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Interactive comment on "Advective mixing in a nondivergent barotropic hurricane model" *by* B. Rutherford et al.

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The unstable breakdown of an annular ring of vorticity is examined in this paper to demonstrate the utility of various Eulerian and Lagrangian diagnostics of stirring/mixing, with possible application to hurricanes. An important message is that the widely-used finite-time Lyapunov exponent (FTLE) measure of particle separation works poorly in environments with strong radial shear. A strongly contrasting picture is obtained with a Lagrangian measure of particle separation in the direction perpendicular to the Lagrangian velocity (the so-called R-field). With a moving time-window the evolution of stirring/mixing can be described without reliance on a single long-time diagnostic.

The paper's message regarding FTLE is timely, in my opinion, but overlabored and

could be delivered more concisely, leaving room for further discussion of the Q- and R-fields. Enough is said regarding the limitation of FTLE for waves on a parallel or concentric shear flow (perhaps this paper will be a defining moment for that message) but more needs to said about the morphology of stirring/mixing in relation to the temporary, but spatio-temporally coherent, structures that emerge en route to monopole collapse.

After an initial linear growth of perturbations on the ring, their finite amplitude evolution can be divided into two stages: (i) crystallization (formation of eyewall mesovortices) and adjacent filamentation, and (ii) vortex interaction and merger, resulting is the loss of mesovortex symmetry and final collapse. Mesovortex formation alone does not destroy symmetry if by "symmetry" is meant "symmetric eddy statistics". Four nearly identical mesovortices form the vertices of a polygon. These are certainly Lagrangian coherent structures, albeit temporary ones. Their resemblance is lost as one or more vortices begin to dominate the others, in a not overwhelming way, with enough competitive balance to ensure that none of the original mesovortices survive intact. This renders the monopole final location very close to (if not exactly at) the original storm center. Here the notion of "predator-prey" has more the feel of "mutual annihilation" owing, in part, to the circular geometry. Vortex pairing in Cartesian geometry is analogous but less destructive.

It should be noted that the nature of stirring/mixing in the two stages is quite different. In the first stage, the flow resembles a nonlinear critical layer for dry barotropic instability of an annular ring. Fluid is homogenized within the four cat's eyes (mesovortices) but otherwise protected from mixing with the outer flow. Filamentation occurs between adjacent cat's eyes and on either side of the critical layer (Dunkerton et al., 2009). In the second stage, protection of mesovortex air is lost systematically, from one mesovortex to the next, and in-mixing of PV increases dramatically. Most of the in-mixing occurs at 8-12 hours, during which the central angular velocity increases 2-3 fold.

A positive focus on the dynamics and success of Q- and R-field diagnostics would help the presentation and lead hopefully to an improved understanding. The submitted paper is more utilitarian than scientific. The opportunity exists for a contribution that goes well beyond a technical examination of Lagrangian transport in 2D flows. I emphasize the point because technical issues tend to obscure the science. The motivation for the R-field is clear, notionally, but its exact meaning and implications less so. The non-symmetric/asymmetric dynamics introduces complexity, to be sure, but makes the problem substantially interesting. As can be seen from the above discussion, the implication of a meridional displacement hinges initially on where in azimuth the displacement is occurring. In one azimuth, a parcel is caught inside a mesovortex; in another, it may circle around the exterior before peeling off as a filament, or move aimlessly around the interior for a time, until propelled violently by the breakdown of mesovortices. Time plays an important role: persistence of LCSs is needed in order to influence stirring/mixing substantially. Here "persistence" refers to the longevity of eye mesovortices in relation to axial rotation. It is the relation of Q- and R-fields to one another and to the mesovortices that is interesting here. Discussion of this relationship would be more illuminating than merely to note the presence of R-structures and the tendency for parcels to move this way and that... unless, of course, such details shed light on the bigger issues.

In hindsight the problems of FTLE might have been avoided if contemporary authors were aware of the classic work of Andrews and McIntyre (1978) on a Generalized Lagrangian Mean (GLM) for nonlinear waves, and the development and refinement, subsequently, of a notional Modified Lagrangian Mean (MLM) using Eulerian adiabatic invariants such as PV and entropy to quantify and distinguish stirring (topological displacement of PV-theta tubes) from irreversible mixing (fate of parcels in relation to these tubes). Two such references are McIntyre (1980) and Dunkerton (1980). Additional aspects of GLM/MLM were developed or applied in recent years by Noburo Nakamura, Oliver Buehler and others; some fruitful work in this area remains to be done. A key advance of GLM/MLM is the introduction of a nonlinear perturbation displacement field, defined with respect to a Lagrangian mean flow, sheared or otherwise. Here I emphasize "perturbation" to note that differential advection by mean shear is subtracted out.

The resulting growth and dispersion of perturbation fields quantifies stirring/mixing in a way that is conceptually satisfying, equally if not more so than the method of Haller and Iacono (2003).

The nonlinear critical layer illustrates nicely the complexity of flow manifolds for near-monochromatic waves in shear, as well as the mathematical pathologies that arise in such topologies. I am not saying that Andrews and McIntyre solved the transport problem in nonlinear wave critical layers (or vortex merger) but their formalism provides a good framework for understanding, and might have provided useful guidance for development of FTLE-like methodology in such problems.

In the breakdown of vorticity annulus, a key question is whether dynamical structures can develop and survive long enough in order to affect transport systematically. The answer in this case, evidently and obviously, is "yes"... but the question could be posed by the authors nonetheless; it would NOT be foolish to do so. Consider the implications: on the one hand, transient sheared disturbances highlight the radius of minimum shear where radial exchange might be favored. Long-lived or quasi-modes, on the other hand, highlight the critical radius where wave and mean flow speeds are equal initially. We have in the latter case a persistent coherent Lagrangian structure – critical layer and cat's-eye pattern within – that pushes radial shear aside, redefining the local flow kinematics in a complicated way (azimuthal variation of Q-field, etc) and setting the stage for their mutual annihilation and monopole collapse. In either scenario, the role of radial shear is not merely to highlight differential advection, nor to expose the limitations of FTLE, but to make the dynamics positively interesting in several ways.

Specific comments:

1. p.16086, l.10: “Clear ridges” are associated with “manifolds” in FTLE. The following sentence advocates the R-field for “distinct ridges”, but their connection with manifolds is less clear than their connection to Rossby waves.

The paper seems to walk a fine line between promoting the R-field as a Lagrangian diagnostic while noting its attachment to propagating waves that aren't Lagrangian entities in general. Noting the role of wave critical layer, as appropriate, would help bring these ideas together insofar as the co-moving Lagrangian mean flow is zero at the critical radius of the waves (Dunkerton et al., 2009).

2. p.16086, l.18: The eyewall region contains a strong variation of Q- & R-fields once finite mesovortices have formed. More needs to be said about precisely where and how such mixing is occurring, how its consequences depend (initially vs ultimately) on azimuthal location.

3. p.16087, l.12: For a contemporary discussion of the axisymmetric model see Wirth and Dunkerton (2006).

4. p.16088, l.3: The non-uniqueness of Eulerian streamlines was discussed by Dunkerton et al. (2009). A co-moving frame that follows the azimuthal propagation of vortex Rossby waves would be considered optimal for flow visualization in this context.

5. p.16088, l.22: I cannot tell from this comment and later statements (e.g., p.16089, l.16) whether the authors advocate the Q-field for visualization of stirring/mixing, or not. I would like to see more discussion of the relation of Q- & R-fields and what this says about the azimuthal dependence of stirring/mixing.

6. p.16088, l.26: See major comment above.

7. p.16089, l.24: "show high mixing" as if you'd prefer to say "give spuriously a false sense of mixing" owing to azimuthal separation in shear, whereas "after polygonal eyewall formation is when the true mixing really occurs".

8. p.16090, l.22: See major comment above.

9. p.16093, l.22: The two terms on the rhs of (4) suggest attraction/repulsion (near a saddle of stream function) and rotation (about a gyre center) respectively. The language of "separation" and "contraction" seems vague if not misleading.

10. p.16094, l.22: Although the vector notation got off to a good start, here and hereafter it becomes incomprehensible. I recommend strict adherence to notational convention, either through symbols (e.g., use of dot product) or a summation convention using subscripts.

Additional references

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McIntyre, M.E., 1980: Towards a Lagrangian-mean description of stratospheric circulations and chemical transports. Phil. Trans. Roy. Soc. London, Series A.

Wirth, V., and T.J. Dunkerton, 2006: A unified perspective on the dynamics of axisymmetric hurricanes and monsoons. J. Atmos. Sci., 63, 2529–2547.

Interactive comment on Atmos. Chem. Phys. Discuss., 9, 16085, 2009.