First, we wish to thank the reviewer for his/her careful review. Below is our response to the comments (in blue) on a point-by-point basis. The text referring to the article is indicated in italic.

Answers to major comments

1 The methods described in this paper are not a "sub-grid turbulence model". This is simply a RANS Lagrangian particle dispersion model. Essentially, the authors average out the grid-scale turbulent motions, and instead use the mean fields to drive the Lagrangian dispersion model (at least, this seems to be how it is described in the text). Thus, the mean field is the 'resolved' component and the Lagrangian evolution equation is used to add in the 'fluctuating' component, or in other words the authors are using a Reynolds decomposition :

$$u_i =< u_i > +u_i' \tag{1}$$

where $< u_i >$ is an ensemble average, and u_i' is the fluctuation from that average. If I am understanding the paper correctly, the authors then prescribe $< u_i >$ using the Eulerian simulations, and u_i' is (indirectly) determined using the Lagrangian stochastic evolution equation.
In order for this to be considered a "subgrid scale" model, the following decomposition would be used

$$u_i = \tilde{u}_i + u_i'' \tag{2}$$

where now $\tilde{u}_i$ is the filtered component (filtered at the grid scale), and u_i'' is the unresolved or subfilter component, which would be determined by solving the evolution equation in terms of the subfilter component :

$$du_i = a_0 dt - a_1(u_i - \tilde{u}_i)dt + b\ dW \tag{3}$$

where dW is an increment in a Weiner process, and coefficients a_0, a_1, and b are determined such that Eq. 3 satisfies the Navier-Stokes Equations. For examples of determining the coefficients see Weil et al. (2004), Shotorban and Mashayek (2006), or Vinkovic et al. (2006). These examples are applied to traditional LES applications, but the principles are the same as in the present manuscript. Thus, the turbulence calculated at the grid-scale is retained, and modeling is reduced to specifying only the subgrid scale turbulence. Why filter out valuable resolved turbulence by averaging ? The point of subgrid scale modeling is to retain as much information as possible, such that modeling is simplified in that we only have to model the smaller, more 'universal' scales.

The suggested downscaling method is derived from the Particle-In-Cell methods developed by Harlow [3] and used, for instance, in plasma modeling [6]. These methods combine Eulerian grid-point models and Lagrangian particles to model subgrid fields. The downscaling method uses the same approach : large scales are resolved by an Eulerian model, and Lagrangian particles inside the grid cells are used to model sub-grid scales.

Within this framework, the article aims at presenting two different points : first a new stochastic downscaling method and then an application of the method to a BLLAST case. The downscaling method has been designed to model sub-grid

turbulence starting from a grid point simulation. The general idea is to force a sub-grid particle system with the resolved components of the grid point fields. In addition, to model the unresolved / sub-grid components, a stochastic Lagrangian model is used. For turbulence modeling, we suggest to use the following model, detailed in the article :

$$X_{k+1} = X_k + \mathbf{V}_k \; \delta t + \sigma^X \Delta B^X_{k+1} \quad (4)$$

$$V_{k+1} = V_k - \nabla_x \overline{p} \; \delta t \; - \; C_1 \frac{\varepsilon_k}{K_k} \; [V_k - < V >] \; \delta t + \; \sqrt{C_0 . \varepsilon_k} \; \Delta B^V_{k+1} \quad (5)$$

$$W_{k+1} = W_k + \Delta_k W \; - \; C_1 \frac{\varepsilon_k}{K_k} \; [W_k - < W >] \; \delta t + \frac{g}{\beta} \Gamma^\theta_k \; \delta t + \sqrt{C_0 . \varepsilon_k} \; \Delta B^W_{k+1} \quad (6)$$

where the control parameters (in blue) are given by the grid point fields.

Then the downscaling method has been applied to LES simulations performed for the BLLAST experiment. The ideal approach should apply the method to LES simulations on a coarse grid and then validate the sub-grid simulations by comparison with finer simulations of the same case performed with the same model. Unfortunately, this was not possible. Because of the high computational cost of the LES simulations performed with the Meso-NH model, simulations were available for only one grid size. These available simulations have been chosen as the reference simulations. Instead of simulations performed on a coarser grid, we have chosen to average the reference simulations on several grid points following a methodology used in [4]. The drawback of this solution is that the coarse fields include not only the components resolved on the grid, but also the average of the sub-grid components, being similar to RANS Lagrangian particle model. To conclude the answer to the first comment, we suggest to present the method and the application separately. The limits of the application are also now underlined.

Lines 142-146 : *The particle systems are forced with a large scale grid-point meteorological fields. The large scale used in our work is later described in details. The forced particles are used to model the sub-grid fields for the large scale model. To validate the sub-grid modeling a higher resolution model is used. In theory, the turbulent fields represented by the particles should be compared to the same fields simulated by a high resolution grid-point simulation. For computational reasons we do not have access to different high resolution simulations. Thus the large scale simulations have been built from the available simulations.*

Lines 155-161 : *Now that we have a reference simulation, a coarser simulation is built in order to force the particle system. To this end, we have chosen to average the grid-point fields on few cells. To be consistent, we have also applied a temporal average. The obtained coarse Meso-NH fields have thus lower spatial and temporal resolutions than the reference simulation while being consistent with it. However, due to the average, the coarse fields include not only the components resolved on the grid, but also the average of the sub-grid components. This limitation will be discussed section 8.*

Caption of figure 1 : *The downscaling experience. Coarse fields are used to force a sub-grid particle system. The sub-grid fields are compared to a reference Meso-NH simulation.*

2 The authors state that the novelty of the proposed modeling methodology is that (Lines 77-78) "The method we suggest differs from these previous works : no assumption is made on the pdf shape." This is not true. By using a Langevin-based equation for the particle motions that is forced by a Weiner process, the authors are effectively specifying the shape of the pdf. Although Sects. 2.2 and 2.4 of Pope (1994) (for example) states that these methods do not assume a pdf, the form used by the author does assume a pdf. Particularly, it is assumed that the pdf is Gaussian with variance of $\sigma^2 = \frac{C_0 K}{2C_1}$ (where of course K is the turbulent kinetic energy). The authors can verify this for themselves. Take the Lagrangian evolution equation, simulate some particles, and calculate the pdf. I have attached some sample MATLAB code to do this in an appendix of this review. For simplicity I have assumed that ϵ and K are constant, and that there is no mean drift (i.e., $\nabla p =< U >= 0$). Means could be included, but they should only shift the pdf and not affect its shape. We should find that an ensemble of particles should have a velocity pdf of

$$P = (2\pi\sigma^2)^{-1/2} \exp(-\frac{u^2}{2\sigma^2})$$

where again $\sigma^2 = \frac{C_0 K}{2C_1}$ is the prescribed velocity variance. As a side note, this is not exactly true for the authors' equation for W given the arbitrary way that they have included buoyancy (i.e., it is random and thus will generate erroneous additional variance). If it was correctly modeled, it should create a skewed velocity distribution and thus none of the analysis above or in the manuscript is valid. See Cassiani et al. (2015) and the references therein for examples.

We agree that a Langevin-based equation assumes the shape of the velocity pdf. The stochastic Lagrangian model that we used (equations 4 to 6) describes actually a Gaussian pdf when it is applied to a whole particle system with the same forcing and when $< \cdot >$ is the ensemble average over the particle system. The question becomes more complex when the model is locally applied, that is to say when the forcing varies in space and the average $< \cdot >$ is computed using only a subset of the particle system. In such case, the pdf is more difficult to describe.

The difference between the given MATLAB code and our code is that in our case, the forcing varies from one coarse cell to an other. Besides, the average $< \cdot >$ is a local average computed over the particles around the point of calculation, and the particles evolves freely in the whole domain. Therefore, at a given time in a given cell, particles with different characteristics are mixed, and pdf are only locally Gaussian. It leads that, the velocity pdf described by the particles in one fine cell is obtained by mixing Gaussian pdf but it is not necessarily Gaussian. To illustrate this, we have plotted on figure 1 the histograms of the first component of the velocity for one fine cell and for five consecutive time steps. One can see that the pdf may be far from the Gaussian distribution.

In the article, we will clearly detail this point. The misleading sentence "no assumption is made on the pdf shape" will be replaced by an explanation about the initial Gaussian assumption. A discussion about its validity in our case will also be added.

Lines 76-81 : *Nowadays, this kind of approximation is still widely used for subgrid modeling [7, 9, 8, 5, 2]. The method we suggest differs from these previous*

works : the Gaussian assumption on the pdf shape is only locally made. Thus the particles give access to a discrete representation of the pdf which is not necessarily Gaussian at the scale of the grid cell. The pdf time evolution is thus given by the particle evolution.

Following line 782 : *Concerning the SLM, another point has to be discussed. The Wiener processes used for the dispersion terms involve a locally Gaussian assumption of the pdf described by the particles. In our work, the Gaussian assumption is not valid at the grid cell scale. Indeed, at a given time in a given cell, particles with different characteristics are mixed. This is partially due to the free evolution of the particles in the domain. Thus the velocity pdf described by the particles in one fine cell is obtained by mixing Gaussian pdf but it is not necessarily Gaussian.*

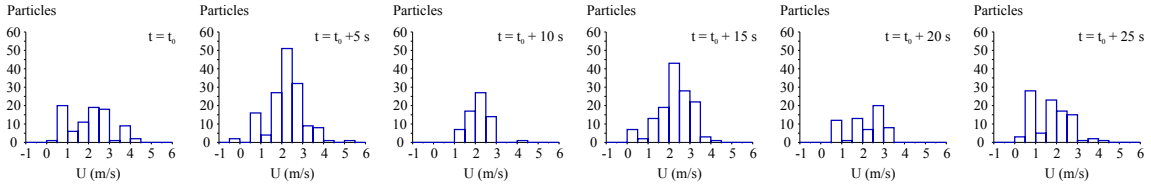

Fig. 1: Histogram of the first component of the velocity (U) at five time steps for one cell.

3 In the (unnumbered) equations following line 348 (note that line numbers are messed up here, there are more than 5 lines between labels 335 and 340), why is an additional noise term added to the position evolution equation (X) ? This is unconventional. Typically, the stochastic noise comes in through the velocity. By adding additional noise to the position statistics will not agree with the velocity statistics. By definition, a particle moves according to its velocity (see for example : Pope, 1994, Eq. 17).

The added term in the particle location has been included to take into account the velocity integration errors considering the Euler scheme used for the velocity equation.

Indeed the stochastic Lagrangian model (SLM) is a McKean-Vlasov equation. $dV_t = b(V_t, \eta_t)dt + g(V_t)dW_t$, where η_t is the V_t probability law. As for Itô processes a Euler integration scheme is the most convenient. For Itô processes a Stratonovitch representation is possible, but for the McKean Vlasov, due to the mean term, a Euler scheme is most adapted.

The errors of Euler scheme for such processes are known (see [1]). These scheme errors are in root square of the time step $\sqrt{\delta t}$.

In the position evolution, the errors on the velocity evolution has to be taken into account. The location is the first integration of the velocity and a error term is added to the location : $X_{(k+1)\Delta t} = X_{k\delta t} + V_{k\delta t}\Delta t + \text{errors}$.

The error term is a stochastic processes taking into account the path dispersion. With errors in $\sqrt{\delta t}$-like, we choose a Wiener process to represent the dispersion process and $X_{k+1} = X_k + V_k\ \delta t + \sigma^X \Delta B_k^X$ where ΔB_k^X is a standard Wiener process.

The choice of the variance σ^X is not anecdotal. We have already performed (unpublished work) a sensitivity analysis, and the variance σ^X is of importance.

For the work presented in this article, σ^X has been settled to 1.

Following line 346 : *One can notice the noise term at the end of the position evolution equation. The added term in the particle location has been included to take into account the velocity integration errors considering the Euler scheme used for the velocity equation (see [1]).*

4 Did the numerical solutions produce the so-called "rogue trajectories" (e.g., Yee and Wilson, 2007 ; Postma et al., 2012 ; Wilson, 2013), and if so, what was done to deal with them ? And how might this affect results, particularly the energy spectra ?

The comment about rogue trajectories raises an interesting point. For dispersion models, one must ensure that modeled trajectories are coherent with the surrounding flow. In our case, particles are used to carry information about the modeled fluid. We are more interested in following the information than in following the particles themselves. Thus a specific particle management system has been designed to ensure that the particle density in each cell is high enough – a lack of particles would lead to a low quality modeling of the sub-grid fields. Because of this management system, particle trajectories are difficult to follow. Indeed, in average, particles are displaced every 3 or 4 time steps by the management system. Thus the trajectories are too short to be rogue. If we look at the longest trajectory without displacement observed during the simulation, we can see that it seems quite coherent with the flow (figure 2).

Lines 445-459 : *As the particles evolve freely in the domain, we also have to ensure a homogeneous repartition of the particles inside the domain. To do so, for each cell of the fine grid we keep the particle number between a minimal value and a maximal value which are given at the beginning of the simulation. By displacing particles, this method of particle management limits trajectory length and prevents rogue trajectories described by [12, 10, 11].*

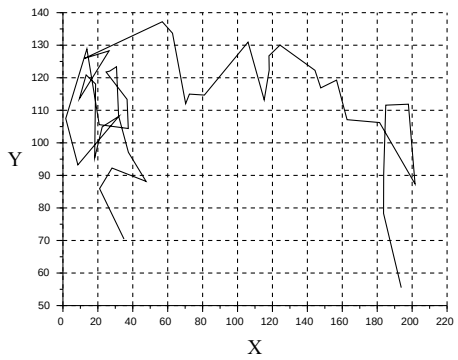

Fig. 2: The longest particle trajectory (45 time steps of 5 s).

5 One problem with validating these subgrid models is that we often don't know what the correct answer is, and many times using no model is better than using a bad model. My opinion is that a more careful validation should be performed before

making claims that the model is performing well. Based on the exercises presented, I do not feel such claims could be made. If the authors wish to test the model more closely, perhaps an a priori test, or even a toy problem may be a better means of testing the model.

As said in the comment, a major problem with validating the sub-grid model is that the correct answer is often unknown. In the present application, the reference is a high resolution grid-point simulation which is not directly used to force the particle system. One of the limitations for the validation has been that the resolution of the reference was not high enough. To tackle it, the ideal solution would be to use direct numerical simulations (DNS) of turbulent atmosphere to validate the sub-grid modeling. Such simulations are very costly and could not be done for the present case.

Concerning the use of toy models, we underline that a model sophisticated enough to model atmospheric turbulent flows is needed. That is why the Meso-NH model has been chosen. We agree that the downscaling method could be applied to an idealized case. However, at the beginning of this work, the resolution problem was not known and a real case –documented by turbulence observations and other studies– seemed a good application case to test the method.

The use of DNS or of a toy model would be interesting perspectives to complete this preliminary work. They should be added to the discussion part of the article. We will also precise that the article presents a first work and not a complete validation of the downscaling method. We suggest the following modifications :

Lines 761-764 : *This article presents a first work on a new way to model sub-grid processes using particle systems. One of the major improvement is the use of a simple turbulence model instead of complex model such as LES or DNS. However, to fully validate the method, one of the first steps should be to use a DNS or to apply the downscaling method to a toy model to know exactly the sub-grid fields. Unfortunately, such a validation could not have been done yet.*

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