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## Interactive comment on "Comment on "Reinterpreting aircraft measurements in anisotropic scaling turbulence" by Lovejoy et al. (2009)" by E. Lindborg et al.

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Abstract: [Lindborg, et al., 2009] claim that it is unreasonable to question the empirical large horizontal wind fluctuation statistics obtained with the help of commercial jetliner trajectory deviations (GASP and Mozaic databases), more precisely to hypothesize, as done by [Lovejoy, et al., 2009b], that the estimated spectrum power law  $E(k)\approx k$  3 on scales  $\geq 600$  km could be spurious. Their main theoretical argument is that this result corresponds to a "well known theory of quasi-geostrophic turbulence developped by [Charney, 1971]" and whose "earlier limitations would have been relaxed in many of the modern models of atmospheric turbulence". We show that in spite of the prime

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historical importance of the quasi-geostrophic approximation this claim is irrelevant.

A first difficulty with the argument of [Lindborg, et al., 2009], is that it relies on the surprising suggestion that models may overcome limitations of a theory, whereas the models are obtained by introducing constraints into a given theoretical framework, e.g. : boundary conditions discretization of partial differential equations, subgrid modelling and other parametrizations. Unfortunately, there are indeed intrinsic limitations of quasi-geostrophic (QG) theory. Indeed, QG is based on a series of approximations, including the hydrostatic approximation and the fact that the "horizontal" (isobaric) material derivative is approximated with the help of the geostrophic velocity. Furthermore, the (relative) vorticity of the atmosphere is assumed to be small with respect to the (local) Coriolis parameter (i.e. the Rossby number is small), the static stability to be nearly uniform. For the sake of the pioneering development of dynamical meteorology, these assumptions were considered acceptable for the large scale dynamics. Furthermore, the QG approximation was attractive because it introduced a new invariant of motion (at least for the geostrophic part of the flow): the potential vorticity. The tricky and exciting result is that the 3D-QG vorticity equation does permit some vortex stretching (a fundamental distinguishing feature of 3D turbulence) for 2D-like motions. However, this QG vortex stretching is quite moderate compared to the 3D one. Furthermore, this invariant does not hold for the generating equations, i.e. it depends on the validity of the QG approximation.

The theoretical problem that LTNCG largely underestimates is that an approximation on a nonlinear system may hold over a given range of large scales only if the smaller scale activity does not destroy the conditions of applicability the approximation. The latter requires in general a separation of scales, this is particularly important for turbulence. This can be understood by the vortex stretching mechanism: 3D turbulence can easily destabilize ("three dimensionalize") 2D turbulence by vortex stretching at larger and larger scales. Indeed, small scale 3D turbulence cannot be understood as only dissipating large scale structures by eddy viscosity, because it also generates larger scale structures by backscattering [Lesieur and Schertzer, 1978] or renormalized forcing [Fournier and Frisch, 1983]. The crucial importance of the separation of scales for numerical weather forecasts was explicitly stated and discussed by [Monin, 1972] and it explains why the concept of "mesoscale gap" [Van der Hoven, 1957] was so cherished during the early history of weather forecasting. It is remarkable that [Charney, 1971]as emphasized by [Schertzer and Lovejoy, 1985] - readily admitted the limitations of the QG approximation and expressed the question of separation of scales in terms of temperature gradients that must remain quite moderate.

The limitation of scale separation must not be reduced to a problem of boundary layer modeling as suggested by LTNCG, because this problem must be solved over all horizontal levels. Nevertheless, let us consider whether the unique QG model simulation cited by LTNCG may be of help in overcoming the limitations of QG theory. In this study [Tung and Orlando, 2003, TO hereafter], simulations were not performed with the help of a 3D discretisation of 3D-QG equations, but rather with the help of a two-layer QG model [Welch and Tung, 1998]. Therefore, this model corresponds in fact two 2D-QG models interacting through the vertical gradient vector potential. Overall, the fundamental variables are the set of values of the velocity potential on the two layers. Furthermore, the Cartesian geometry and linearisation of the Coriolis parameter are used with the help of similar considerations for the  $\beta$ -plane approximation, i.e. the model is restricted to a rather narrow mid-latitude strip.

In any case, with only two layers, not much can be done about either the boundary layer nor the horizontal / vertical anisotropy. Therefore, LTNCG's reference to this study is quite inappropriate. Indeed, as emphasised by TO, their goal was quite different and rather ambitious: to generate a composite horizontal spectrum of the type Ak-5/3+Bk-3, an issue that can be traced back to [Charney, 1971], with the help of a rather simple model that nevertheless would have all the necessary elements. This model indeed gave some numerical evidence in the direction of such a composite spectrum. However, this numerical evidence was thoroughly criticized by [Smith, 2003]

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who convincingly pointed out that the estimated "meso-scale" spectrum slope  $\approx$ 5/3 may well be spurious and correspond to an artificial build up of enstrophy (and therefore of energy) at the smallest explicit model scales. This is due to the fact that the hyperviscous dissipation scale is not meridionally resolved (TO used indeed the 10th power of the Laplace operator for small scale dissipation). In any case, TO did not explain which model mechanism could generate a forward energy cascade over the meso-scale, whereas there are two fundamental obstacles to overcome. The first is that the QG approximation is fundamentally irrelevant for the mesoscale range: all the necessary approximations are no longer justified (e.g. the Rossby number becomes much larger than unity). Therefore, it remains unreasonable to hope that some 3D-like behaviour would occur over this range in the model, whereas it might occur in nature. Furthermore, the directions of cascades can be inferred from inviscid statistical equilibria of the systems [Kraichnan, 1971] and they are known to give for both 2D and QG turbulence upward energy and downward enstrophy cascades. Therefore, we can safely conclude - contrary to LTNCG - that there is no model evidence in favour of two downscale cascades and that furthermore, the existence of a possible QG generating mechanism is nontrivial. The fact that pressure coordinates are commonly used in meteorology - as advocated by LTNCG - does not prevent them from theoretically introducing biases in statistical analyses because their possible dynamical significance strongly depends on the validity of a number of approximations that we questioned above.

We believe that these elements of clarification plead in favour of the unifying theory of 23/9-D atmospheric turbulence over a wide range of scales [Schertzer and Lovejoy, 1984, 1985]. It indeed seems to result from the application of Occam's Razor applied to today's large quantity of unbiased atmospheric data. In this direction, we need to emphasize that - contrary to LTNCG subsume potential with kinetic energy does not solve the problem of addressing the vertical statistics. Indeed, the naïve scaling exponents, i.e. without including intermittency effects or logarithmic corrections [e.g. Kraichnan, 1967] are obtained by arguments à la Kolmogorov [Kolmogorov, 1941] that

are ultimately based on dimensional analysis of the relevant fluxes. Therefore in order to get different horizontal and vertical scaling exponents (scaling anisotropy) one needs a turbulent flux with physical dimensions different from energy. The theoretical choice of the buoyancy force variance flux [Schertzer and Lovejoy, 1984, 1985] still seems reasonable, although it was a bit bold when first proposed. Indeed, since the pioneering studies of [Adelfang, 1971; Endlich and Mancuso, 1968] and especially since the 1980's, there has been a growing body of evidence that the scaling is indeed anisotropic with at least roughly the predicted exponents c.f. the numerous references in our previous comment.

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