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Interactive Comment

Interactive comment on "On the validity of representing hurricanes as Carnot heat engine" *by* A. M. Makarieva et al.

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In this comment we respond to three issues raised in the comments of Dr. Meesters (S8916, S9060, hereafter SCM1 and SCM3, respectively), namely (1) the estimate of dissipative heating, turbulent and molecular viscosity, (2) Bernoulli's equation and (3) the dissipative heat engine concept.

1. On turbulent and molecular viscosity and estimates of dissipative heating

In the discussion paper (DP, p. 17432) we wrote that "In the result of the replacement of molecular kinematic viscosity by eddy viscosity in the work of Bister and Emanuel (1998) and subsequent papers the magnitude of dissipative heating was overestimated by about 10^8 times." Dr. Meesters objected this statement by noting that in the stationary case turbulent dissipation equals thermal dissipation (SCM1), so that equating the

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two viscosities is justified. We responded (S8923) that hurricane is not a stationary system, so the rate of dissipation of turbulent eddies must not be equal to the rate of dissipative heating. That is, hurricane serves as an engine producing small turbulent eddies that accumulate within the hurricane area and are then transported far away from the hurricane. In the resulut, turbulent kinetic energy dissipates to heat on a large area at a significantly smaller rate than the rate of turbulent dissipation within the hurricane.

In his further comment on the issue (SCM3) Dr. Meesters stated that "since small eddies have a short lifetime, the consequences should not be exaggerated. The dissipation as calculated in the criticized papers may be inexact, but this is in the nature of their approach: there is still no good reason to assume that the (variable) correction factor would differ strongly from one."

As we show below, a good reason exists and can be quantified. In the stationary state, large turbulent eddies dissipate into smaller ones, the smaller ones into yet smaller ones and so on, with the smallest eddies dissipating to heat. Small eddies have a short lifetime, but large eddies do not, so each step in this sequence of dissipation events is characterised by its own rate depending on the properties of the eddy of a given size and the number (spatial density) of eddies of that size. As the dissipation proceeds with time, the characteristic air velocity of eddies diminishes. For this reason, in its later stage, the dissipation of turbulent kinetic energy of the hurricane goes through the same steps as dissipation of turbulent kinetic energy of a stationary, globally averaged circulation with horizontal mean velocities of the order of several m s^{-1}. Since this circulation is stationary, in this case rates of turbulent dissipation and thermal dissipation (dissipative heating) indeed coincide.

The volume-specific power of turbulent dissipation, according to formula 31.1 in Landau and Lifshitz (1987), which precedes formula 31.3 referred to in (SCM1), can be estimated as $\rho(\Delta v)^3/l$, where Δv is the characteristic change of air velocity in the maximum eddy corresponding to linear size l that is set by the geometry of the con-

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sidered problem. Considering the vertical dimension of the atmosphere h one can write for turbulent dissipation rate $\rho w^3/h$, where $w \sim (0.1 - 1)$ m s^{-1} is vertical velocity that changes by $\Delta w \sim w$ along the linear size h of the maximum eddy that is set by the atmospheric height scale. In the stationary case of large-scale circulation, the rate of turbulent dissipation, if estimated along the same formula, is $\rho \overline{w}^3/h$, where $\overline{w} \sim 10^{-3}$ m s^{-1} is the stationary vertical velocity of ascending/descending air masses. The ratio of these rates is $(w/\overline{w})^3 \sim 10^6 - 10^9$, an interval to which the factor of 10^8 estimated in the DP belongs. Dissipation rate per unit area of the Earth's surface is ρw^3 and $\rho \overline{w}^3$ for hurricane and stationary circulation, respectively.

There is another linear scale in the considered problem, namely the horizontal one. Writing the above formulae for the horizontal dimension L_H of the hurricane and L_E for large-scale stationary circulation we have for dissipation rates per unit volume $\rho u^3/L_H$ for hurricane and $\rho \overline{u}^3/L_E$ for stationary circulation, where $u \sim 50$ m s^{-1} is typical hurricane velocity, $\overline{u}$ is typical horizontal velocity of stationary circulation, $u/\overline{u} \sim 10$, $L_H/L_E > 10$. The ratio of these estimates is in the order of 10^4. For hurricane turbulent dissipation per unit area of the Earth's surface we have $\rho(u^3/L_E)h = \rho u^2 w \sim 10^3$ W m^{-2}. To the accuracy of coefficients of the order of unity namely this formula is used in the estimates of atmospheric turbulent dissipation (which is incorrectly equated to dissipative heating rate), see, e.g., (Businger and Businger (2001) J. Atm. Sci. 58: 3793, Fig. 1). (One makes use of the phenomenological drag coefficient C_D, which is in fact proportional to the ratio between vertical and horizontal velocities, $C_D \sim h/L_E$.)

However, the discrepancy between the two formulae, the one for horizontal linear scale and the one for vertical linear scale, indicates that this approach is not justified for the considered problem. Indeed, while in a turbulent eddy change Δv of velocity v along the eddy's characteristic linear scale can be considered as a dissipative change, change of horizontal velocity in the hurricane circulation has a different physical meaning. It is governed by the continuity equation $wL_H = uh$: similar to how liquid in the wide part of the tube flows more slowly than in the narrow part of the tube, so air flows

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in the horizontal direction via the vertical atmospheric cross-section of area hL_H more rapidly than in the vertical direction via the horizontal atmospheric cross-section of area $L_H^2 \gg hL_H$. This change of velocity magnitude when the flow changes its direction is not related to turbulent dissipation.

As discussed in the Authors Comment (AC S8904), turbulent friction force consists of two parts, the one depending on the weight of atmospheric column and the one proportional to the cube of velocity (identical to the formulae discussed above), see Eq. (7) in AC (S8904). Using the estimates obtained for the stationary circulation, the dissipative heating rate equal to turbulent dissipation rate is determined by the first – larger – term and can be estimated as $\rho g z_T \overline{u}/2 \sim 4$ W m^{-2}, where z_T is a linear scale characterizing surface roughness, g is the acceleration of gravity. For turbulent dissipation within the hurricane we have an order of magnitude larger value $\rho g z_T u/2 \sim 40$ W m^{-2}, which shows that the rate of turbulent dissipation in the hurricane is not equal to the rate of dissipative heating. (The aforementioned value of $\rho u^2 w/2 \sim 10^3$ W m^{-2} is the drag power of the hurricane, which is at least two orders of magnitude larger than the power of turbulent dissipation (AS S8912).) Instead, the hurricane produces turbulent eddies that are spread along the sea and land surface and ultimately dissipate to heat at a rate of several W m^{-2}. Remarkably, the globally and annually averaged hurricane power was found to be precisely of this order of magnitude (Trenberth and Fasullo (2007) J Geophys Res 112: D23107). To summarize, the flux of dissipative heating is negligible compared to other energy fluxes (e.g., of sensible or latent heat or of turbulent dissipation) that occur within the hurricane.

2. Integration of Bernoulli's equation

We stated (DP, p. S17428) that Bernoulli's equation (E1) in the work of Emanuel (1991)

$$d\left(\frac{1}{2}|\boldsymbol{V}|^2\right) + d(gz) + \alpha dp + \boldsymbol{F}\, d\boldsymbol{l} = 0 \qquad \text{(E1)}$$

was integrated incorrectly along the horizontal part of the streamline $a - c$ from the

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outer environment (a) to the hurricane center (c). Specifically, term $\int_a^c d(|\boldsymbol{V}|^2/2)$ for squared velocity was dropped from the integral despite that this term makes the major contribution to the integral as demonstrated in (AC S8923). Dr. Meesters first objected to our criticism and estimated this term to be negligible (SCM1), but subsequently re-evaluated the issue and admitted that this term was dropped from the integral by Emanuel (1991) incorrectly (SCM3). Nevertheless, the importance of this error and its implications remained unclear to Dr. Meesters, who remarked that he did not "understand why so much noise is made about this neglecting" (SCM3), mentioning, in particular, that in the discussion paper it was claimed that the hurricane "does not exist". However, putting $V = 0$ essentially means that velocity is zero and there is no hurricane.

More specifically, Bernoulli's equation contains a variable of velocity which is not present in the thermodynamic equations describing the two isotherms and two adiabates of Carnot cycle. The incorrect integration of Bernoulli's equation has led to the incorrect formula for work of Carnot cycle, formula (E16) of Emanuel (1991):

$$A \approx RT_s \ln \frac{p_a}{p_c}. \qquad (E16)$$

contradicting the one derived from explicit consideration of the processes of Carnot cycle, see formula (7) in Authors Comment (S7325).

For the warmer isotherm one has (see Eq. (1) in (AC S7325) and formula (8) in Emanuel (1991))

$$\Delta Q_s = A/\varepsilon = RT_s \ln \frac{p_a}{p_c} + L_v(q_c^* - q_a), \qquad (E8)$$

where $q_c^* = q_c^*(T_s, p_c)$ is water mixing saturated ratio in the hurricane center, q_a is water vapor mixing ratio outside the hurricane. Combining this equation with the incorrect formula (E16) allowed Emanuel (1991) to calculate pressure p_c in the hurricane center

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from the known p_a, q_a, T_s and T_0 (the two temperatures define Carnot efficiency $\varepsilon = (T_s - T_0)/T_s$). This is the main result of of Emanuel (1991).

In the meantime, correct consideration of Carnot cycle does not allow one to do that. As we noted in our response to Referee 3 and elsewhere in this discussion (AC S7325), consideration of Carnot cycle only allows one to know pressure difference $p_c - p_a$ if one knows heat input Q_s or vice versa. In other words, if written correctly, Eqs. (E16) and (E8) do not solve for p_c but yield a trivial equality, because the value of ε is itself derived from consideration of these equations.

This can be most transparently illustrated for "dry" Carnot cycle, where $A = R(T_s - T_0)\ln\frac{p_a}{p_c}$ and $Q_s = RT_s \ln\frac{p_a}{p_c}$, see (AC S7325). The equation $A = \varepsilon \Delta Q_s$, where $\varepsilon = (T_s - T_0)/T_s$, cf. Eq. (E8), is in fact not an equation but a trivial equality. This means that all conclusions of Emanuel (1991) are based on a mathematical and physical error that resulted in Eq. (E16) and, hence, that the framework does not exist.

3. Dissipative heat engine

In his third comment (SCM3) Dr. Meesters noted, in relation to the dissipative heat engine concept, (1) that he found "no time to look at the hurricane literature, and little time for reading the previous comments", (2) that he "cannot follow the accompanying arguments" in the discussion of the dissipative heat engine concept made in Authors Comment (S8193), (3) that the value of efficiency that can be infinite for the dissipative heat engine "is new" to Dr. Meesters and he would be interested in knowing more literature on the phenomenon.

It also appears from the comment, "in daily life we are surrounded by installations containing dissipative heat engines of some kind", that Dr. Meesters consider a dissipative heat engine to be the one where some dissipation (e.g., friction) occurs. In reality, dissipative heat engine is a theoretical concept, which implies recirculation of the dissipated energy back into mechanical work at one and the same temperature. Such engines do not exist.

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We have dwelt on the issue in great detail in our answers to Referee 1. Additionally, Dr. Sherman (S8953) provided a brilliant analogy between the jumping ball converting kinetic energy to potential one and vice versa (possible) and dissipative heat engine designed to convert heat to work and back (prohibited). The problem with the dissipative heat engine lies in the fact that it presumes conversion of work to heat and back at one and the same temperature. Specifically, this occurs at the warmer isotherm of Carnot cycle where all heat added to the cycle (including heat that just appeared from dissipation of mechanical work) is converted to work.

This can be traced in the equation for entropy change that was first introduced by Referee 1 (S7915, Eq. 2)

$$\Delta S = 0 = \frac{\Delta Q_s}{T_s} - \frac{\Delta Q_0}{T_0} + \frac{\Delta Q_A}{T_A}.$$

Here ΔQ_s is heat received by the engine from the heat source at the warmer isotherm and converted to work performed by the gas during its expansion, $\Delta Q_0 = \Delta Q_s$ is heat lost by the engine at the colder isotherm equal to work that was performed on the gas during its contraction. $\Delta Q_A = A$ is interpreted as heat added to the engine due to dissipation of mechanical work A at the warmer isotherm. However, in order that this term had the same physical meaning as the first term and could be interpreted within Carnot framework, this heat should be, similar to ΔQ_s, also converted to work during extension of the gas along the warmer isotherm, i.e. at the same temperature at which work dissipated to heat, $T_A = T_s$. This is another (among the many already made available in this discussion) illustration of the physical impossibility of the dissipative heat engine.

We emphasize that the flaw in this concept becomes immediately visible as soon as one considers the particular processes making up Carnot cycle (two isotherms and two adiabates) and traces what happens with heat and work during those processes. Such an analysis has not been attempted by Bister and Emanuel (1998) and subsequent papers that discussed this concept (neither was the issue mentioned in this discussion

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by Referee 1, Referee 3 or Dr. Meesters, who criticized our results). Instead, as pointed out by Dr. Sherman (S8953), Carnot cycle was indeed effectively treated as a mathematical black box at the level of proportionality between work and heat input (to which dissipated work is formally added) with proportionality coefficient formally equal to Carnot efficiency, $A = \varepsilon(\Delta Q_s + A)$. The neglect of the real physical processes behind Carnot cycle resulted in the formulation of the dissipative heat engine concept equivalent to the perpetual motion machine of the second kind.

Since the existence of such an engine is impossible, the above equation does not exist either. The main results of Bister and Emanuel (1998, Meteorol. Atmos. Phys., 65: 233), Emanuel (1999, Nature, 401: 665), Emanuel (2003, Annu. Rev. Earth Planet. Sci., 31: 75, Emanuel (2005, Divine wind: The history and science of hurricanes, OUP), Emanuel (2006, Physics Today, 59: 74) that are based on this equation are therefore physically unsound.

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