describe the fisheye camera model, the applied camera calibration techniques, as well as the epipolar rectification method and the triangulation. Sec. 4 introduces the employed dense stereo algorithm to obtain corresponding image points. Sec. 5 presents our stereo setup, a geometric uncertainty analysis and two case studies of our reconstructions stereo reconstruction case studies. One case shows a reconstructed stratocumulus layer which serves as a validation for the achieved geometric accuracy including comparisons with lidar-ceilometer and cloud radar observations. The second case analyzes the cloud development within under strong convection and wind shear and illustrates the quality of the cloud morphology reconstruction.

2 Related Work

Recent studies employed cameras with wide-angle lenses in a stereo setup to recover 3D information of clouds. For example, using feature-based methods, Seiz (2003) exploited a stereo pair of consumer cameras with a 800 m baseline to derive the cloud base height within a field of view of about 100° with errors well below 5 %. Hu et al. (2010) used a stereo camera system with a spatial offset of 1.5 km oriented towards the Santa Catarina Mountains near Tuscon, Arizona to reconstruct the evolution, to observe the diurnal cycle of orographic convection over the diurnal cycle in three dimensions. Recently, Öktem et al. (2014) used a stereo camera setup for the observation of maritime clouds near Biscayne Bay, Florida, with a distance baseline of 873 m between the cameras. Their results show a good agreement with heights obtained with a lidar, yielding errors of mostly below 2 % for shallow clouds and up to 8 % for high cirrocumulus clouds. They also compared the derived cloud motion speed of the individual cloud layers with wind speed measurements from radiosondes.

In Öktem et al. (2014) they extend their approach to marine convection. The studies of Seiz (2003) and Öktem et al. (2014) show that an accurate cloud reconstruction is possible with a stereo camera system. To the best of our knowledge, only Hu et al. (2010) and Roms and Öktem (2015) used stereo vision to reconstruct a convective cloud.

Experiments involving sky imagers focused on the derivation of the cloud base height.

Allmen and Kegelmeyer (1996) used two Whole-Sky-Imagers (WSI) to derive cloud base heights. ~~They experienced that~~ a standard ordinary stereo matching method failed due to the rather large distance between the cameras of 5 km. ~~They report that more~~ More than 50 % of the estimated cloud heights deviated ~~with by~~ less than 5 % from heights obtained by the lidar-ceilometer. Kassianov et al. (2010) compared cloud base heights derived from two virtual and two real fisheye cameras located at the ARM site in the Southern Great Planes, with a ~~spatial offset of the cameras of~~ baseline of 540 m. They used stochastic simulations to create a virtual cloud field and used the virtual fisheye projections for stereo vision. Comparisons with micropulse lidar observations showed that typical errors were about 10 % for low-level clouds up to 2 km high. Recently, Nguyen and Kleissl (2014) used a plane-sweep-like approach with a baseline of 1230 m between the sky imagers. Although a plane-sweep technique is also capable to produce a dense 3D geometry (Häne et al. (2014)), their implementation assumes

10 a horizontal cloud field without any vertical structures and is aimed at computing the cloud base height for short-term solar radiation forecasting.

In our work, we use two hemispheric sky imagers in a stereo setup ~~and apply dense stereo methods to recover a~~ with a baseline of 300 m. In contrast to previous studies, we use a dense stereo method to recover dense 3D cloud ~~geometry~~ geometries (Figure 1). Dense stereo methods obtain more geometric information than feature-based methods, especially in image regions

15 with low contrast, which is a general problem in cloud photogrammetry. This additional geometric information can prove beneficial in cloud evolution and radiation closure studies where the cloud geometry is modeled explicitly. We evaluate our results by comparing them with cloud base height observations from a lidar-ceilometer and reflectivity profiles of a cloud radar.

3 Camera Calibration and Stereo Reconstruction

In this section, we describe the projection model for fisheye cameras and we formulate the geometric relationship between

20 two sky imagers, which is required for 3D stereo reconstruction. We also introduce an epipolar image rectification scheme for fisheye stereo cameras that allows to identify corresponding image points between the two images using a dense stereo matching algorithm. Finally, we describe the employed camera calibration methods and the triangulation of 3D points using corresponding image points in the epipolar rectified images.

3.1 Interior Orientation of a Fisheye Camera

25 The interior orientation of a camera describes the camera specific projection of light onto the image plane. In ~~case of~~ a central projection each ~~sensed~~ ray passes through ~~a certain the~~ projection center in one particular direction. ~~When the geometry of the camera is known, that is, when the camera is calibrated, one can compute this direction for each measured~~ A camera is calibrated when its calibration parameters are known and this direction can be computed for every image point.

The camera model contains a projection function, which should be close to the projection of light in the optics. The interior

30 orientation ~~consist~~ consists of the camera calibration parameters of this model, describing the camera specific projection on the image plane. The projection can be split up into a mapping of a 3D point \mathbf{P} to a 2D point \mathbf{x}' on the model image plane, and a mapping of \mathbf{x}' to \mathbf{x} to the actual pixel coordinates on the sensor plane (Figure 2). While most cameras follow the pinhole

camera model (Sonka et al., 1999; Stockman and Shapiro, 2001), fisheye cameras have lenses with a different projection function and follow the omnidirectional camera model (Kannala and Brandt, 2006; Bakstein and Pajdla, 2002)., visualized in Figure 2 visualizes the omnidirectional camera model. Each projection ray passes through the projection center C and intersects the image sphere in the point \mathbf{x}'' , which can be considered as determines the ray direction. The optical axis intersects the image plane in the principal point $\mathbf{x}_0 \mathbf{x}_C$.

The ray direction \mathbf{x}'' can be mapped to \mathbf{x}' on the image plane using e.g. one of used projection functions $r(\theta)$ provided by Abraham and Förstner (2005). Each symmetric projection function $r(\theta)$ defines the distance between \mathbf{x}' to and the principal point \mathbf{x}_0 only \mathbf{x}_C as a function of the zenith angle $r(\theta) - \theta$ between the incoming projection ray and the optical axis as depicted in Figure 2 (a). Accordingly, the pixel-coordinates of \mathbf{x}' are on the image plane are a function of the azimuth angle φ and $r(\theta)$ and are given by

$$\mathbf{x}' = \begin{bmatrix} \cos(\varphi)r(\theta) \\ \sin(\varphi)r(\theta) \end{bmatrix}. \quad (1)$$

The mapping of \mathbf{x}' in Cartesian image coordinates to \mathbf{x} in pixel coordinates is usually described as an affine transformation

$$\mathbf{x} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} c & s & u_0 \\ 0 & c & v_0 \end{bmatrix} \begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix} \quad (2)$$

with the parameter of the interior orientation, namely the principal distance which depends on the camera constant c , the shear factor s and the principal point $\mathbf{x}_0 = (u_0, v_0)^\top$, i.e. the projection-center-principal point \mathbf{x}_C in pixel coordinates. Note that the origin of the sensor coordinate system lies in the upper left corner of the image as depicted in Figure 2 (b).

Due to lens imperfections real camera projections do not follow a projection model perfectly. In our case the The lenses of the sky imagers are camera might be shielded by an additional glass dome as in our setup which additionally refracts the light before it enters the lens. Radial symmetric distortions result in either a barrel or pillow like stretching or bending of the image with increasing distance from the projection-center-principal point. Such distortions can be compensated by adding even-powered polynomials to the radial distance function following Brown (1971)

$$\Delta u = L(\hat{r})\hat{u} = A_1 \hat{u} \hat{r}^2 + A_2 \hat{u} \hat{r}^4 + A_3 \hat{u} \hat{r}^6 \quad (3)$$

$$\Delta v = L(\hat{r})\hat{v} = A_1 \hat{v} \hat{r}^2 + A_2 \hat{v} \hat{r}^4 + A_3 \hat{v} \hat{r}^6 \quad (4)$$

with $\hat{u} = u - u_0$, $\hat{v} = v - v_0$ and $\hat{r} = \sqrt{\hat{u}^2 + \hat{v}^2}$. A_1, A_2 and A_3 denote the respective coefficients of the polynomial.

In summary, we formulate the mapping into (distorted) image point coordinates $\tilde{\mathbf{x}} = (\tilde{u}, \tilde{v})^\top$ on the sensor plane as

$$\begin{aligned} \tilde{u} &\equiv c \cos \varphi r(\theta) + s \sin \varphi r(\theta) + u_0 + \Delta u \\ \tilde{v} &\equiv c \sin \varphi r(\theta) + v_0 + \Delta v \end{aligned} \quad (5)$$

~~and the~~ The reverse mapping of a distortion-corrected image point \mathbf{x} in pixel coordinates to the 3D direction vector \mathbf{x}'' with unit length is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \approx \begin{bmatrix} \cos \varphi \sin \theta \\ \sin \varphi \sin \theta \\ \cos \theta \end{bmatrix} \approx \begin{bmatrix} \frac{x'}{r} \cos r \\ \frac{y'}{r} \cos r \\ \sin r \end{bmatrix}$$

where x' and y' are normalized image coordinates

$$\begin{aligned} x' &= \frac{u - \Delta u - u_0 - s(u - u_0)}{c} \frac{u - u_0 - s(u - u_0)}{c} \\ y' &= \frac{v - \Delta v - v_0}{c} \frac{v - v_0}{c} \end{aligned}$$

and $r = \sqrt{x'^2 + y'^2}$ the respective value of the radial projection function $r(\theta)$. The equidistant projection $r(\theta) = \theta$ fits the projection of our sky imagers best. ~~We will~~ Sec. 3.5 and Sec. 3.6 describe the calibration procedure to determine the parameters of interior orientation with distortion parameters as well as the calibration of a stereo camera pair ~~in~~.

10 3.2 Exterior Orientation and Epipolar Geometry

The omnidirectional camera model refers to the local camera coordinate system with the projection center as the origin and the sensor plane defining its orientation. The exterior orientation of a camera, which ~~consist of~~ is described by three rotation angles and three translation shifts, is described in a common world reference system Ω_W and allows to derive the geometric relationship between two or more cameras. We choose one camera as the reference camera, which is considered as the left camera and the other as the right camera, which simplifies the following notation and avoids misconceptions. The choice of the reference camera has no impact on the reconstruction results.

Figure 3 illustrates the principal stereo configuration with two hemispheric cameras, making the world reference system Ω_W and the two camera reference systems Ω_L and Ω_R explicit. Let \mathbf{C}_L be the world coordinates of the left camera and \mathbf{P}_L an object point in the left camera reference frame. The transformation of \mathbf{P}_L into world coordinates reads ~~then~~ as

$$20 \quad \mathbf{P} = (R_L \mathbf{P}_L) + \mathbf{C}_L \quad (6)$$

with the rotation matrix $R_L = R_x(\alpha_L) R_y(\beta_L) R_z(\gamma_L)$ and $\mathbf{C}_L \in \mathbb{R}^3$. Here α_L, β_L and γ_L are the Eulerian angles (roll, pitch, yaw) and $R_x(\alpha_L), R_y(\beta_L)$ and $R_z(\gamma_L)$ the respective rotation matrices. Considering R_L and R_R as the rotation matrices and \mathbf{C}_L and \mathbf{C}_R as the world coordinates of the left and right camera, we obtain the relative orientation ~~from the left to~~ between the left and the right camera ~~with~~ via a rotation matrix R and the baseline vector \mathbf{t} ~~with~~ via

$$25 \quad R = R_L^T R_R \quad \text{and} \quad \mathbf{t} = R_L^T \mathbf{C}_R - \mathbf{C}_L. \quad (7)$$

The determination of an accurate relative pose is crucial, as errors in the estimated exterior orientations may sum up to larger errors in the relative orientation which compromises the image correspondence algorithm and the triangulation of 3D point coordinates as investigated by Hirschmüller and Gehrig (2009).

The two camera centers \mathbf{C}_L , \mathbf{C}_R and the object point \mathbf{P} span the ~~epipolar plane~~epipolar plane. This geometry can be expressed with the coplanarity equation and holds when

$$\mathbf{x}_L''^\top E \mathbf{x}_R'' = 0 \quad \text{with} \quad E = [\mathbf{t}]_\times R, \quad (8)$$

where E is the essential matrix obtained by a matrix multiplication of R with the skew symmetric matrix $[\mathbf{t}]_\times$ of \mathbf{t} .

- 5 Given two direction rays \mathbf{x}_L'' and \mathbf{x}_R'' of corresponding image points \mathbf{x}_L and \mathbf{x}_R in the left and the right camera images, $E^\top \mathbf{x}_L''$ defines the normal vector of the epipolar plane spanned by \mathbf{x}_L'' and \mathbf{t} , and hence requires its correspondence \mathbf{x}_R'' to lie on the intersection circle between the epipolar plane and the image sphere. The same holds for $E \mathbf{x}_R''$ and \mathbf{x}_L'' . In case of deviations from this constraint, Eq. (8) defines the cosine of the angle between the two epipolar planes spanned by \mathbf{t} together with $E^\top \mathbf{x}_L''$ and $E \mathbf{x}_R''$ respectively, which we are using as an error measure to estimate E using image point correspondences—
- 10 ~~∴ The essential matrix E can be expressed with five parameters, therefore has five degrees of freedom (independent parameters,~~ two for the baseline direction \mathbf{t} and three for the rotation angles). We use an adapted version of the direct method of Longuet-Higgins (1981) to compute E with eight correspondences, which exploits that the eigenvalues of E are $\lambda_1 = \lambda_2 = 1$ and that we use spherically normalized ray directions.

- The estimated essential matrix E can be decomposed to obtain the relative rotation R and baseline vector \mathbf{t} , which can be
- 15 used for triangulation and epipolar image rectification, as described in Sec. 3.3. However, a 3D reconstruction based on R and \mathbf{t} alone takes place only in the coordinate system of the reference camera and only up to scale because of the scale ambiguity. A meaningful reconstruction in world coordinates requires the absolute length of the baseline (distance between the cameras) and the absolute orientation of the reference camera. ~~presents~~ Sec. 3.5 and Sec. 3.6 present methods to estimate both, the parameters of exterior and interior orientation.

20 3.3 Epipolar Rectification

- Once the epipolar geometry and the interior orientation is known, the input images can be transformed ~~so in such a way~~ that corresponding image points lie on the same image row, which reduces the search for corresponding image points from two dimensions (image) to one (image row). In the frame of pinhole-type cameras, epipolar image rectification refers to the computation and application of a homography which maps epipolar lines (projections of epipolar planes on the image plane)
- 25 to image rows. In the omnidirectional camera model however, epipolar lines become epipolar curves due to the non-linear projection and thus cannot be mapped by a homography because of its line-preserving character. Therefore, we employ the rectification scheme following Abraham and Förstner (2005) which is sketched in Figure 4. The epipolar rectification allows to rectify a fisheye image over a broad spectrum of the angle θ , which allows to use the complete image content of a fisheye image, which is not possible via perspective rectification. However, epipolar rectification leads to lower accuracies at the margins as
- 30 the image ~~content~~ is stretched in these areas, cf. to Schneider et al. (2016). ~~The epipolar rectification scheme following is shown in—~~

~~Note, that the following formulas~~ The following derivations with respect to β and ψ are only valid for an epipolar rectified image pair. For each real camera we can define a virtual camera (subscript v), such that the virtual cameras are in a canonical

stereo setup, i.e. both have a common x -axis, equal-orientation-the same orientation ($R_{L,V} = R_{R,V} = I$) and are only shifted along the virtual x -axis $\mathbf{t}_V = (\|\mathbf{t}\|, 0, 0)^\top$. An object point \mathbf{P}_V in the virtual world coordinate system is then defined by the angle β which denotes the respective epipolar plane, and the two angles ψ_L and ψ_R that define the angle of the projection ray within the epipolar plane ~~see~~ (Figure 4 (b)). Based on this geometry, we are able to define a rectification scheme ~~see~~

- 5 (Figure 4 (d), ~~to cover~~), which covers the whole 3D space: the image rows correspond to the angle β , which represents the rotation of the epipolar plane, while the image columns represent the respective angles ψ_L and ψ_R in the rotated epipolar plane

$$\mathbf{x}'_V = \begin{bmatrix} \psi \\ \beta \end{bmatrix} = \begin{bmatrix} \text{atan2}(z_V, y_V) \\ \text{atan2}(x_V, y_V) \end{bmatrix} \quad \text{with} \quad \mathbf{x}''_V = \underline{(x_V, y_V)}, \begin{bmatrix} x_V \\ y_V \\ z_V \end{bmatrix}, \quad \text{atan2}(z_V, y_V) = \begin{cases} \underline{\arctan \frac{z_V}{y_V}} & \text{for } y_V > 0, \\ \underline{\text{sgn}(z_V) \cdot \frac{\pi}{2}} & \text{for } y_V = 0, \\ \underline{\arctan \frac{z_V}{y_V} + \pi} & \text{for } y_V < 0 \wedge y_V \geq 0, \\ \underline{\arctan \frac{z_V}{y_V} - \pi} & \text{for } y_V < 0 \wedge y_V < 0 \end{cases}$$

where \mathbf{x}''_V corresponds to a projection ray within the frame of a virtual camera.

- Let \mathbf{x}''_L and \mathbf{x}''_R be the projection rays of an object point \mathbf{P} in the left and the right camera coordinate system (Ω_L, Ω_R) respectively, and $\mathbf{x}''_{L,V}$ and $\mathbf{x}''_{R,V}$ the corresponding projection rays in the virtual coordinate systems ($\Omega_{L,V}, \Omega_{R,V}$). The mapping between the real and virtual coordinate system follows a two-step procedure: In the first step, \mathbf{x}''_L and \mathbf{x}''_R are mapped from the local camera coordinate systems (Ω_L, Ω_R) to the world coordinate system Ω_W . If we do not have knowledge about the world coordinate system Ω_W , we choose $\Omega_W = \Omega_L$. Given From the essential matrix E , we ~~are able to~~ extract the rotations $R_L = I$ and $R_R = R$, which map from camera coordinates to world coordinates. This leads to an equal coordinate system orientation, see step 1 in Figure 4 (a). In the second step, we ~~have to~~ construct an appropriate rotation matrix R_V in order to align each systems x -axis with the baseline \mathbf{t} , see step 2 in Figure 4 (a).

- Since the matrix columns of R_V are the images of the base vectors \mathbf{e}_i , the first column is the normalized baseline vector. We can freely choose the other two coordinate axes as long as they form an orthonormal system, because each realization aligns the x -axis with the baseline. This means, that the rectification scheme is defined up to a rotation about the baseline \mathbf{t} , which corresponds to a shift of the range of the angle β and a vertical translation in the rectified image. We define the virtual y -axis in the x - y -plane of the world coordinate system, which also determines the virtual z -axis.

Thus we finally get

$$R_V^{-1} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \quad \text{with} \quad \begin{aligned} \mathbf{e}_1 &= \mathbf{t} \|\mathbf{t}\|^{-1}, \\ \mathbf{e}_2 &= (-y_T, x_T, 0)^\top \|(-y_T, x_T, 0)\|^{-1}, \\ \mathbf{e}_3 &= \mathbf{e}_1 \times \mathbf{e}_2. \end{aligned}$$

- Given the angular information β , ψ_L and ψ_R as well as principal distance-the camera constant c , we get the ~~final~~-rectified image coordinates by

$$\begin{aligned} u_V &= c \psi + u_{0,V} \\ v_V &= c \beta + v_{0,V} \end{aligned} \quad \text{with} \quad u_{0,V} = c \pi/2 \quad \text{and} \quad v_{0,V} = c \pi/2.$$

The reverse mapping, from rectified image coordinates to world coordinates is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_V \begin{bmatrix} \sin(u_V^*) \\ \cos(u_V^*) \sin(v_V^*) \\ \cos(u_V^*) \cos(v_V^*) \end{bmatrix} \quad \text{where} \quad \begin{aligned} u_V^* &= (u_V - u_{0,V})/c \\ v_V^* &= (v_V - v_{0,V})/c. \end{aligned} \quad (9)$$

3.4 Triangulation for 3D-Reconstruction

Having corresponding image points \mathbf{x}_L and \mathbf{x}_R identified in the image rows of the epipolar rectified images, \mathbf{x}_L'' and \mathbf{x}_R'' can be directly derived from $\mathbf{x}_{L,V}$ and $\mathbf{x}_{R,V}$ using the reverse mapping of the rectification scheme of Eq. (9). Due to the rectification the ray directions are guaranteed to lie in the 3D epipolar plane and do intersect. Considering the geometry shown in Figure 5 we identify the relation $s \cdot \sin(\psi_R) = b \cdot \sin(\gamma) \cdot s \cdot \sin(\psi_R + \frac{\pi}{2}) = b \cdot \sin(\gamma)$. With $\gamma = \psi_L - \psi_R$ and $\mathbf{P} = s \cdot \mathbf{x}_L''$ we have

$$\mathbf{P} = b \left(\frac{\sin(-\psi_R)}{\sin(\psi_L - \psi_R)} \frac{\sin(\psi_R - \frac{\pi}{2})}{\sin(\psi_L - \psi_R)} \right) \mathbf{x}_L''. \quad (10)$$

3.5 Calibration and Parameter Estimation

In the following we present our procedure to estimate the parameters of the interior and exterior orientation, introduced in and 3.2.

3.4.1 Parameters of the Interior Orientation

3.5 Parameters of the Interior Orientation

For the estimation of the parameters of the interior orientation in Eq. (5) we employ a test field with markers that encode a geometric relationship. Such a test field can be a sophisticated setup in a laboratory (Seiz, 2003) or - as in our case - a pattern printed or fixed on a plane or inside an open half-cube as depicted in Figure 6. The calibration generally proceeds in two steps: The first step provides sample images of the pattern in different poses covering the field of view. In-For each image an image processing routine detects and extracts the image coordinates of the pattern geometry. In the second step, the extracted image points are used to estimate the optimal parameters of the camera model with an adjustment procedure.

We employ a software developed by Abraham and Hau (1997), that accepts input images of a calibration cube with a fixed white dotted pattern. Each inner cube side has a fingerprint pattern to make sure the detected dots are properly identified as lying on the x-, y- or z-plane, which determines their corresponding absolute 3D coordinates with respect to the cube reference system. The extraction stage results in a set of correspondences between 2D image points and 3D cube points, which then-are are then used in a nonlinear bundle adjustment that iteratively minimizes the reprojection error between the observed image points and the reprojections of the 3D points of the pattern using the respective parameter estimation according to Eq. (5) and (6).

3.5.1 Parameters of the Exterior Orientation

3.6 Parameters of the Exterior Orientation

First, we describe how to estimate the absolute location and orientation of each camera in the world reference system. This information ~~can then be~~ is then used to derive a first estimate of the essential matrix E^* , which will then be iteratively refined

5 using point-feature correspondences obtained from the stereo images according to the epipolar constraint in Eq. (8).

Employing a satellite navigation system like GPS allows us to derive the geographic position of the cameras with an accuracy of about 2–3 m. Accuracies in the range of centimeters can be achieved by using additional correction information broadcasted by terrestrial reference stations (D-GPS). The obtained coordinates can be mapped from the global reference system, e.g. WGS-84, to a local reference system using a suitable projection in order to get the exact baseline length and

10 ~~vector~~ direction.

A more challenging task is the estimation of the ~~orientation of the cameras~~ camera orientation. Hu et al. (2010) uses geographic landmarks with known coordinates, Öktem et al. (2014) use the horizon and Seiz (2003) exploits stars as geometric references. As Seiz, we use sensed stars in the images, see Figure 7, as observations to estimate the absolute camera orientations.

15 This requires a set of reference stars which can be observed by the cameras in the local night sky. The coordinates of the stars can be obtained from a star catalog like Stellarium or from online sources, e.g. of the NAOJ¹. The coordinates are usually provided with the (north-aligned) azimuth angle φ_n and altitude angle θ and have to be converted to 3D unit vectors according to

$$\mathbf{x}_s = \begin{bmatrix} \varphi_n \\ \theta \end{bmatrix} \longrightarrow \mathbf{x}''_s = \begin{bmatrix} \cos(\varphi_n + \pi/2) \cos(\theta) \\ -\sin(\varphi_n + \pi/2) \cos(\theta) \\ \sin(\delta\theta) \end{bmatrix}.$$

20 In order to get the coordinates of the each reference star in the recorded image, we first take several long-exposure night sky images, compute the median image and subtract the median image from the original night sky images. As a result, only the moving stars are left and we can compute the respective centroid coordinates. The correct identification of the stars in the image is currently done manually by adjusting the rotation angles α , β and γ until the projections are close enough to the centroids to be attributed. After the conversion of the stars image coordinates and catalog coordinates to 3D unit vectors (x''_s and x''_{cat}),
 25 they ~~can be~~ are used to estimate the rotation R_{abs} of the camera ~~by~~ via a Levenberg-Marquardt minimization (Madsen et al., 2004) of the angular error

$$\arg \min_{R_{abs}} \left\{ \sum_{i \in stars} (1 - (\mathbf{x}''_{cat}{}^\top (R_{abs} \mathbf{x}''_s)))^2 \longrightarrow \text{minimize} \right\},$$

where R_{abs} can be parametrized as a unit quaternion or as axis-angle representation.

¹ http://eco.mtk.nao.ac.jp/cgi-bin/koyomi/cande/horizontal_rhip_en.cgi, last accessed April 2016.

From the absolute location and orientation of the cameras we can derive the relative orientation using Eq. (7) and a first estimate E^* of the essential matrix can be composed according to Eq. (8).

For a further refinement of the essential matrix E^* , we collect SIFT-point-feature correspondences (Lowe (2004)) that are consistent with the epipolar constraint in Eq. (8): For each detected feature in the left image, we can select all features in the right image that are consistent with E^* up to a predefined error threshold, e.g. $\angle(E^{*\top} \mathbf{x}_L'', E^* \mathbf{x}_R'') < 2^\circ$, and then find the best match via K-Nearest-Neighbor using the SIFT feature descriptor. The same is done in the other direction, i.e. from the right to the left image, so that only mutually consistent matches are selected.

A couple of image pairs are enough to collect plenty of evenly distributed correspondences, as is shown in Figure 8. Since this set of correspondences will contain mismatches that would lead to a flawed refinement of E^* , we use the robust parameter estimation technique RANSAC (Fischler and Bolles, 1981) to filter out those likely mismatches.

Finally, we employ Levenberg-Marquardt minimization of the cost function

$$\arg \min_{\mathcal{E}} \left\{ \sum_{i \in \text{inliers}} \sin^2 \left(\angle(\mathbf{x}_L'', \hat{\mathbf{x}}_L'') \right) + \sin^2 \left(\angle(\mathbf{x}_R'', \hat{\mathbf{x}}_R'') \right) \rightarrow \text{minimize} \right\}. \quad (11)$$

Because the observations \mathbf{x}_L'' and \mathbf{x}_R'' are always subject to ~~some measurement error~~ measurement errors, they will not lie exactly within an epipolar plane. $\hat{\mathbf{x}}_L''$ and $\hat{\mathbf{x}}_R''$ denote the estimated true locations of \mathbf{x}_L'' and \mathbf{x}_R'' that do lie exactly within an epipolar plane and ~~at the same time~~ are closest to the observations in an angular sense. As the estimations and the observations are unit vectors and lie on the image sphere, Eq. (11) formulates a meaningful angular error measure and its minimization provides an optimal maximum likelihood solution, see Oliensis (2002).

4 Stereo Matching

To calculate the 3D information of a point \mathbf{P} , we need to know the coordinates of the projected point on ~~the~~ both images planes. Only if such correspondences are known, ~~the its~~ 3D information location can be computed. The aim of stereo matching algorithms is to compute such correspondence information.

The visual appearance of a scene point \mathbf{P} in each camera determines if stereo matching is successful or not. Automatic stereo matching is likely to fail if there are occlusions, specular reflections, varying illumination or large scale and pose differences between the images, so that either corresponding object points are not visible in both images or differ significantly in their appearance with respect to shape and size. Also objects may lack sufficient texture or contrast, or a unique surface does not exist that has a consistent visual appearance when observed from different perspectives. Especially the latter poses a problem in cloud photogrammetry, ~~and~~ Hence, depending on the cloud situation stereo reconstruction has limitations.

In practice, one either aims at finding the correspondences between distinct points in the images (sparse stereo) or between all pixels (dense stereo). A good overview is given in Scharstein and Szeliski (2002).

We only employ sparse stereo during the estimation of the essential matrix (Figure 8) as described in Sec. 3.6.

Dense stereo ~~is can be~~ advantageous when dealing with complex ~~geometries, but also~~ and dynamic scenes that have limited texture, because it effectively delivers reasonable results for image regions with low-contrast. ~~In these cases, It propagates~~

information from high-contrast regions ~~is propagated~~ into the low-contrast regions assuming similar depth at nearby pixels with similar intensity ~~or structural values~~. In such regions local methods may deliver few or no information leading to a sparse point cloud, which makes further analysis like segmentation or classification difficult.

To simplify the search for correspondences, dense methods usually require epipolar rectified images, see Sec. 3.3. As a result of that, corresponding pixels are restricted to lie on the same image row, which reduces the search space from 2D to 1D.

The correspondence information is stored in the so-called *disparity map* D , that contains for each pixel in the rectified reference image the horizontal sub-pixel distance d to its corresponding image point shifted in the same row in the other image, see Figure 9. Hence, for the two corresponding points $\mathbf{x}_{L,V}$ in the left and $\mathbf{x}_{R,V}$ in the right image, we have for each pixel position in the disparity map $D(\mathbf{x}_{L,V}) = |u_L - u_R|$ $D(\mathbf{x}_{L,V}) = |u_{L,V} - u_{R,V}|$ and therefore have the relation

$$\mathbf{x}_{R,V} = (u_R, v_R)^\top = (u_L - d, v_R)^\top \quad \mathbf{x}_{R,V} = (u_{R,V}, v_{R,V})^\top = (u_{L,V} - d, u_{R,V})^\top.$$

In our current approach, we rely on a dense matching algorithm that is based on the Semi-Global Matching (SGM) proposed by Hirschmüller (2005) and is called Semi-Global Block-Matching (SGBM). It produces accurate results while being deterministic and ~~fast to compute~~ computationally efficient. In this work we use the implementation provided in OpenCV.

For a detailed algorithmic description, we refer to the original paper by Hirschmüller (2005) ~~or~~ and to the OpenCV documentation. Here, we present only a short summary.

The basic problem of finding an optimal disparity map can be formalized as an energy minimization problem involving an energy functional and an appropriate minimization technique (Scharstein and Szeliski, 2002). A good disparity map should satisfy at least the following two aspects:

1. Two corresponding pixels should have similar intensity or structural values (data consistency).
2. Neighboring pixels with similar intensity or structural values should have similar disparity values (smoothness assumption).

Both aspects can ~~then~~ be combined to form the global energy $E(D) = E_{data} + E_{smooth}$ and are realized in SGBM as follows:

To achieve data consistency (aspect 1), $E_{data}(\mathbf{x})$ is computed for each pixel \mathbf{x} independently using a window-based similarity measure such as sum of absolute/squared differences or normalized cross-correlation (NCC), yielding one matching cost per pixel for each valid disparity value $d \in [d_{min}, d_{max}]$. Note, that using a larger window will smooth the disparity map since small details have a smaller influence on the measure. This causes fine structures to disappear, but also reduces errors caused by image noise. However, relying only on a minimum in E_{data} will cause mismatches, especially in regions with low-contrast or repeating patterns, which results in a flawed disparity map. This can be ~~encountered~~ countered by introducing an additional smoothness term E_{smooth} , that penalizes larger disparity differences of neighboring image points across eight linear paths.

Figure 9 illustrates the computation of E_{data} using epipolar rectified images and the final disparity map after incorporating ~~also~~ E_{smooth} . One can clearly see the difference in the disparity values between clouds that are closer (high disparity) and more distant clouds (smaller disparity).

In our application, we use a window-size of 11×11 pixels. To achieve a successful matching in larger low-contrast regions and reduce the variability in the reconstruction due to the noisy image signal, we ~~furthermore~~ scale the input images to one quarter size. This causes an oversmoothing near cloud boundaries, but this way we obtain smoother cloud surfaces.

5 Stereo Setup and Results

5 In this section we present our stereo setup deployed at the Forschungszentrum Jülich GmbH, Germany. We also give a geometric uncertainty analysis of the current setup, discuss common error sources and how they affect the calibration and reconstruction results ~~;~~ with a focus on asynchronous recordings. Finally, we present our experimental results of two case studies. The first case shows that our dense stereo approach is able to achieve a geometric accuracy that is comparable with those of previous studies using sparse stereo methods, like Seiz (2003) and Öktem et al. (2014). The second case illustrates the capability of our
10 approach to successfully reconstruct the complex 3D cloud structure and dynamics of convective clouds.

5.1 Camera Setup

We exploit two sky imagers installed at the Forschungszentrum Jülich GmbH, Germany, which also hosts the Jülich Observatory for Cloud Evolution (JOYCE Löhnert et al. (2014)), which has been developed in the framework of the Transregional Collaborative Research Center TR32 (Simmer et al. (2015)), ~~to for more information~~. For the evaluation of our results we use
15 observations from a local lidar-ceilometer and cloud radar (Figure 10). The first ~~cameras~~ camera is located at 50.90849° N, 6.41342° E and the second at 50.90613° N, 6.41144° E, resulting in a baseline length of approximately 300 m. This Compared to previous studies that mention a baseline between 500 m (Allmen and Kegelmeyer (1996)) and 900 m (Öktem et al. (2014)), this is a rather short distance and results in higher geometric uncertainty of the estimated 3D points on the clouds, but reduces occlusions and enhances the ratio of mutually visible cloud regions in both images. Furthermore, the short baseline increases
20 the similarity of the cloud appearance in both images, which is beneficial for stereo matching. A more in-depth analysis of these aspects is presented in the next section.

Both cameras are IDS network-cameras of type uEye GigE UI-2280SE with a $2/3''$ CCD sensor consisting of 2448×2048 pixels and ~~have~~ are equipped with a Fujinon FE185C057HA-1 C-Mount Fisheye adapter, providing a 185° field of view ~~;~~ and have ~~a~~ and fixed focus. The cameras are mounted in a box ~~with~~ and point towards the sky. An acrylic glass dome protects the cam-
25 eras against environmental effects. A power supply and a fan distribute heat to prevent the condensation of water on the glass dome. Each camera is connected to a small computer that hosts a self-developed camera control application, which bases based on the IDS C++ SDK (IDS, 2013) ~~;~~ and which allows us to control the cameras remotely, e.g. for scheduled recordings with settings as exposure time, recording interval (e.g. 15 seconds) or modes like long-exposure (night mode) or High Dynamic Range (HDR). The images are currently saved locally and transferred if needed. Synchronization is done by frequent requests
30 to a local NTP service.

5.2 Geometric Uncertainty

First, we discuss the general spatial accuracy of a 3D reconstruction assuming correct orientation parameters, but a flawed disparity estimate. In order to understand the the individual contribution of each parameter to the depth uncertainty, we use the standard formulation for pinhole cameras (Kraus (2004)), where the disparity is modeled by the parallax p_x and its uncertainty σ_{p_x} . The parallax is the angle between two corresponding projection rays, i.e. γ in Figure 5, which can also be formulated as the distance between corresponding projections on the image plane (Figure 12). Given a stereo system of identical cameras with camera constant c (cf. Sec. 3.1) and baseline length t , as illustrated in Figure 12, we have the horizontal coordinate X and the depth D given by

$$X = x'_L \frac{t}{p_x} \quad D = c \frac{t}{p_x} \quad (12)$$

- 10 We focus on the absolute depth uncertainty σ_D . From the relation between the relative depth uncertainty σ_D and the relative parallax uncertainty σ_{p_x}

$$\frac{\sigma_D}{D} = \frac{\sigma_{p_x}}{p_x} \quad (13)$$

we can formulate the depth accuracy in several ways

$$\sigma_D = \frac{D}{p_x} \sigma_{p_x} = \frac{c t}{p_x^2} \sigma_{p_x} = \frac{D}{c t / D} \sigma_{p_x} \quad (14)$$

- 15 The first identity simply states that the (nominal) uncertainty grows linear if the whole setup is scaled up (increasing depth and baseline), the second term indicates that the uncertainty is proportional to the squared inverse of the disparity. As a consequence, deviations at higher disparities are less significant to σ_D than deviations at smaller disparities. An analogous statement is that at smaller angles γ in Figure 5, deviations in ψ_L or ψ_R cause higher errors. The last identity shows that uncertainty is inverse to the ratio of baseline length to depth value. In other words, increasing the baseline t while maintaining a fixed distance to an object will double the accuracy (increased γ).

- 20 These considerations assume that the image points can be identified and matched with a specific parallax uncertainty σ_{p_x} . However, larger baselines (or disparities) usually affect the stereo matching because increasing parts of the object might not be visible by both cameras. Additionally, the object will have significantly different geometric appearance in each camera. Thus a tradeoff between accuracy and geometric completeness and consistency is necessary to get the best results. Compared to previous studies, the small baseline of our stereo setup leads to noisy and inconsistent reconstructions beyond 5 to 6 km. However, our current focus lies on boundary layer clouds and their lateral morphology, which usually have a horizontal spacing of just a couple of kilometers between each other. Also, distant clouds are often occluded by others so that a larger baseline does not always offer the desired benefits.

- 25 Figure 11 shows the reconstruction error for a virtual cloud layer at 1500 m and 3000 m height over a 10×10 km² area around the cameras, assuming an error of 1 pixel in the disparity map after the matching phase (Sec. 4), which corresponds to

a directional error of approximately 0.1° . The values represent absolute errors within the epipolar plane. Therefore, depending on the horizontal distance to the cameras, the error has a larger vertical component (small distance) or a larger horizontal component (larger distance). The error grows larger with increasing distance to the cameras and with increasing co-linearity between object point and the camera centers. In both cases, the angle γ between the projection rays becomes very small, yielding larger triangulation errors. Thus, sky imagers do not provide hemispheric 3D reconstruction with homogeneous accuracy. However, this deficit can be ameliorated by employing a third camera in a triangle configuration. A successful integration of a third camera into the dense stereo matching scheme including epipolar rectification is explained for perspective cameras in Heinrichs and Rodehorst (2006). An adaption to omnidirectional cameras has, to the best of our knowledge, not been addressed yet, but seems to be possible.

Next, we compare the spatial resolution of a sky imager with a wide-angle camera used in previous studies. Fisheye lenses cover a substantially larger field of view than normal perspective lenses, but at a reduced effective angular resolution. As a consequence, the stereo depth resolution is lower for fisheye lenses compared to perspective ones. This drawback limits the effective range for a high quality reconstruction, especially for distant clouds. A comparison of our fisheye cameras with one of the wide-angle cameras from Öktem et al. (2014) highlights the differences: While our camera has a field of view (FOV) of about 180° and the circular view field covers a 3.5 Megapixel region (from a 5 Megapixel sensor), the wide-angle camera has a FOV of 67° and takes 1-Megapixel images from a 5-Megapixel sensor. To compare the view fields, the respective solid angle Ω_{fish} for the sky imager and Ω_{wa} for the wide angle camera – given in steradians – must be derived. Assuming a field of view of 180° for the sky imager and 67° for the wide-angle camera leads to $\Omega_{fish} = 6.28$ and $\Omega_{wide} = 1.04$.

FurtherFurthermore, the solid-angle per pixel is $6.28/3.5 \cdot 10^{-6} = 1.8 \cdot 10^{-6}$ for the fisheye and $1.04/1.0 \cdot 10^{-6} = 1.04 \cdot 10^{-6}$ for the wide angle camera, resulting in a 43 % lower spatial resolution of the fisheye camera. Using the full resolution of the wide-angle camera with 5 Megapixel, the ratio would be 11 %. Hence, one must use a sensor almost 10 times higher resolution to compete with the wide-angle camera in this respect. Depending on the fisheye projection function and the location in the image, the spatial resolution will of course vary due to the different degree of distortion.

~~Next, we demonstrate the spatial variability of the 3D reconstruction error based on our real stereo setup by assuming a constant error in the disparity map. Based on the distance between our cameras, we can estimate the uncertainty of the reconstruction using the triangulation method presented in . shows the reconstruction error for a virtual cloud layer at 1500m and 3000m height assuming an error of 1 pixel in the disparity map after the matching phase, which corresponds to a directional error of approximately 0.1° . The values represent absolute errors within the epipolar plane. Therefore, depending on the horizontal distance to the cameras, the error has a larger vertical component (small distance) or a larger horizontal component (larger distance). The error grows larger in two cases: First, with increasing distance to the cameras, and second, with increasing co-linearity of the object point and the camera centers. In both cases, the angle γ between the projection rays becomes very small, yielding larger triangulation errors. Thus, the sky imagers do not provide a hemispheric 3D reconstruction with homogeneous accuracy. However, the accuracy can be ameliorated by employing a third camera in a triangle configuration.~~

The imaging process of a sensor adds a random noise signal, which can be limited, but not avoided. In principle, this also ~~effects-affects~~ parameter estimation, because both localization and measurement are disturbed. Given a large number of measurements for the calibration, the signal noise can be compensated in a maximum likelihood estimation as the redundancy is high. The stereo matching is also ~~effected-~~affected by the noisy image signal and causes a disturbed 3D reconstruction ~~as~~
5 ~~shown-in-~~.

~~The following analysis is designed to~~ In the following analysis we investigate the effects of an asynchronous recording of the stereo images during the observation of a dynamic cloud scene.

Despite frequent requests to an NTP service, we sometimes experience asynchronous system times on the local computers in the range of a few seconds. Consequently, the whole cloud scene shifts between the single shots and thus causes a displacement
10 in the images, which leads to a biased or flawed disparity map. We investigate the effects of a cloud field displacement of $\Delta = \pm 15$ m along the baseline (x-direction) and perpendicular to the baseline (y-direction) in a virtual sky imager setup together with a virtual cloud layer at 3 km, using the 3D rendering software Blender ~~cite~~blender (Blender Foundation, 2016). The virtual sky imagers have identical internal camera geometries comparable to the real ones, and a relative pose of $R_L = R_R = I$ and $\mathbf{t} = (300, 0, 0)^\top$ m. Figure 13 shows the cross-sections of the respective reconstruction along the baseline (x-axis) and
15 perpendicular to the baseline (y-axis). A displacement along the x-axis results in a lower (2875 m for $\Delta = -15$ m) or a higher (3183 m for $\Delta = +15$ m) cloud base compared to the unaffected reconstruction (3025 m for $\Delta = 0$ m), while a displacement along the y-axis just causes an overall higher standard deviation of the reconstruction without a systematic error in the mean base height ($\sigma(\Delta = +15 \text{ m}) = 45 \text{ m}$, $\sigma(\Delta = -15 \text{ m}) = 48 \text{ m}$ and $\sigma(\Delta = 0 \text{ m}) = 35 \text{ m}$). The results confirm that a displacement in x-direction is equivalent to a change in the length of the baseline \mathbf{t} . Hence, the reconstructed cloud base \hat{h} compared to the
20 real cloud base h can be derived according to $\hat{h} = h \cdot [\|\mathbf{t}\| / (\|\mathbf{t}\| + \Delta)]$.

5.3 Evaluation of the 3D Reconstruction

The following two case studies are designed to evaluate our approach using observations from a lidar-ceilometer and a cloud radar, and to show its capability to capture the complex 3D shapes and dynamics of convective clouds.

We compare some of the reconstructions with observations of cloud base heights from a lidar-ceilometer and reflectivity
25 measurements by a cloud radar, both deployed in the vicinity of the cameras at the JOYCE observation site. The lidar-ceilometer is a Vaisala CT25K which was operated with a range between 60 m and 7500 m and an angular resolution of 0.84° , which corresponds to 4.1 m at 3000 m height. The cloud radar is a Metek polarimetric Doppler Radar (MIRA) operating at 35 GHz with a similar angular resolution over a range between 150 m and 15 km and a maximum sensitivity of -45 dBZ at 5 km. For our comparisons we average observations over an integration time of 0.15 seconds for each range resolved beam. A complete
30 cross-section scan then takes approximately one minute for all elevation angles, that range between 15° and 165° . For the empirical evaluation, we use the cloud base height observations from the lidar-ceilometer. Since the lidar-ceilometer offers only point measurements every 15 seconds, we use observations from a 10 minute period around the measurement time of the cameras to get a meaningful comparison. We compare the mean reconstructed height values from a near-zenith rectangular area of 9 km^2 . The cloud radar offers a direct comparison between the reflectivity signal from an RHI cross-section scan

and the respective cross-section of the 3D cloud shape derived by the stereo method. In order to cover the region of best geometric accuracy of the stereo method, we orientate the radar scans ~~towards a~~ along an almost perpendicular direction to the baseline ~~vector~~, compare Figure 10 and 11). After image acquisition, the input images are preprocessed, which includes a resampling to one quarter size and a contrast enhancement using the *Contrast Limited Adaptive Histogram Equalization* (CLAHE). For stereo matching with SGBM we use a window-size of 11×11 pixels and a cloud mask to remove some artifacts. After triangulation of the 3D point cloud, we create a cloud surface mesh using methods from the open-source *Point Cloud Library* (Library, 2016).

5.3.1 Analysis of the 3D Reconstruction of Stratocumulus-layer Clouds

We present two cases with stratocumulus clouds, which we will use to evaluate our result by comparing it against observations from the lidar-ceilometer and the cloud radar.

Figure 14 shows the results from the 11th of August, 2014, at 14:12:00 UTC, right after a shower moved over the JOYCE site with a trailing stratocumulus layer, that was also captured by the cloud radar. The mean cloud-base heights from the reconstructions are 2881 m while the lidar-ceilometer observations result in 2897 m. Note also, that the cloud radar is – due to its measuring frequency in the microwaves – not as sensitive as the lidar-ceilometer or the camera to the visible outer cloud boundaries, with the consequence that especially the boundaries of a cloud might not be detectable by the radar, but by the camera or the lidar.

The second case has been recorded on the 5th of August, 2014, which is interesting for two reasons: First, a typical boundary layer evolution has been observed, starting with small to medium size cumulus convection along with a steady rise of the cloud base. Second, from early noon on the cloud scenery shifted towards stratocumulus layers, resulting in less convection and thus providing again good conditions for a comparison against observations from the lidar-ceilometer and the cloud radar. ~~The results shown in~~ (Figure 15 ~~are organized as described in the previous case~~). Comparing the reconstruction result with the cloud radar indicates a reasonable estimation of the bases of the higher cloud layer and the cumulus cloud below it. However, differences between 100 to 150 m are noticeable and the lidar-ceilometer observations report a cloud-base mean height of 2922 m compared with the height of 2766 m derived from stereo reconstruction. ~~We assume a systematic error in the reconstruction, which is discussed in~~ See Sec. 5.3.3 for a discussion of the potential reasons for this discrepancy.

5.3.2 Analysis of the 3D Reconstruction of Cumulus Clouds

The 24th of July, 2014 showed strong convection with rapid cloud development and decline, providing excellent conditions to apply our dense stereo reconstruction and to capture their complex shapes and dynamics. Figure 16 shows a cumulus mediocris forming approximately 3 km away from the stereo camera pair. One can observe the convective updraft and the rising cumulus turret. While reaching a height of about 4000 m, the turret enters the higher wind field, resulting in a ~~skew~~ skewed shape of the cloud due to ~~the~~ wind shear. Figure 17 shows a cross-section of the reconstructed 3D cloud surface at 12:07:00 UTC together with the cloud base measured with the the lidar-ceilometer.

Figure 18 shows ~~another example of~~ a smaller convective cloud, but also with a rather complex morphology from 11:28:00 UTC to 11:32:00 UTC on the same day. One observes how the developing convective turret covers parts of the cloud, which results from its increasingly concave shape. The temporally closest cloud base height values ~~of from~~ the lidar-ceilometer ~~are~~ between 12:14:38 UTC and 12:17:08 UTC ~~and~~ report 1487 m on average. Figure 19 shows a cross-section of the reconstruction ~~the~~ ~~same way similar~~ as in Figure 17.

5.3.3 Discussion of the Experimental Results

Our cloud reconstructions show an overall good agreement with the cloud base height observations from the lidar-ceilometer and the cloud radar. Mean near-zenith cloud base heights for the stratocumulus cases are within 1 % for the 11th of August and 5 % for the 5th of August of ~~the~~ lidar-ceilometer mean values, and the stereo method is able to capture the geometric shapes of the cloud bases as in Figure 15. A possible explanation for the deviation ~~from occurring on~~ the 5th of August is a shift between the ~~computers system time~~ ~~computer system times~~ as is described in Sec. 5.2. Although errors in the orientation parameters cannot be excluded, the relative orientation estimation shows a standard deviation of 0.04° ~~and thus which~~ is comparably accurate. On the other hand, a bias in the computers system time could be observed at times and confirms this assumption. Based on the observations from the nearby wind-lidar, which reported a wind speed of about 5 m s^{-1} in a direction almost collinear to the baseline, a time difference of about +3 seconds between the left and the right camera would lead to a scaled reconstructed cloud base of $2922 \text{ m} \cdot (302 \text{ m}/317 \text{ m}) = 2783 \text{ m}$ which is close to the actual reconstructed cloud base height of 2766 m.

The results from the 24th of July show that the dense stereo method is able to almost fully reconstruct the visible outer shape of a convective cloud. In both cases the concave and increasingly ~~skew skewed~~ shape of the cloud is nicely captured by the dense stereo method, as is illustrated in the cross-sections in Figure 17 and Figure 19. Also the cloud base is clearly visible and matches the lidar-ceilometer value of 1495 m quite well, considering that temporally close measurements are only available from 12:12:07 UTC to 12:17:08 UTC ranging from 1448 m to 1585 m.

6 Conclusions

In this paper, we ~~investigated the problem of computing a~~ ~~investigate the reconstruction of the~~ 3D ~~reconstruction geometry of~~ ~~clouds~~ from fisheye cameras using dense stereo approaches from photogrammetry. We ~~presented~~ ~~present~~ a complete approach for stereo cloud photogrammetry using hemispheric sky imagers. Our approach combines calibration, epipolar rectification, and block-based correspondence search for dense fisheye stereo reconstruction for clouds. We ~~showed~~ ~~show~~ that cloud photogrammetry is able to compute the cloud envelope geometry and ~~demonstrated~~ ~~demonstrate~~ the potential of such methods for the analysis of detailed cloud morphologies. By applying an epipolar rectification together with a dense (semi-)global stereo matching algorithm ~~instead of using the original images~~, we are able to compute clouds shapes that are more complete and contiguous than reconstructions relying on regular feature-based methods. Once the cameras are calibrated, the method can be fully automated to deliver real-time information of the cloud scene.

~~Our~~The proposed technique requires accurate camera calibration parameters and synchronously triggered cameras. Although the validation of our results with cloud radar observations should be extended to convective clouds, the reconstructions have shown to be stable over time, yielding robust cloud base and motion estimates. The validation for stratiform clouds show acceptable deviations from the lidar-ceilometer and radar measurements.

- 5 The system will be permanently installed at JOYCE and record cloud evolution on an automated basis, which will provide a large data basis for more extended analysis. We will add further camera pairs at larger distances from JOYCE to enable the reconstruction of more complete cloud boundaries. Future work will also focus on the combination of cloud photogrammetry with different sensors. ~~Furthermore we want to use two additional hemispheric sky imagers outside the research center to obtain more closed surfaces by fusing different views.~~
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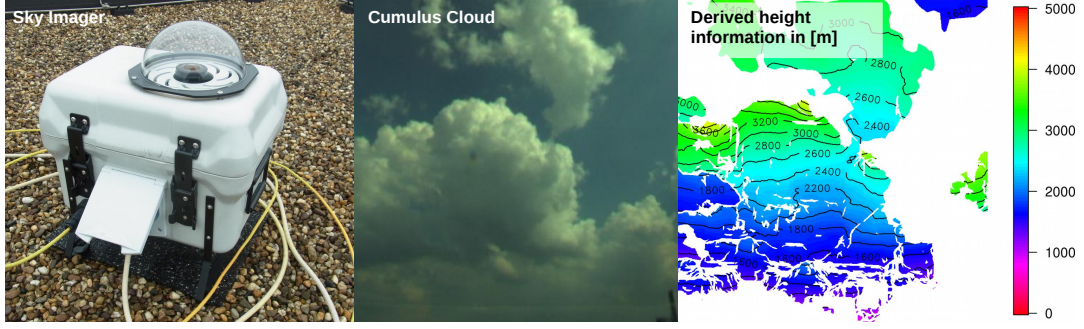


Figure 1. We employ two hemispheric sky imagers (left) in a stereo setup with a baseline of 300 m to derive dense and detailed geometric information such as height and morphology of clouds within a range of about 5 km using the simultaneously recorded image pairs. The derived heights (right) of an exemplary recorded cloud (middle) using the fisheye images, as shown in Figure 4 (c), of the two sky imagers.

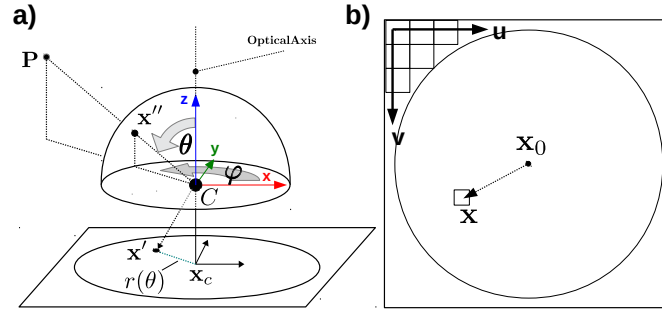


Figure 2. (a) In the omnidirectional camera model the 3D object point \mathbf{P} is mapped to \mathbf{x}' on the image plane. Several radial-symmetric projection functions $r(\theta)$ can be used, which define the distance to the projection center \mathbf{x}_c ; (b) The final projection \mathbf{x} in pixel coordinates is determined by the camera calibration parameters and additional distortion coefficients.

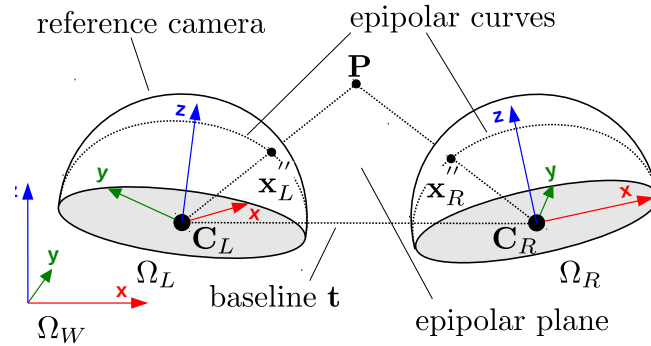


Figure 3. The two hemispheric cameras are located at \mathbf{C}_L and \mathbf{C}_R with independent orientation in the world coordinate system Ω_W . The projections \mathbf{x}_L'' and \mathbf{x}_R'' of a 3D point \mathbf{P} on the image hemisphere in each camera system Ω_L and Ω_R span together with the baseline \mathbf{t} the epipolar plane and can be used to reconstruct \mathbf{P} via triangulation.

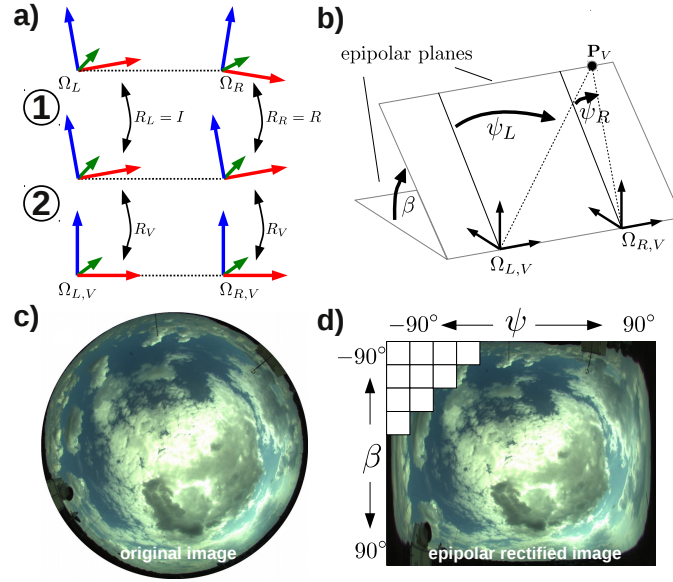


Figure 4. Epipolar rectification for omnidirectional cameras: The two-step rotational mapping between real and virtual cameras (a) results in a canonical camera setup of virtual cameras during rectification (b). A fisheye image (c) is rectified such that the angles β and ψ correspond to the lines and rows of the image (d).

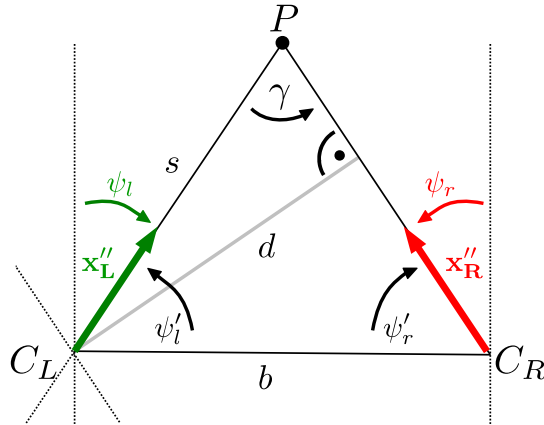


Figure 5. Two corresponding ray directions \mathbf{x}''_L and \mathbf{x}''_R are defined by the angles ψ_L and ψ_R within the epipolar plane. With baseline length b the distance s between left camera and 3D Point \mathbf{P} can be derived, which allows to determine 3D point coordinates $\mathbf{P} = s \mathbf{x}''_L$.



Figure 6. Camera calibration with cube: the pattern on the inside of the cube defines a set of 3D points with respect to the cube coordinate system and can be used as reference data to solve for the fisheye projection parameters.

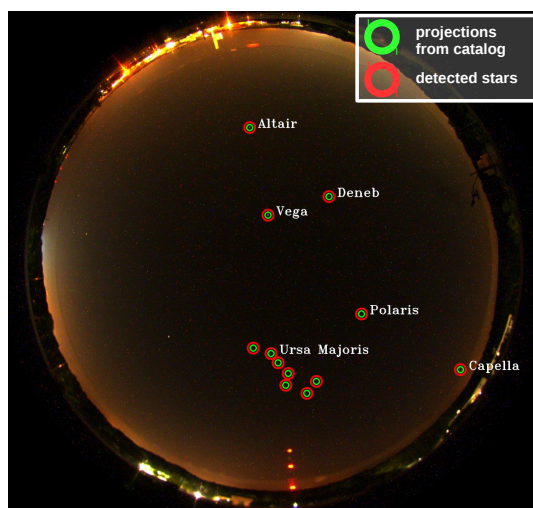


Figure 7. Absolute orientation estimation via stars from long-exposure images. In case of an accurate orientation estimation, the projected coordinates from the star catalog should match the detected stars in the image.

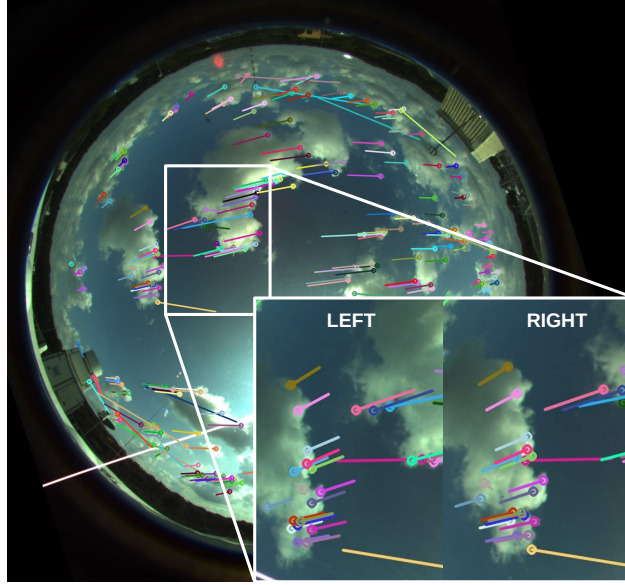


Figure 8. Detected interest points can be matched across the stereo images and are marked by the same color. At least 5 correspondences are needed to compute the relative orientation between the cameras and each also provides one 3D point.

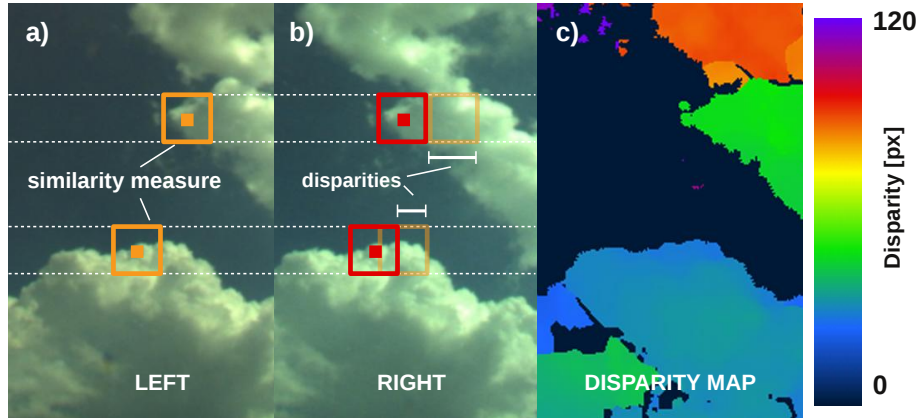


Figure 9. Illustration of dense stereo matching using epipolar rectified images. The correspondence information is stored in a disparity map. Each disparity $D(\mathbf{x}) > 0$ then defines a correspondence between two image points \mathbf{x}_L and \mathbf{x}_R and can be used for triangulation, which results in a dense 3D point cloud.

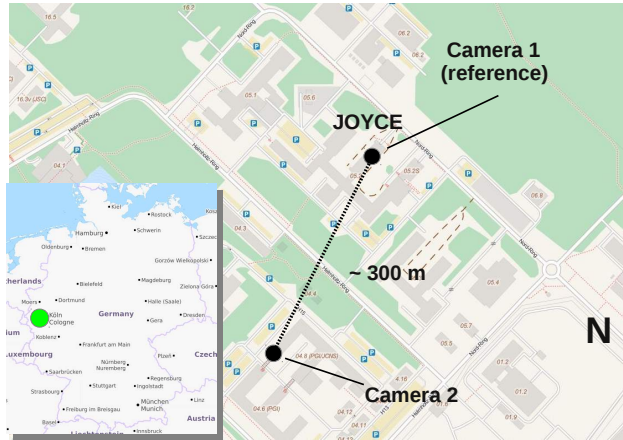


Figure 10. Camera setup at the Research Center Jülich and RHI-scanning directions of the cloud radar (map source: *OpenStreetMap*).

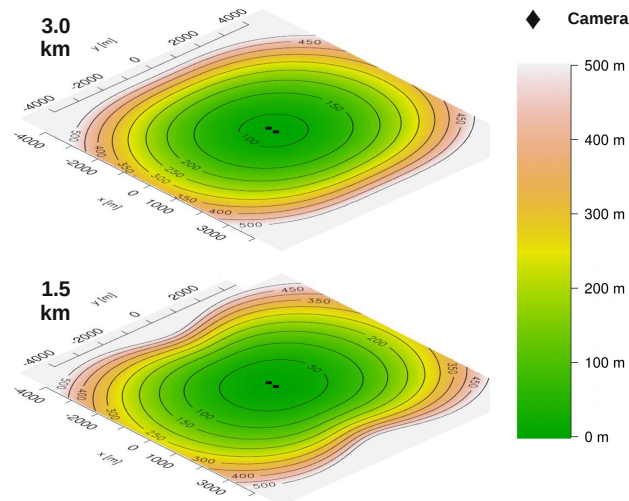


Figure 11. Errors in the disparity values cause a directional deviation of the projection rays within the epipolar plane. Assuming an error of $\Delta\psi_R = 0.1^\circ$, we can compute the absolute geometric error within the epipolar plane for a hypothetical cloud layer at 1.5 km and 3 km height in an area of 5 km around the cameras.

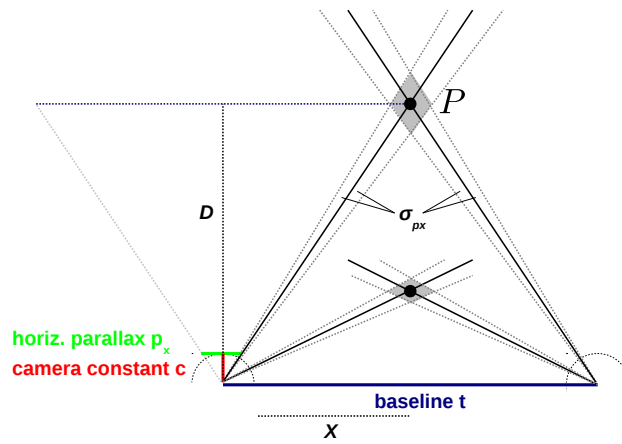


Figure 12. Illustration of geometric uncertainty of the reconstruction within an epipolar plane. An uncertainty in the projection rays σ_{p_x} introduces an uncertainty in the estimated location of P indicated by the gray region: A smaller/higher depth value D (or higher/smaller parallax p_x) reduces/increases the depth uncertainty σ_D , but reduces/increases the horizontal uncertainty σ_x

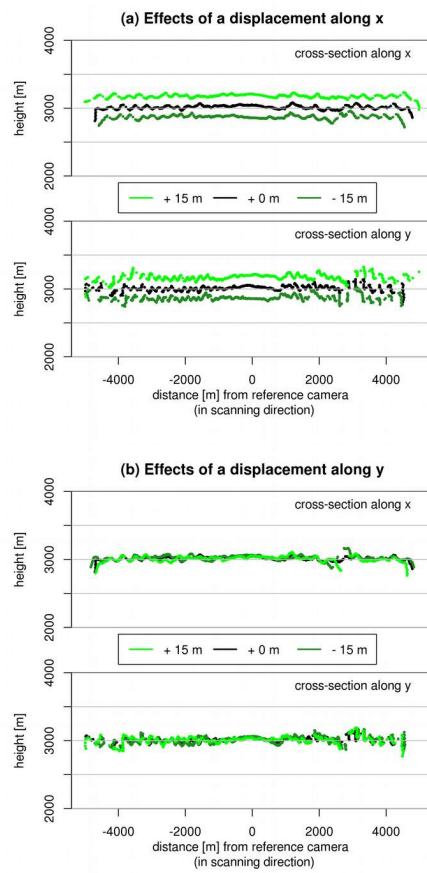
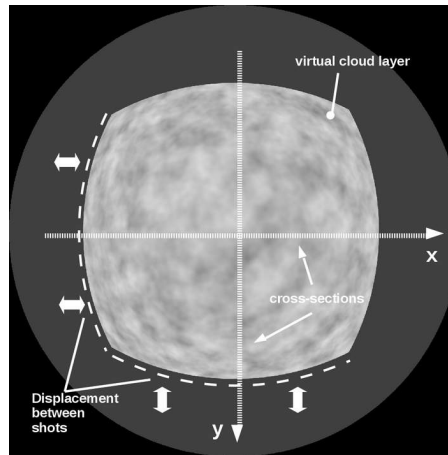


Figure 13. Simulation of an asynchronous recording by the stereo cameras [for the same area size](#). A virtual cloud layer at 3 km height was displaced between the two recordings of the stereo cameras by ± 15 m (a) along the baseline (x-direction) and (b) in perpendicular direction (y-axis). Plots show for each case the respective cross-sections in x- and y-direction of the reconstruction.

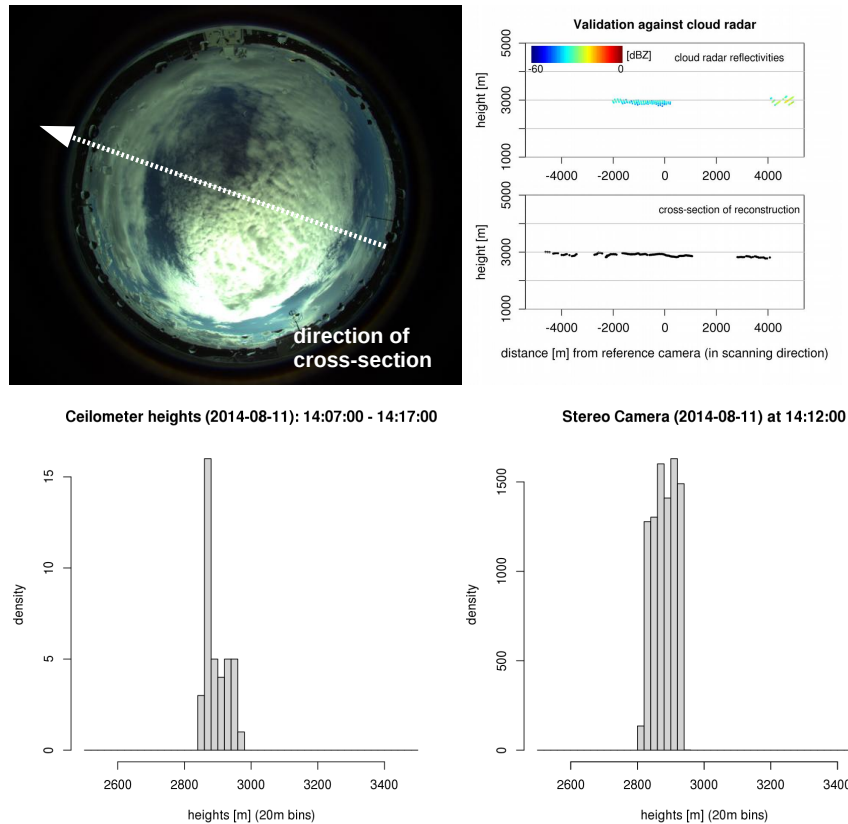


Figure 14. Comparison of the reconstruction with cloud radar (cross-section) and lidar (near-zenith cloud base heights) at 11 August 2014, 14:12:00 UTC. The original fisheye image showing the respective direction of the cross-sections (top left). The cross-section of the reconstruction compared with the reflectivities from the cloud radar (top right). Histogram of the cloud base heights observed by the stereo camera (bottom left) and the lidar-ceilometer (bottom right).

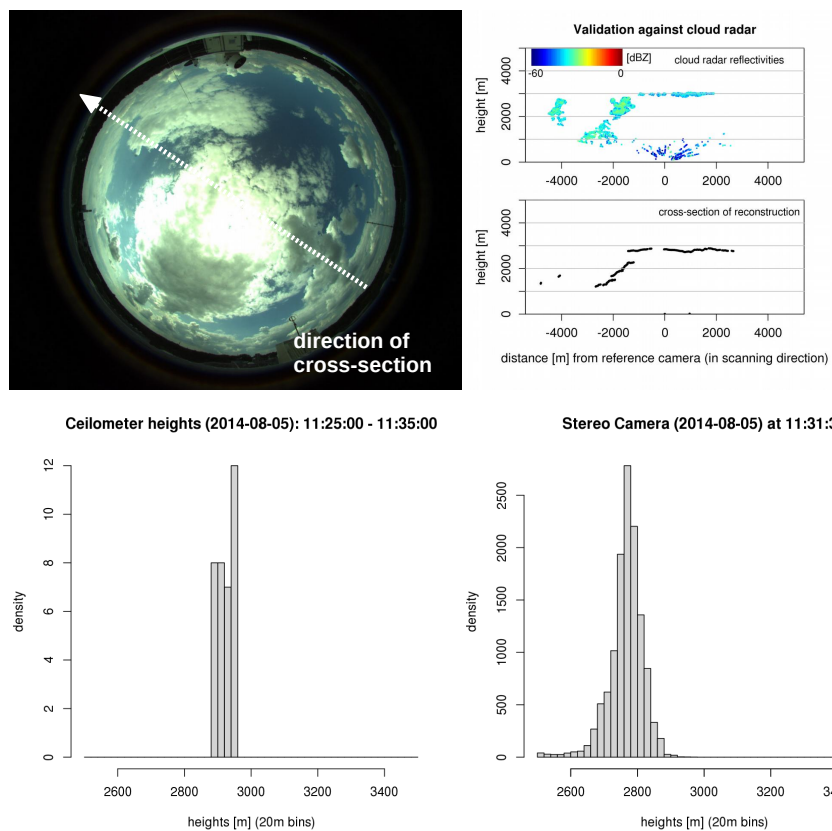


Figure 15. Comparison of the reconstruction with cloud radar and lidar at 5 August 2014, 11:31:30 UTC, as in Fig.14

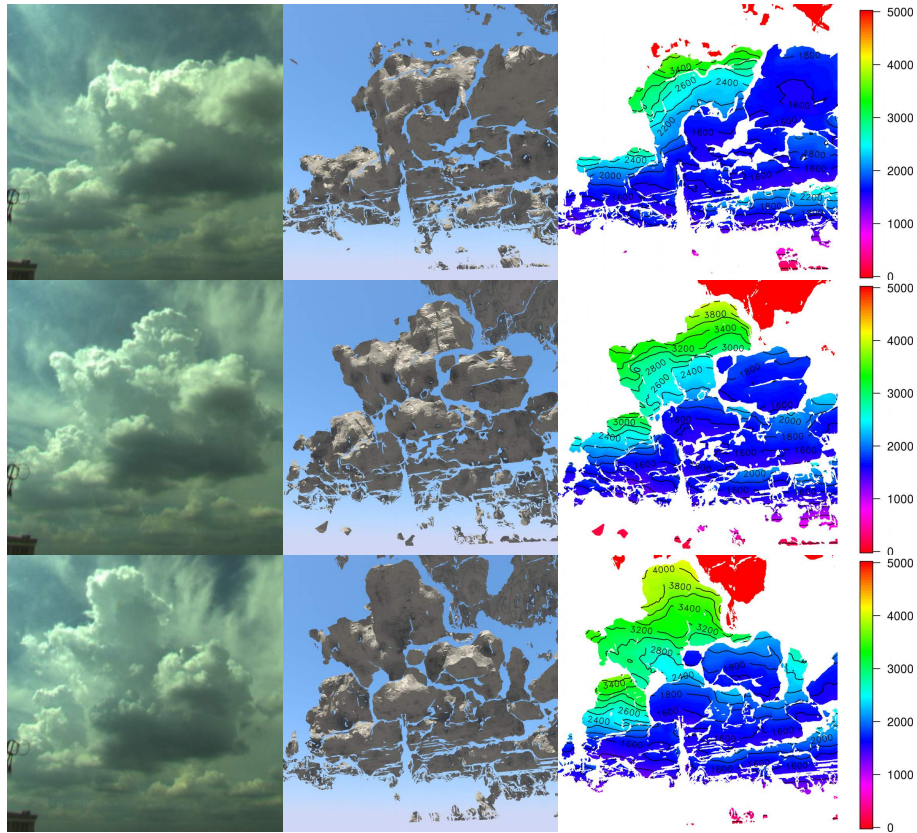


Figure 16. 3D reconstruction of cumulus mediocris at 24 July 2014 approximately 3 km away: The left column shows a subsection of the images obtained from Camera 1. The central column visualizes the reconstruction as an untextured triangulated surface mesh. The right column shows the color-coded height of the reconstruction in meters with contour lines (right). Results are shown for 12:03:00 UTC (top row), 12:05:00 UTC (middle row) and 12:07:00 UTC (bottom row).

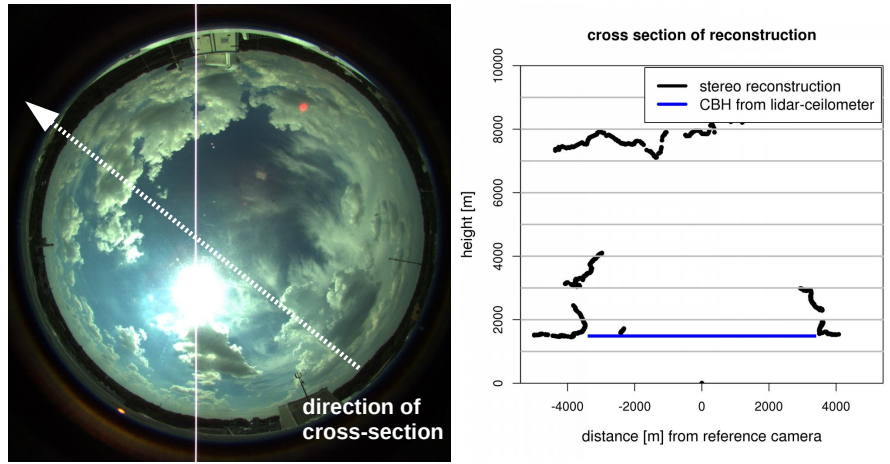


Figure 17. Cross-section of the reconstruction from 24 July 2014, 12:07:00 UTC, and highlighted cloud base height from the lidar-ceilometer.

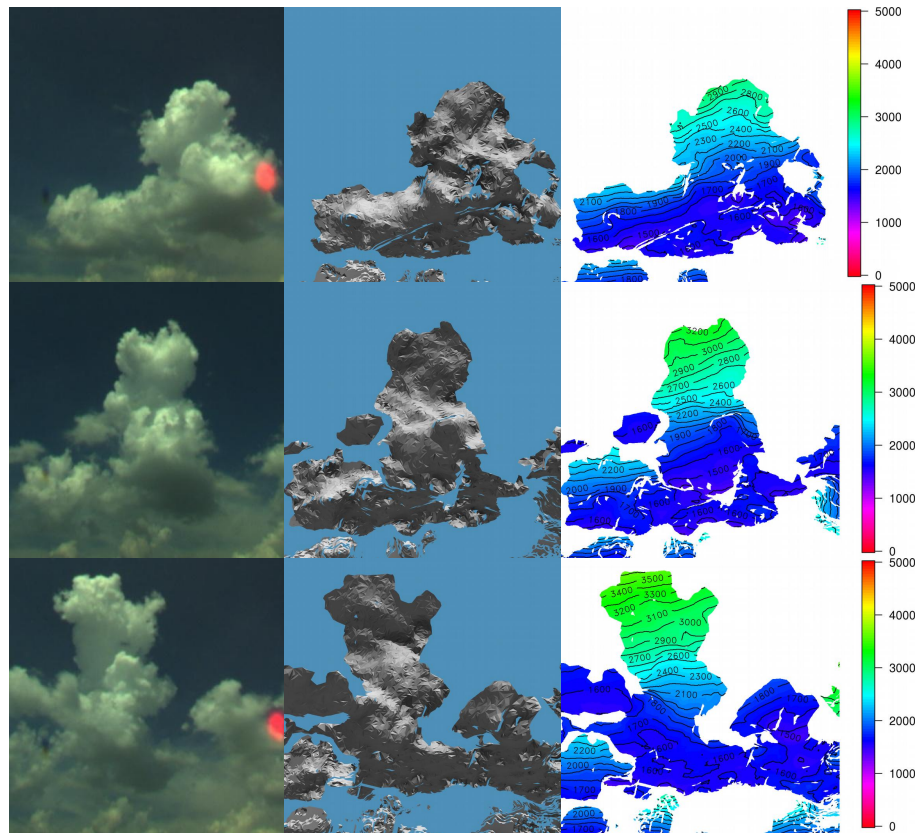


Figure 18. 3D reconstruction of cumulus cloud from 24 July 2014, organized as in Figure 16. Results are shown for 11:28:00 UTC (top row), 11:30:00 UTC (middle row) and 11:32:00 UTC (bottom row). The cloud is approximately 3 km away.

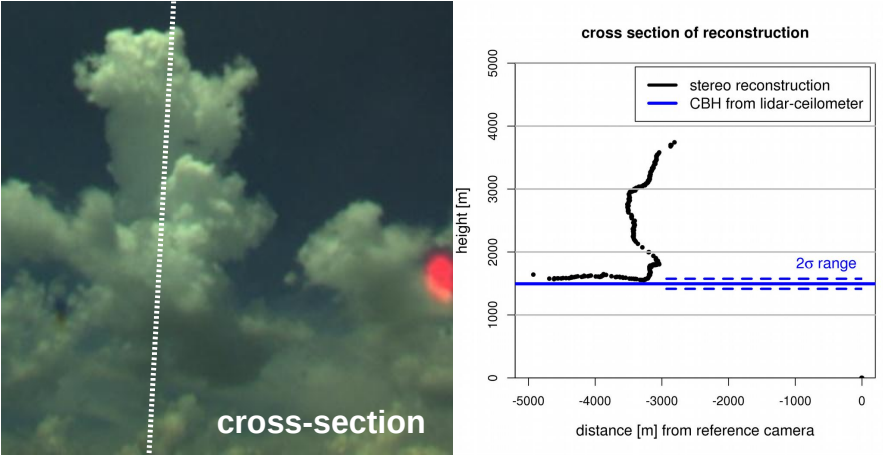


Figure 19. Cross-section of the reconstruction from 24 July 2014, 11:32:00 UTC.