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Mean wind profile in the ABL

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A semi-analytical solution for the mean wind profile in the Atmospheric Boundary Layer: the convective case

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Received: 26 June 2009 – Accepted: 25 August 2009 – Published: 23 September 2009

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Published by Copernicus Publications on behalf of the European Geosciences Union.

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Abstract

A novel methodology to derive the average wind profile from the Navier-Stokes equations is presented. The development employs the Generalized Integral Transform Technique (GITT), which joints series expansions with Integral Transforms. The new approach provides a solution described in terms of the quantities that control the wind vector with height. Parameters, such as divergence and vorticity, whose magnitudes represent synoptic patterns are contained in the semi-analytical solution. The results of this new method applied to the convective boundary layer are shown to agree with wind data measured in Wangara experiment.

1 Introduction

The Navier-Stokes equations provide the framework for the interpretation of atmospheric boundary layer flows. However, their analytical solution requires approximations, which are, in many cases, idealized and distant from the physical reality. Particularly, the mean wind profile is a solution of the governing equations whose derivation can be applied to a wide variety of natural processes.

The classical Ekman expression for the mean wind profile is, probably, the most famous example of an analytical solution of the simplified Navier-Stokes equations (Sorbjan, 1989; Stull, 1988). Such solution needs, however, the strong, non-realistic assumption that the vertical eddy diffusivities (K) are constant with height. In fact, Grisogono (1995) argues that "...it is a complicated, nonlinear function of the flow structure and there is no explicit relation between the boundary-layer profiles and K ".

The search for an analytical solution for the mean wind profiles in the atmospheric boundary layer under more realistic conditions has been a major focus of mathematical and physical research for a long time. For a given, imposed eddy diffusivity profile, solutions have been derived in a wide set of studies, reviewed by Zilitinkevich (1970), Monin and Yaglom (1971) and Grisogono (1995).

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Other studies have expanded the solution to include baroclinic and advective conditions (Miles, 1994; Bannon and Salem, 1995). Berger and Grisogono (1998) extended the results obtained by Grisogono (1995) for the baroclinic case and a generic vertical eddy diffusivity profile. Tan (2001) proposed a semi-geostrophic Ekman layer solution for variable eddy diffusivities and baroclinicity. This model united the solutions presented by Wu and Blumen (1982) and Grisogono (1995), finding that the mean wind structure depends on the inertial acceleration, eddy diffusivity and baroclinic pressure gradient. The study concluded that anti-cyclonic wind shear accelerates the flow, while cyclonic shear has the opposite effect. Wilson and Flesch (2004) used a three-layer simplified model that provided a good comparison to observed wind profiles.

In the present study, using the Generalized Integral Transform Technique (GITT), we derive a semi-analytical solution of the Navier-Stokes equation to obtain the mean wind profile in the atmospheric boundary layer. Such technique is a hybrid numerical-analytical method applied to the treatment and solution of partial differential equations (Özişik, 1993; Mikhailov and Özişik, 1984; Cotta, 1993). It provides a systematic, direct and efficient approximation to the solution of homogeneous and non-homogeneous, stationary and non-stationary, linear and non-linear boundary-value problems. The technique combines series expansion and an integration employing an inverse-transform pair. The PBL is discretized into N sub-intervals in such manner that inside each sub-region the eddy diffusivity is the average value (Moreira et al., 1999), that allows the use of realistic eddy diffusivity profiles, which depend on the physical characteristics of the energy-containing eddies. The nonlinear terms are written in terms of kinematical properties of the flow, such as divergence and vorticity, allowing the solutions to be interpreted in terms of large-scale synoptic conditions. The model results are compared to observed wind profiles obtained from the classical Wangara experiment (Clarke et al., 1971).

2 Semi-analytical model

2.1 Basic equations

Considering that the turbulent momentum fluxes can be parameterized by a first order closure (K-theory), the mean horizontal wind spatial distribution is given by the Navier-Stokes equation in the following form:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = f_c v - f_c v_g + \frac{\partial}{\partial x} \left(K_x \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial u}{\partial z} \right), \quad (1a)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = f_c u_g - f_c u + \frac{\partial}{\partial x} \left(K_x \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial v}{\partial z} \right), \quad (1b)$$

with $u \equiv u(x, y, z)$, $v \equiv v(x, y, z)$, $z_0 < z < z_i$,
 $0 < x < L_x$ and $0 < y < L_y$.

The equations above assume stationarity, no mean vertical motion, and that the molecular dissipation terms are neglectable. On the other hand, the flow is allowed to vary horizontally. The eddy diffusivities in each of the directions are represented by K_x , K_y and K_z .

To realistically reproduce the wind profile, it is important to consider the vertical variation of the eddy diffusivities. As a consequence, in the present approach, the planetary boundary layer (PBL) is discretized into N sublayers (Vilhena and Barichello, 1991; Moreira et al., 1999; Degrazia et al., 2001). In each of the sublayers, the eddy diffusivities and horizontal wind components assume vertically averaged values. To overcome the difficulties that arise from the nonlinear character of Eqs. (1a) and (1b), the advective terms are written in terms of the large-scale kinematical properties of the flow

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(Bluestein, 1992), namely, divergence and vorticity:

$$u_n \frac{\partial u_n}{\partial x} + v_n \frac{\partial u_n}{\partial y} = \frac{\delta}{2} u_n - \frac{\zeta}{2} v_n, \quad (2a)$$

$$u_n \frac{\partial v_n}{\partial x} + v_n \frac{\partial v_n}{\partial y} = \frac{\delta}{2} v_n + \frac{\zeta}{2} u_n. \quad (2b)$$

5 Where the index n refers to the different sublayers considered. With the assumptions above, Eqs. (1a) and (1b) can be written as:

$$\frac{\delta}{2} u_n - \frac{\zeta}{2} v_n = f_c v_n - f_c v_{gn} + K_{xn} \frac{\partial^2 u_n}{\partial x^2} + K_{yn} \frac{\partial^2 u_n}{\partial y^2} + K_{zn} \frac{\partial^2 u_n}{\partial z^2}, \quad (3a)$$

$$\frac{\delta}{2} v_n + \frac{\zeta}{2} u_n = f_c u_{gn} - f_c u_n + K_{xn} \frac{\partial^2 v_n}{\partial x^2} + K_{yn} \frac{\partial^2 v_n}{\partial y^2} + K_{zn} \frac{\partial^2 v_n}{\partial z^2}, \quad (3b)$$

with $z_n \leq z \leq z_{n+1}$, $0 < x < L_x$, $0 < y < L_y$ and $n=1, 2, \dots, N$.

10 Multiplying Eq. (3b) by i ($i \in \mathbb{C}$), and adding term by term to Eq. (3a), yields

$$K_{xn} \frac{\partial^2 w_n}{\partial x^2} + K_{yn} \frac{\partial^2 w_n}{\partial y^2} + K_{zn} \frac{\partial^2 w_n}{\partial z^2} - \left[\frac{\delta}{2} + \left(f_c + \frac{\zeta}{2} \right) i \right] w_n = -f_c w_{gn} i, \quad (4)$$

where $w_n = u_n + v_n i$, $w_{gn} = u_{gn} + v_{gn} i$, $z_n \leq z \leq z_{n+1}$, $0 < x < L_x$, $0 < y < L_y$ and $n=1, 2, \dots, N$.

15 Equation (4) is a differential equation on the complex variable function $w_n = u_n + v_n i$, whose solution provides an expression to the mean wind profile in terms of the flow divergence and vorticity.

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2.2 Boundary and interface conditions

The horizontal wind speeds are assumed to be constant at the lower boundary at $z=z_0$, and to be geostrophic at the upper boundary, the PBL top. Laterally, a horizontal domain is assumed, with dimensions $L_x \times L_y$. At the lateral boundaries, the wind components are given by the imposed divergence and vorticity, so that:

$$w_n = w_0 \text{ in } z=z_0 \text{ and } n=1, \quad (5a)$$

$$w_n = w_{gn} \text{ in } z=z_i \text{ and } n=N, \quad (5b)$$

and

$$w_n = \frac{1}{2} (-\zeta + \delta i) y \text{ in } x=0, \quad (6a)$$

$$w_n = \frac{1}{2} (\delta + \zeta i) L_x + \frac{1}{2} (-\zeta + \delta i) y \text{ in } x=L_x, \quad (6b)$$

$$w_n = \frac{1}{2} (\delta + \zeta i) x, \text{ in } y=0, \quad (6c)$$

$$w_n = \frac{1}{2} (\delta + \zeta i) x + \frac{1}{2} (-\zeta + \delta i) L_y \text{ in } y=L_y. \quad (6d)$$

At the interfaces between neighbor vertical layers, it is necessary to assume continuity of both the eddy fluxes and horizontal wind components.

$$w_n = w_{n+1}, \quad (7a)$$

$$K_{zn} \frac{\partial w_n}{\partial z} = K_{z(n+1)} \frac{\partial w_{n+1}}{\partial z}, \quad (7b)$$

with $z=z_n$ and $n=1, 2, \dots, (N-1)$,

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3 Solution

Equation (4) can be solved using the Generalized Integral Transform Technique – GITT (Mikhailov and Özişik, 1984; Özişik, 1993; Cotta, 1993). In this method, the solution function is expanded in terms of the eigenfunctions corresponding to the auxiliary problem (Sturm-Liouville), associated with the original problem. The eigenfunction orthogonality condition is used to determine the expansion coefficients, hence originating the integral transform and its inverse. Applying the integral transform, the partial derivatives in relation to variables x and y are removed, reducing the problem to an ordinary second-order differential equation on variable z . Therefore, once the transformed problem is solved, the inverse formula is used to obtain a solution to the original problem. The truncation order is selected according to the desired precision.

The auxiliary problem associated with w_n is:

$$\frac{K_{xn}}{K_{zn}} \frac{\partial^2 \psi(\lambda_{pq}, x, y)}{\partial x^2} + \frac{K_{yn}}{K_{zn}} \frac{\partial^2 \psi(\lambda_{pq}, x, y)}{\partial y^2} = -\lambda_{pq}^2 \psi(\lambda_{pq}, x, y), \quad (8)$$

with boundary conditions:

$$\psi = 0 \quad \text{in } x=0, \quad \text{and } x=L_x, \quad (9a)$$

$$\psi = 0 \quad \text{in } y=0, \quad \text{and } y=L_y. \quad (9b)$$

The solution to the problem above is (Mikhailov and Özişik, 1984; Özişik, 1993):

$$\psi_1(\beta_p, x) = \sin(\beta'_p x) \quad \text{for } p=1, 2, \dots \quad (10a)$$

$$\psi_2(\gamma_q, y) = \sin(\gamma'_q y) \quad \text{for } q=1, 2, \dots \quad (10b)$$

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$$\beta_p = \frac{p\pi}{L_x} \sqrt{\frac{K_{xn}}{K_{zn}}} \quad \text{for } p=1, 2, \dots \quad (11a)$$

$$\gamma_q = \frac{q\pi}{L_y} \sqrt{\frac{K_{yn}}{K_{zn}}} \quad \text{for } q=1, 2, \dots \quad (11b)$$

5 where $\beta'_p = \sqrt{\frac{K_{zn}}{K_{xn}}} \beta_p$ and $\gamma'_q = \sqrt{\frac{K_{zn}}{K_{yn}}} \gamma_q$ are the positive roots of equations $\sin(\beta'_p L_x) = 0$ and $\sin(\gamma'_q L_y) = 0$, respectively. Eqs. (10) and (11) are known, respectively, as the eigenfunctions and eigenvalues associated with the Sturm-Liouville problem. From the normalization integral:

$$N(\lambda_{pq}) = \int_0^{L_x} \int_0^{L_y} [\psi(\lambda_{pq}, x', y')]^2 dy' dx', \quad (12)$$

10 the corresponding norms can be obtained:

$$\frac{1}{N(\beta_p)^{1/2}} = \sqrt{\frac{2}{L_x}} \quad \text{for } p=1, 2, \dots \quad (13a)$$

$$\frac{1}{N(\gamma_q)^{1/2}} = \sqrt{\frac{2}{L_y}} \quad \text{for } q=1, 2, \dots \quad (13b)$$

From the GITT formalism, the solution to Eq. (4) is given by the expansion:

$$15 \quad w_n(x, y, z) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\psi(\lambda_{pq}, x, y)}{N(\lambda_{pq})^{1/2}} w_n(\lambda_{pq}, z). \quad (14)$$

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The integral transform is given by:

$$\overline{w}_n(\lambda_{pq}, z) = \frac{1}{N(\lambda_{pq})^{1/2}} \int_0^{L_x} \int_0^{L_y} [\psi(\lambda_{pq}, x', y') w_n(x', y', z)] dy' dx', \quad (15)$$

with $N(\lambda_{pq}) = N(\beta_p) N(\gamma_q)$.

Applying the operator

$$\frac{1}{N(\lambda_{pq})^{1/2}} \int_0^{L_x} \int_0^{L_y} \psi(\lambda_{pq}, x', y') dy' dx'$$

to Eq. (4) yields:

$$\frac{1}{N(\lambda_{pq})^{1/2}} \int_0^{L_x} \int_0^{L_y} \psi(\lambda_{pq}, x', y') \frac{\partial^2 w_n}{\partial z^2} dy' dx' + \frac{1}{N(\lambda_{pq})^{1/2}} \int_0^{L_x} \int_0^{L_y} \psi(\lambda_{pq}, x', y') \quad (16)$$

$$\left[\frac{K_{xn}}{K_{zn}} \frac{\partial^2 w_n}{\partial x'^2} + \frac{K_{yn}}{K_{zn}} \frac{\partial^2 w_n}{\partial y'^2} \right] dy' dx' - \left[\frac{\delta}{2K_{zn}} + \left(\frac{f_c}{K_{zn}} + \frac{\xi}{2K_{zn}} \right) i \right] \frac{1}{N(\lambda_{pq})^{1/2}} \int_0^{L_x} \int_0^{L_y} \psi$$

$$(\lambda_{pq}, x', y') w_n dy' dx' = - \frac{f_c w_{gn} i}{K_{zn}} \frac{1}{N(\lambda_{pq})^{1/2}} \int_0^{L_x} \int_0^{L_y} \psi(\lambda_{pq}, x', y') dy' dx', \quad (10)$$

with $w_n \equiv w_n(x', y', z)$ and $\lambda_{pq}^2 = \beta_p^2 + \gamma_q^2$.

Each term in Eq. (16) is solved by a different method. In the first integral, Leibniz rule is used. The second integral is solved by using Green's theorem, employing the eigenvalue problem (8), along with the boundary conditions (6). The third integral is directly substituted by definition (15). Finally, in the right-hand side, results (10) and (11) are used. This procedure yields in the following ordinary differential equation:

$$\frac{d^2 \overline{w}_n}{dz^2} - (\alpha_{1n} + \alpha_{2n} i) \overline{w}_n = - \frac{f_c w_{gn} G i}{K_{zn}} - C_n, \quad (17)$$

where

$$\alpha_{1n} = \lambda_{pq}^2 + \frac{\delta}{2K_{zn}}, \quad (18a)$$

$$\alpha_{2n} = \frac{f_c}{K_{zn}} + \frac{\zeta}{2K_{zn}}, \quad (18b)$$

5

$$G = \frac{2}{\sqrt{L_x L_y}} \left[\left(\frac{1 - \cos(p\pi)}{\beta'_p} \right) \left(\frac{1 - \cos(q\pi)}{\gamma'_q} \right) \right], \quad (19)$$

$$C_n = C_3 + C_6, \quad (20)$$

$$C_3 = \frac{2}{\sqrt{L_x L_y}} \frac{K_{xn}}{K_{zn}} \left[C_1 L_y \frac{\cos(q\pi)}{\gamma'_q} \right. \quad (21a)$$

$$\left. - C_2 \left(\frac{1 - \cos(q\pi)}{\gamma'_q} \right) \right], \quad (21b)$$

10

$$C_1 = -\frac{1}{2} (-\zeta + \delta i) \beta'_p [1 - \cos(p\pi)], \quad (21c)$$

$$C_2 = \frac{1}{2} (\delta + \zeta i) L_x \beta'_p \cos(p\pi), \quad (21d)$$

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$$C_6 = \frac{2}{\sqrt{L_x L_y}} \frac{K_{yn}}{K_{zn}} \left[C_4 L_x \frac{\cos(p\pi)}{\beta'_p} \right] \quad (21e)$$

$$-C_5 \left(\frac{1 - \cos(p\pi)}{\beta'_p} \right) \right], \quad (21f)$$

$$C_4 = -\frac{1}{2} (\delta + \zeta i) \gamma'_q [1 - \cos(q\pi)], \quad (21g)$$

$$C_5 = \frac{1}{2} (-\zeta + \delta i) L_y \gamma'_q \cos(q\pi). \quad (21h)$$

5 The boundary and interface conditions associated with the transformed problem in z direction can be determined applying definition (15) to Eqs. (5) and (7a) given, respectively, by:

$$\overline{w}_n = G w_0 \quad \text{in } z = z_0 \quad \text{and } n = 1, \quad (22a)$$

$$10 \overline{w}_n = G w_{gn} \quad \text{in } z = z_i \quad \text{and } n = N, \quad (22b)$$

$$\overline{w}_n = \overline{w}_{n+1}, \quad (23a)$$

$$K_{zn} \frac{\partial \overline{w}_n}{\partial z} = K_{z(n+1)} \frac{\partial \overline{w}_{n+1}}{\partial z}, \quad (23b)$$

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with $z=z_n$ and $n=1, 2, \dots(N-1)$.

The solution to Eq. (17) is given by:

$$\overline{w}_n(\lambda_{pq}, z) = A_n \exp[r_{1n} z] + B_n \exp[r_{2n} z] + \overline{w}_{nP}(\lambda_{pq}, z), \quad (24)$$

where

$$r_{1n} = (\alpha_{1n}^2 + \alpha_{2n}^2)^{1/4} \left[\cos\left(\frac{\theta_n}{2}\right) + i \sin\left(\frac{\theta_n}{2}\right) \right], \quad (25)$$

$$r_{2n} = (\alpha_{1n}^2 + \alpha_{2n}^2)^{1/4} \left[\cos\left(\frac{\theta_n}{2} + \pi\right) + i \sin\left(\frac{\theta_n}{2} + \pi\right) \right], \quad (26)$$

$$\alpha_{1n} \neq 0 \text{ e } \alpha_{2n} \neq 0, \theta_n = \arctan\left(\frac{\alpha_{2n}}{\alpha_{1n}}\right); \theta_n \neq \frac{\pi}{2} + \kappa\pi, \kappa=0, 1.$$

$$\overline{w}_{nP}(\lambda_{pq}, z) = \frac{1}{(\alpha_{1n}^2 + \alpha_{2n}^2)} \left[(\alpha_{2n} + \alpha_{1n} i) \frac{f_c w_{gn} G}{K_{zn}} + (\alpha_{1n} - \alpha_{2n} i) C_n \right], \quad (27)$$

$$A_n, B_n, r_{1n}, r_{2n}, \overline{w}_{nP} \in \mathbb{C}.$$

The boundary and interface conditions lead to the determination of constants A_n and B_n . Therefore, for each p and q , the resulting system is solved numerically.

Using the previous results in the Eq. (14), results

$$w_n(x, y, z) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{2}{\sqrt{L_x L_y}} \sin(\beta'_p x) \sin(\gamma'_q y) \overline{w}_n(\lambda_{pq}, z), \quad (28)$$

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where $\overline{w_n}(\lambda_{pq}, z)$ is given by the Eq. (24).

Finally, we obtain the components of the average wind, u_n and v_n , from the fact that $w_n(x, y, z) = u_n(x, y, z) + v_n(x, y, z) i$. In that case,

$$u_n(x, y, z) = \Re w_n(x, y, z), \quad (29a)$$

$$v_n(x, y, z) = \Im w_n(x, y, z), \quad (29b)$$

where \Re represents the real part of w_n , and \Im represents the imaginary part of w_n .

4 Eddy diffusivity

The eddy diffusivity vertical profiles employed in this study have been proposed by De-
 grazia et al. (2000). Such eddy diffusivities are based on Taylor's statistical diffusion
 theory, in which the shear buoyancy PBL spectra are modeled by means of a linear
 combination of the convective and mechanical forcings. Therefore, in the present case,
 such parameterization allowed reproducing the realistic case of a convective boundary
 layer where shear-generated turbulence occurs. The eddy diffusivities for such condi-
 tions are given by the following expression:

$$K_\alpha = 0.11 \sqrt{c_i} \left[\frac{z \psi_\epsilon^{1/3} w_* (z/z_i)^{1/3}}{(f_m^*)_i^{4/3}} + \frac{u_* z \phi_\epsilon^{1/3}}{(f_m)_i^{4/3}} \right], \quad (30)$$

where $c_i = \alpha_i \alpha_u^{-2/3}$ with $\alpha_u = 0.5 \pm 0.05$ (Champagne et al., 1977; Sorbjan, 1989) and
 $\alpha_i = 1, \frac{4}{3}, \frac{4}{3}$ for u, v and w components, respectively. $w_* = (u_*)_0 \left(\frac{z_i}{\kappa |L|} \right)^{1/3}$ is the convective
 velocity scale; $u_* = (u_*)_0^2 \left(1 - \frac{z}{h} \right)^{\alpha_1}$ is the local friction velocity, in which $\alpha_1 = 1.7$ (Wyn-
 gaard et al., 1974); $(u_*)_0$ is the superficial friction velocity; z_i is the convective PBL

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height; z is the height above the surface; $\psi_e = \frac{\epsilon_b z_i}{w_*^3}$ is the adimensional dissipation rate

functions, $\epsilon_b = (0.75)^{3/2} \left(w_*^3 / z_i \right)$ is the buoyant rate of TKE dissipation; $\phi_e = \frac{\epsilon_s k z}{u_*^3}$ is

the adimensional molecular dissipation rate functions associated with mechanical pro-

ductions, $\epsilon_s = \frac{u_*^3}{kz} \left(1 - \frac{z}{z_i} \right)$ is the mechanical rate of TKE dissipation (Højstrup, 1982); L

5 is the Monin-Obukov length and $\kappa = 0.4$ is the von Kármán constant; $(f_m^*)_i = \frac{z}{(\lambda_m)_i}$ is the

reduced frequency of the convective spectral peak, where $(\lambda_m)_i$ is the peak wavelength

of the turbulent velocity spectra. According to Kaimal et al. (1976) and Degrazia and

Anfonssi (1998), $(\lambda_m)_u = (\lambda_m)_v = 1.5z_i$ and

$(\lambda_m)_w = 1.8z_i \left[1 - \exp\left(\frac{-4z}{z_i}\right) - 0.0003 \exp\left(\frac{8z}{z_i}\right) \right]$.

10 The reduced frequency of the neutral peak, $(f_m)_i$, with

$(f_m)_u = 0.045 \left(1 + 117 \frac{f_c z}{(u_*)_0} \right)$, $(f_m)_v = 0.16 \left(1 + 33 \frac{f_c z}{(u_*)_0} \right)$ and

$(f_m)_w = 0.35 \left(1 + 15 \frac{f_c z}{(u_*)_0} \right)$; $f_c = 2\Omega \sin \phi$ is the Coriolis parameter.

5 Result

5.1 Model convergence analysis

15 Equation (28) expresses, in terms of many independent parameters, the mean wind profiles. Among these parameters are: the size of the horizontal area defined by L_x and L_y , the thickness Δz of the vertical sublayers over which the PBL was divided, the truncation order (p and q), and the values of large-scale divergence δ and vorticity ζ , which affect the lateral boundary conditions.

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The size of the horizontal domain has an appreciable impact on the solution, but only for small areas (Fig. 1). As L_x and L_y are successively increased from 1 km to 100 km, the solution becomes independent of the domain size for $L_x=L_y\geq 50$ km. It means that the present solution is meaningful only over horizontal areas as large as 50 km. From this point on, the solutions shown were obtained with $L_x=L_y=50$ km.

A similar convergence analysis was applied to the sublayer thickness Δz and the truncation orders p and q , leading to the conclusion that $\Delta z=5$ m and $p=q=9$ are values that warrant convergence of the mean wind profiles. The model results depend on the horizontal position within the domain, even when no large-scale divergence and vorticity are considered (Fig. 2). The horizontal variation is larger close to the domain boundaries, so that there is a good portion of the domain, near its center, for which the wind profiles do not vary largely in the horizontal. The following analysis considers the vertical profiles at the domain center only.

5.2 Comparison to observational data from the Wangara experiment

The Wangara experiment was conducted in Hay, New South Wales, Australia, from July to August 1967 (Clarke et al., 1971). Wind profiles were obtained every hour up to a 2-km height, using pilot balloons. A 16-m tower provided micrometeorological surface observations. In the present study, two convective days were chosen for comparison to the proposed model: days 33 and 40 (Table 1).

The wind magnitude profiles simulated by the model are similar to those observed in Wangara (Fig. 3). The simulated values for different large-scale synoptic conditions (in terms of divergence and vorticity) cover a range of wind magnitudes, generally in agreement with the observations. Far from the surface, the condition without divergence and vorticity is the one which departs mostly from the measurements. This same condition, on the other hand, provides the best match to observations at the lowest levels (inlet). Different combinations of vorticity and divergence have been applied for the comparison. There is no clear distinction among most of them, as can be seen in Fig. 3. However, an analysis based on statistical indices reveals that, while the

results are very similar for the wind magnitude (Table 2), the approximation for wind direction (Table 3) is improved when both the divergence and vorticity are positive.

Similar results were obtained for day 40 (Fig. 4). Indeed, the statistical indices (Tables 4 and 5) indicate that the model reproduced the observations even better than for day 33. In this case, the condition without divergence and vorticity showed the largest departure from the observations for wind magnitude both at upper and lower levels. Again, the best representation offered by the model for both wind magnitude and direction occurred with positive values of divergence and vorticity. Furthermore, the solutions with negative vorticity provided the worst approximation for wind direction.

In both days 33 and 40, the best approximations to the observed profiles were obtained for positive vorticity and divergence, consistent with the occurrence of anti-cyclonic large-scale flow at the period. Indeed, the synoptic surface pressure charts (Clarke et al., 1971) indicate the presence of a high-pressure system at the region, for both days 33 and 40. Such consistency is further evidence that the model is able to reproduce the wind field realistically.

6 Conclusions

In the present study, a novel approach was used to obtain the average wind profile from the Navier-Stokes equations. The method is based on the Generalized Integral Transform Technique (GITT), applied to the convective boundary layer discretized in sublayers. Such discretization allows using eddy diffusivities that vary vertically. GITT is a procedure that combines series development and integral transforms, leading to a final solution (Eq. 28) that contains the physical parameters determining the wind variability with height. Large-scale kinematical flow properties, such as divergence and vorticity are included in the solution, through the boundary conditions and the nonlinear advective terms of the original equations.

The results obtained are comparable to those found in the literature (Wilson and Flesch, 2004; Stull, 1988). Furthermore, the model provided a good comparison to

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observed data from Wangara experiment. The best approximations were obtained considering values of divergence and vorticity consistent with the synoptic charts from the experiment.

The main aim of this study is to establish an alternative method to determine the mean wind profiles. The method has been shown in detail, as well as its validation in comparison to observed data. From this point, it can be used for a further examination of a more generalized problem. As an example, in the approach taken here no temporal evolution is considered. The non-stationary problem can be solved using GITT along with Laplace transform applied to the time. Besides, the examples here were restrained to the convective case, but the development allows the use of the same approach for any stability condition. The use of appropriate eddy diffusivity profiles may lead to the determination of wind profiles under stable conditions as well.

Acknowledgements. This work was partially supported by CNPq and CAPES.

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Day	Hour	$-L$ (m)	z_i (m)	u_* (m/s)	u_{g0} (m/s)	v_{g0} (m/s)
33	15:00	2.8	1200	0.155	-5.32	-0.77
40	15:00	10	1200	0.14	1.7	-2.55

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Table 2. Statistical indices evaluating the wind magnitude profiles shown in Fig. 3.

Wind Speed (m/s)	<i>NMSE</i>	<i>FB</i>	<i>FS</i>	<i>R</i>	<i>FA2</i>
$\delta = \zeta = 0$	0.021	-0.107	-0.183	0.685	1.000
$\delta = 0; \zeta = 2.5f_c$	0.009	0.032	0.067	0.688	1.000
$\delta = -f_c; \zeta = 0$	0.007	0.004	0.110	0.719	1.000
$\delta = -f_c; \zeta = -f_c$	0.008	-0.024	0.071	0.711	1.000
$\delta = 0.5f_c; \zeta = -1.5f_c$	0.009	0.010	0.039	0.650	1.000
$\delta = -1.5f_c; \zeta = -0.5f_c$	0.009	0.040	0.193	0.702	1.000

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Table 3. Statistical indices evaluating the wind direction profiles shown in Fig. 3.

Direction (°)	<i>NMSE</i>	<i>FB</i>	<i>FS</i>	<i>R</i>	<i>FA2</i>
$\delta = \zeta = 0$	0.029	0.151	-0.583	-0.526	1.000
$\delta = 0; \zeta = 2.5f_c$	0.123	0.318	-1.121	-0.309	1.000
$\delta = -f_c; \zeta = 0$	0.023	0.129	-0.535	-0.579	1.000
$\delta = -f_c; \zeta = -f_c$	0.010	0.068	-0.446	-0.599	1.000
$\delta = 0.5f_c; \zeta = -1.5f_c$	0.235	0.437	-1.301	-0.172	1.000
$\delta = -1.5f_c; \zeta = -0.5f_c$	0.014	0.090	-0.470	-0.619	1.000

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Table 4. Statistical indices evaluating the wind magnitude profiles shown in Fig. 4.

Wind Speed (m/s)	<i>NMSE</i>	<i>FB</i>	<i>FS</i>	<i>R</i>	<i>FA2</i>
$\delta = \zeta = 0$	0.042	-0.187	-0.179	0.952	1.000
$\delta = -f_c; \zeta = 0$	0.007	-0.028	-0.038	0.941	1.000
$\delta = 0; \zeta = 1.5f_c$	0.008	-0.047	-0.095	0.954	1.000
$\delta = -f_c; \zeta = -f_c$	0.011	-0.071	-0.038	0.946	1.000
$\delta = -0.1f_c; \zeta = f_c$	0.012	-0.083	-0.107	0.960	1.000
$\delta = -1.5f_c; \zeta = -0.5f_c$	0.010	0.022	-0.001	0.914	1.000

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Table 5. Statistical indices evaluating the wind direction profiles shown in Fig. 4.

Direction (°)	<i>NMSE</i>	<i>FB</i>	<i>FS</i>	<i>R</i>	<i>FA2</i>
$\delta = \zeta = 0$	0.004	0.052	0.005	0.493	1.000
$\delta = -f_c; \zeta = 0$	0.002	0.040	0.101	0.604	1.000
$\delta = 0; \zeta = 1.5f_c$	0.011	0.091	-0.364	0.288	1.000
$\delta = -f_c; \zeta = -f_c$	0.001	0.013	0.340	0.850	1.000
$\delta = -0.1f_c; \zeta = f_c$	0.008	0.078	-0.249	0.346	1.000
$\delta = -1.5f_c; \zeta = -0.5f_c$	0.001	0.023	0.244	0.763	1.000

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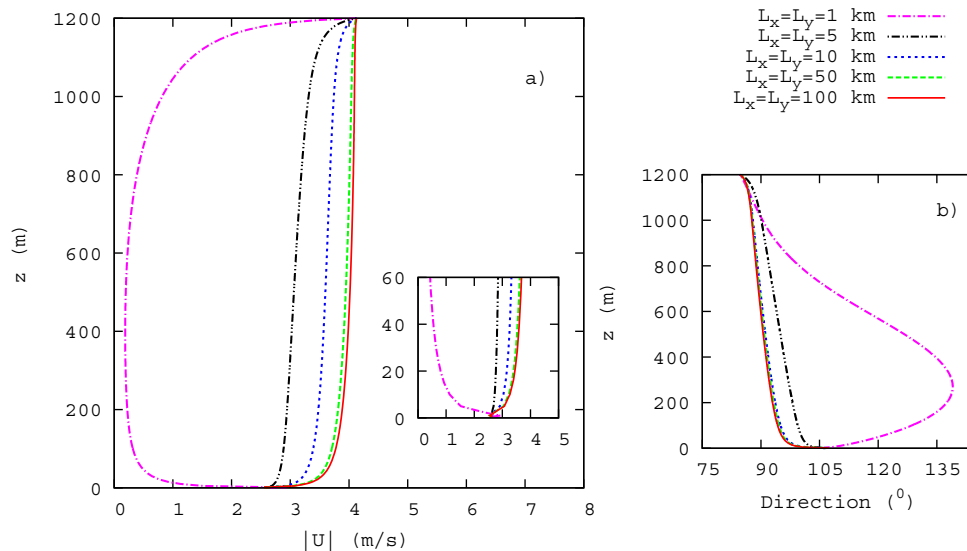


Fig. 1. Simulated vertical profiles of **(a)** wind magnitude and **(b)** wind direction, for different domain sizes, as indicated in legend. Inset shows the wind magnitude vertical profile for the lowest 60 m in detail.

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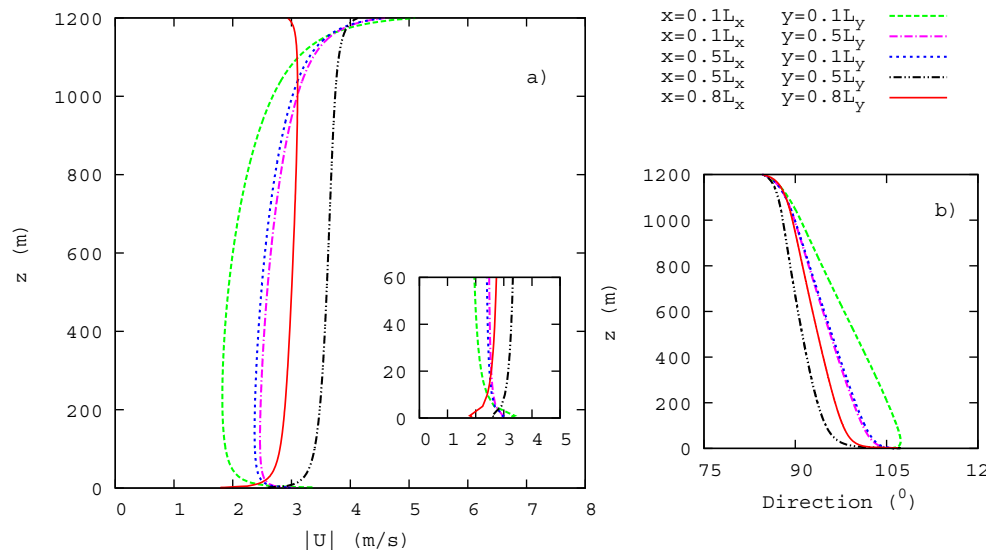


Fig. 2. Simulated vertical profiles of **(a)** wind magnitude and **(b)** wind direction, for different positions within the domain, as indicated in legend. Inset shows the wind magnitude vertical profile for the lowest 60 m in detail.

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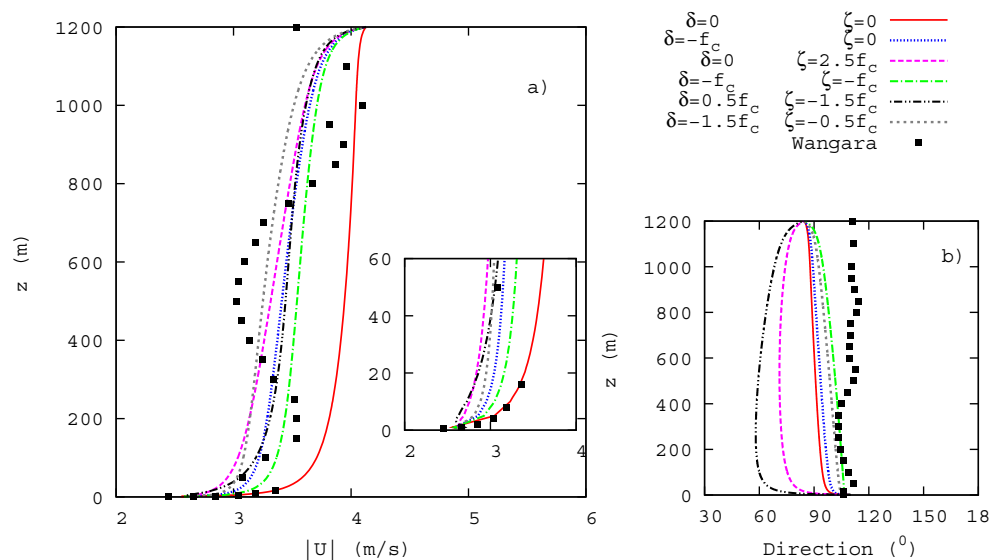


Fig. 3. Simulated vertical profiles of **(a)** wind magnitude and **(b)** wind direction for Wangara day 33, and different values of wind divergence and vorticity, as indicated in legend. Inlet shows the wind magnitude vertical profile for the lowest 60 m in detail. Squares represent data from the Wangara experiment for day 33. At Wangara, $f_c = -8.2 \times 10^{-5} \text{ s}^{-1}$.

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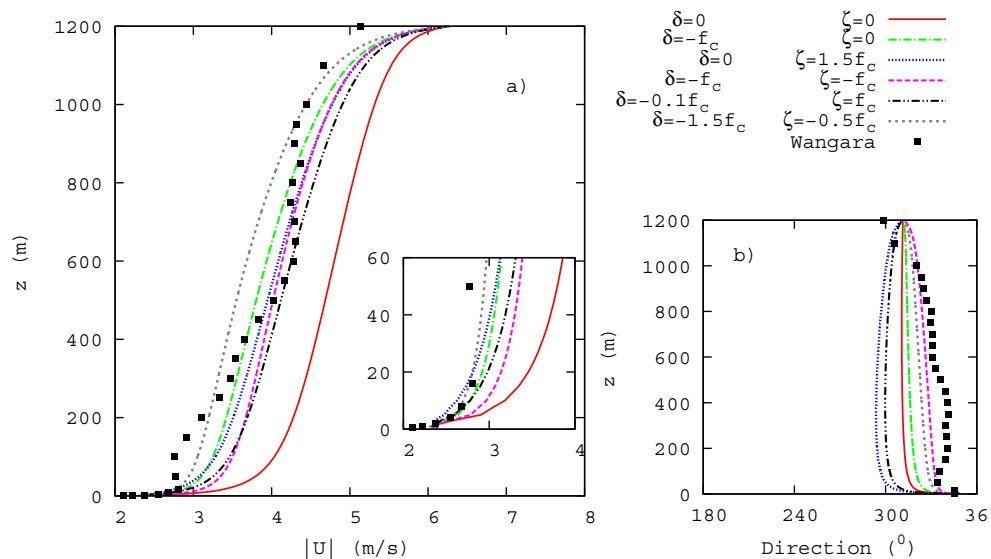


Fig. 4. Simulated vertical profiles of **(a)** wind magnitude and **(b)** wind direction for Wangara day 33, and different values of wind divergence and vorticity, as indicated in legend. Inlet shows the wind magnitude vertical profile for the lowest 60 m in detail. Squares represent data from the Wangara experiment for day 40. At Wangara, $f_c = -8.2 \times 10^{-5} \text{ s}^{-1}$.

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