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# *Interactive comment on* "On the validity of representing hurricanes as Carnot heat engine" by A. M. Makarieva et al.

### A. M. Makarieva et al.

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Following the recommendation of Referee 2 (ACPD 8: S8531) on including "a full account of the new hurricane theory" into the revised version of the paper, here we summarize the theoretical framework that allows one to numerically explain atmospheric circulation events occurring at different spatial and temporal scales on the basis of considering the physical process of water vapor condensation as their major driver. This comment also responds to the question posed by Dr. Nobre (Nobre (2008) ACPD 8: S8669), namely why hurricanes and tornadoes do not arise over large forest areas such as, for example, the Amazon river basin, despite the high intensity of regional evaporation and condensation.

#### 1. Evaporative-condensational pressure gradient force

Condensation of water vapor lowers air pressure and creates a pressure gradient force.





What are the physical determinants of this force? In contrast to dry air, atmospheric water vapor finds itself under the action of two independent physical factors. On the one hand, as all other air gases, water vapor tends to aerostatic equilibrium described by the equation  $\partial p_v/\partial z - \rho_v(z)g = 0$ , where  $p_v$  is partial pressure of water vapor,  $\rho_v$  is its mass density. Using the equation of state for ideal gas  $\rho_v/M_v = p_v/(RT)$ ,  $M_v = 18 \text{ g mol}^{-1}$  is water vapor molar mass,  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$  is the universal gas constant, T is absolute temperature, the condition for aerostatic equilibrium of water vapor can be written as

$$\frac{1}{p_v(z)} \frac{\partial p_v}{\partial z} = -\frac{1}{h_v(z)},$$

$$h_v(z) \equiv \frac{RT(z)}{M_v g}, h_v(0) = 13.5 \text{ km}.$$
(1)

Eq. (1) means that in aerostatic equilibrium partial pressure  $p_v(z)$  of water vapor diminishes exponentially with height, by e times per each 13.5 km or twice per each 9 km.

On the other hand, partial pressure of water vapor cannot exceed the saturated partial pressure  $p_{H_2O}(z)$ ,  $p_v(z) \le p_{H_2O}(z)$ , the latter depending on temperature T(z) as dictated by Clausius-Clapeyron equation. This equation can be written in a form similar to that of Eq. (1):

$$\frac{1}{p_{\mathsf{H}_2\mathsf{O}}(z)} \frac{\partial p_{\mathsf{H}_2\mathsf{O}}(z)}{\partial z} = -\frac{1}{h_{\mathsf{H}_2\mathsf{O}}(z)},\tag{2}$$

$$h_{\text{H}_2\text{O}}(z) \equiv h_v(z) \frac{\Gamma_{\text{H}_2\text{O}}(z)}{\Gamma}, \ h_{\text{H}_2\text{O}}(0) = 2.4 \text{ km}$$

$$\Gamma \equiv -\frac{dT(z)}{dz}, \ \Gamma_{\rm H_2O}(z) \equiv \frac{T(z)M_v g}{L}, \ \Gamma_{\rm H_2O}(0) \equiv \Gamma_{\rm H_2O} = 1.2 \ {\rm K \ km^{-1}}.$$
(3)

Here L is molar vaporization heat of water vapor (latent heat). Note that for the circulation patterns to be considered, the above use of Clausius-Clapeyron equation (derivable from the consideration of Carnot cycle (Makarieva et al. (2008) ACPD 8: S8340)) is the single connection with thermodynamics.

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It follows immediately from Eqs. (1) and (2) that saturated water vapor can only be in aerostatic equilibrium when  $h_v(z) = h_{H_2O}(z)$ , i.e.  $\Gamma = \Gamma_{H_2O} = 1.2 \text{ K km}^{-1}$  (Makarieva, Gorshkov (2007) HESS 11: 1013). At  $\Gamma > \Gamma_{H_2O}$  saturated water vapor cannot be in equilibrium. The difference between the right-hand and the left-hand parts of Eq. (1) is then not equal to zero and represents an upward force  $f_E$  acting on a unit mass of moist air:

$$f_E(z) = \frac{\partial p_{\mathsf{H}_2\mathsf{O}}(z)}{\partial z} - g\rho_{\mathsf{H}_2\mathsf{O}}(z) = \frac{p_{\mathsf{H}_2\mathsf{O}}(z)}{h_{\mathsf{H}_2\mathsf{O}}(z)} \left(1 - \frac{\Gamma_{\mathsf{H}_2\mathsf{O}}(z)}{\Gamma}\right). \tag{4}$$

The observed tropospheric mean temperature lapse rate is  $\Gamma = 6.5$  K km<sup>-1</sup>, which is six times a larger value than  $\Gamma_{\text{H}_2\text{O}}$  (3). Therefore the last term,  $\Gamma_{\text{H}_2\text{O}}/\Gamma$ , in brackets (4) is only 1/6.

Work  $A_E = \int_0^\infty f_E(z)dz$  performed by force  $f_E$  when raising a unit volume of moist air along the entire atmospheric column is, to the accuracy of a few per cent neglecting the change of absolute temperature in the lower part of the atmosphere up to  $h_{\text{H}_2\text{O}}$ ,  $[(T(z) - T(0)]/T(0) \leq \Gamma h_{\text{H}_2\text{O}}/T(0) = 0.05$ , equal to

$$A_E = f_E(0)h_{\mathsf{H}_2\mathsf{O}}(0) = p_{\mathsf{H}_2\mathsf{O}}(0)\left(1 - \frac{\Gamma_{\mathsf{H}_2\mathsf{O}}}{\Gamma}\right) = \Delta p \equiv \rho \frac{u_E^2}{2}.$$
 (5)

Velocity  $u_E$  has the meaning of vertical velocity of air that has been accelerated by force  $f_E$  along the entire atmospheric column. Force  $f_E$  and work  $A_E$  are present everywhere in the atmosphere where there is saturated water vapor. Since force  $f_E$  arises due to condensation of water vapor in the atmosphere, which can only be sustained if there is a compensating process of evaporation from the hydrosphere, it is logical to term this force as the evaporative-condensational force or E-force, for brevity. Note that since  $\Gamma_{\rm H_2O}/\Gamma = 1/6$ , work  $A_E$  does not depend on latent heat to the accuracy of 17%, Eq. (5).

The moist atmosphere of Earth, which exists in contact with liquid hydropshere in the presence of a supracritical vertical lapse rate of air temperature  $\Gamma > \Gamma_{H_2O}$ , is not in S8906

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equilibrium. (As such, it cannot be characterized by the notions of stable, unstable or neutral equilibrium.) Force  $f_E$  that arises due to the absence of equilibrium causes the atmosphere to circulate continuously along a variety of linear scales. The associated vertical mixing of atmospheric layers with different water vapor content results in the fact that the global mean relative humidity  $R_H$  at the surface becomes less than unity,  $R_H \sim 0.8$  (Held, Soden (2000) Annu. Rev. Energy Env. 25: 441). This means that water vapor becomes saturated starting from some height  $z_H \sim 500$  m. This does not change the value of  $A_E$  in any considerable way.

Depending on the rate at which condensation of water vapor occurs, the resulting atmospheric circulation phenomena can be either stationary or episodic. In hurricanes and tornadoes the rate of water vapor condensation is determined by the maximum velocity  $u_E$  of air masses accelerated by force  $f_E$ ; condensation rate exceeds the evaporation rate dictated by solar power by many orders of magnitude. The stationary pattern is therefore composed of long periods of relative calmness (when water vapor slowly accumulates in the atmosphere) intermitted by intense hurricane or tornado events accompanied by rapid condensation of the previsouly accumulated water vapor.

However, under conditions when the turbulent surface friction is sufficient to impede acceleration of air masses, a stationary state is possible when the rate of condensation is equal to the rate of evaporation at any moment of time. Such a situation arises when there are two large adjacent areas of comparable linear size of the order of several thousand kilometers and when the evaporation rate in one of the two areas (the acceptor area) is, for some physical reason, consistently higher than in the other (the donor area). In this case there forms a large-scale atmospheric circulation pattern with surface air flowing from the donor to acceptor area. This can be called the evaporative-condensational pump. This horizontal air flow compensates the ascending and descending motion of air masses in the acceptor and donor areas, respectively. Essentially, in this case pressure difference  $\Delta p$  (5) is distributed along the total length of the streamline, which horizontal part is three orders of magnitude larger

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than the vertical one (atmospheric height) (see Appendix 2 and below). For this reason, the non-equilibrium pressure difference in the vertical dimension of the streamline becomes thousand times smaller than  $\Delta p$  (5), which prohibits the origin of hurricanes and tornadoes (as well as of other circulation irregularities associated with floods and droughts (Makarieva, Gorshkov (2007) HESS 11: 1013)). Such large regions with inherently different evaporation rates can be exemplified by large river basins covered by natural forests (acceptor) + the adjacent ocean (donor); ocean (acceptor) and adjacent desert (donor), as well as by the part of Hadley circulation cell in the Intertropical Convergence Zone. This responds to the question of Dr. Nobre (Nobre (2008) ACPD 8: S8669) on why there are no hurricanes either in the Amazon basin or in the oceanic region immediately adjacent to it.

Hurricanes and tornadoes arise when the linear size of the donor and acceptor areas diminishes to the values when the power of turbulent surface friction becomes sufficiently smaller than the power of the evaporative-condensational force. Consider the following analogy (be warned that it has its limitations). A diesel locomotive pulls a train consisting of 50 carriages at a constant low velocity. In this stationary case the total pulling power of the locomotive is equal to 50 times the power of friction between each carriage and the railway. This is comparable to a stationary large-scale atmospheric circulation. Now if one detached 49 carriages from the train and left the locomotive to haul only one carriage, the locomotive power would appear much greater than the diminished friction power. In the result, the train would move with acceleration until all the fuel were used up. The resulting velocity would be much higher than in the first case of a long train. This situation could be compared to hurricane or tornado. The locomotive pulling force, which is equivalent to the evaporative-condensational force in the atmosphere, is the same in both cases, yet it produces fundamentally different patterns of motion. Below we consider these patterns in quantitative terms.

#### 2. Tornado, hurricane and large-scale stationary circulation

The simplest case, when work  $A_E$  is released and the locally accumulated potential en-

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ergy per unit volume  $p_{H_2O} \approx \Delta p$  is converted to kinetic energy  $\rho u_E^2/2$ , is the case when intense condensation of water vapor occurs in a local area surrounded by dry areas where water vapor concentration is very low, like when tornadoes develop in the semideserts of North America. In this case the ascending air flow in the condensation area sucks in dry air masses from the neighboring areas. The converging air streams approach the condensation area with some non-zero angular momenta, which inevitably results in a spiral-like rotating structure of the tornado. This structure is therefore an immediate consequence of the vertical force  $f_E$  acting in the three-dimensional space. Tornado further moves in the direction of maximum water vapor concentration. When all local water vapor is used up, tornado is extinguished, since the neighboring areas are dry and cannot supply more water vapor. The next tornado occurrence will be due to a moment when, after a prolonged period of water vapor accumulation in the atmosphere, moist air masses again become spatially concentrated amidst the otherwise dry area.

We now consider two extensive (oceanic and/or land) regions where water vapor is present in the atmosphere in approximately equal quantities, but condensation in one region, termed here the acceptor region, is more intense than in the other (donor) region. The ascending air flow caused by condensation in the acceptor region leads to the inflow of moist air masses from the donor region; the imported water vapor serves to sustain the condensation process in the acceptor region. We denote the approximately equal lengths of acceptor and donor regions for  $L_E$ , their width (equal to the length of the border between them) for D, vertical velocity of air masses ascending within the donor region for w, horizontal air velocity for u. The mass-conserving equality between the horizontal air flux entering the donor region via vertical atmospheric cross-section  $Dh_{H_2O}$  with velocity u ( $h_{H_2O}$  is the scale height of the vertical distribution of atmospheric water vapor, see Eq. (2), and, hence, describes the part of the atmospheric column where the evaporative-condensational force  $f_E$  is in action) and the vertical air flux leaving the donor region via horizontal atmospheric area  $DL_E$  (the continuity equation)

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reads as

$$L_E w = h_{\mathsf{H}_2\mathsf{O}} u.$$

In order to understand the structure of atmospheric circulation in these regions it is necessary to take into account the turbulent friction forces that impede acceleration of air masses (that would otherwise occur under the action of force  $f_E$  and work  $A_E$ ) and lead to the formation of a stationary circulation pattern.

Total turbulent friction force  $f_T$  consists of two parts: the one independent of wind velocity – this part is determined by weight of atmospheric column proportional to  $\rho g$ , and the one dependent on vertical velocity of air masses ascending within the atmospheric column. From the dimensional considerations  $f_T$  acting on unit air volume can be written as

$$f_T = \frac{\rho g z_T}{2h_{\rm H_2O}} + \frac{\rho w^2}{2h_{\rm H_2O}}.$$
 (7)

Here  $z_T$  is the characteristic height up to which the horizontal air flow is influenced by surface roughness. The value of  $\rho g z_T/2$  represents turbulent friction force per unit surface area (turbulent friction pressure, multiplier 1/2 is introduced for convenience). The corresponding force per unit air volume averaged over the atmospheric column is therefore  $\rho g z_T/2h_{\rm H_2O}$  (7). The second term describes power loss of the streamflow due to formation of turbulent eddies. In similarity to Eq. (5), the second term represents force which work is equal to kinetic energy  $\rho w^2/2$  of an eddy budding from the main flow and developing along the atmospheric height  $h_{\rm H_2O}$  (Appendix 1).

The circulation is stationary in strict terms when the rate of condensation is equal to the rate of evaporation sustained by solar power for any period of time and when the power of evaporative-condensational force  $f_E$  is equal to the power of turbulent friction force  $f_T$ . These conditions can be written for work  $A_E$  or power  $W_E$ , see Eq. (6):

$$W_E = f_E \overline{w} = f_T \overline{u}, \quad A_E = f_E h_{\mathsf{H}_2\mathsf{O}} = f_T L_E.$$
(8)

(6)

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Here vertical velocity  $w = \overline{w}$  is the average vertical velocity of upwelling moist air masses that corresponds to the mean evaporation rate from the surface,  $\overline{w} \sim 10^{-3} \text{ m s}^{-1}$  (Makarieva, Gorshkov (2007) HESS 11: 1013). Due to the very small magnitude of  $\overline{w}$  the last term in Eq. (7) can be neglected. Along the horizontal part of the streamline turbulent friction force  $f_T$  remains therefore constant and independent of velocity (Appendix 1), so its work  $f_T L_E$  grows linearly proportionally to length  $L_E$  of the acceptor region. Using Eq. (8), for average horizontal velocity  $\overline{u}$  and linear dimension of the acceptor region  $L_E$  we have

$$\overline{u} = K\overline{w}, \ L_E = Kh_{\mathsf{H}_2\mathsf{O}}, \ K \equiv \frac{f_E}{f_T} = \frac{u_E^2}{gz_T}.$$
(9)

In the expression for the dimensionless coefficient K all magnitudes except for  $z_T$  are quantified theoretically, see Eqs. (2) and (5). Height  $z_T$  can be determined phenomenologically by comparing Eq. (9) with the available empirical data. Global mean wind velocity estimates as  $\overline{u} \sim 7 \text{ m s}^{-1}$  (Gustavson (1979) Science 204: 13) and  $\overline{w}$  as  $\overline{w} \approx 1.3 \text{ mm s}^{-1}$  (Makarieva, Gorshkov (2007) HESS 11: 1013), which gives  $K \approx 5400$  in Eq. (9). (For  $u_E = 60 \text{ m s}^{-1}$  we have  $z_T \sim 0.1 \text{ m}$ .) Finally, using Eqs. (9) and (2), we have  $L_E \sim 10^4 \text{ km}$ . This estimate shows that, in order to enjoy a stable stationary circulation continuously sustained by evaporation and solar radiation, the donor and acceptor regions must be very large.

Note an essential detail. According to Eqs. (8) and (5) the turbulent friction force is equal to  $f_T = \Delta p/L_E$ . In the stationary case, when the horizontal acceleration is absent, pressure gradient force should be equal to turbulent friction force. This means that pressure difference  $\Delta p$  is uniformly distributed along the horizontal part of the streamline with a constant pressure gradient equal to  $\Delta p/L_E = f_T = \rho g z_T/2h_{\rm H_2O}$ . As the horizontal part of the streamline is thousand of times longer than the vertical part,  $L_E \sim 10^3 h_{\rm H_2O}$ , the non-equilibrium pressure difference along the vertical part of the streamline is thousand of times smaller than the total pressure difference  $\Delta p$  (see also Appendix 2). In the considered case the deviation from hydrostatic equilibrium for air

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as a whole is practically unobservable. (See (Makarieva, Gorshkov (2007) HESS 11: 1013, Section 3.1) on the difference between the physical notions of aerostatic versus hydrostatic equilibrium.)

With decreasing linear size  $L_E$  of the donor and acceptor regions work  $f_T L_E$  of the turbulent friction force diminishes and becomes smaller than work  $f_E h_{H_2O}$  of the evaporative-condensational force. Work of the turbulent friction force becomes negligibly small when  $L_E$  diminishes by one order of magnitude as compared to the stationary value and becomes less than  $10^3$  km. This is the spatial domain for hurricane development:

$$h_{\rm H_2O} \sim 2 \ \rm km \ll L_E < 10^3 \ \rm km.$$
 (10)

In the view of Eq. (7), vertical velocity w becomes tens and hundreds of times larger than the stationary value  $\overline{w}$ . The rate of condensation is, unlike in the stationary case, no longer related to the evaporation rate. However, since  $L_E \gg h_{H_2O}$  and, hence,  $w \ll u$ , the major part of pressure difference  $\Delta p$  that appears due to water vapor condensation still falls on the horizontal part of the streamline. Thus, to a good approximation ( $h_{H_2O} \ll L_E$ ,  $w \ll u$ ), hurricane represents a two-dimensional wind structure. Note that the velocity-dependent turbulent friction force described by the second term in Eq. (7) remains negligibly small within the hurricane due to  $w \ll u$ . (This term becomes the major one in tornado at  $w \sim u$ , where it limits the stationary value of velocity.) Hurricane intensity is limited not by the turbulent friction force, but by the inertial forces of air masses that are sucked in towards the hurricane center from the neighbouring areas. Radial velocity u developed near the windwall is determined by Bernoulli's equation (Appendix 2):

$$\rho \frac{u^2}{2} = \Delta p \approx p_{\mathsf{H}_2\mathsf{O}} \left( 1 - \frac{\Gamma_{\mathsf{H}_2\mathsf{O}}}{\Gamma} \right). \tag{11}$$

As the converging air masses approach hurricane center they curve as dictated by their initial angular momenta and Coriolis acceleration. This leads to a non-uniform \$8912

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distribution of pressure gradient along the streamline and results in the formation of the hurricane eye where air pressure drops exponentially towards the center compensating the centrifugal forces (see, e.g., Holland (1980) Mon. Wea. Rev. 108: 1212). However, the primary determinant of the hurricane wind structure is the pressure difference  $\Delta p$ , Eq. (11), related to condensation of water vapor. Hurricane arises in the region of maximum intensity of water vapor condensation. Hurricane intensity is therefore largely determined by the intensity of condensation of water vapor accumulated within the very area of hurricane development rather than by the intensity of water vapor import from the neighboring areas.

Finally, when horizontal size  $L_E$  diminishes to the values of the order of atmospheric height  $h_{H_2O}$ , there appears a possibility of tornado formation, as discussed above.

#### Appendix 1. Turbulent friction force

Here we give a more detailed derivation of Eq. (8) that includes turbulent friction force  $f_T$  (7). Power of the upward evaporative-condensational force is equal to  $\rho(u_E^2/2)wDL_E$  for the entire atmospheric column and  $\rho(u_E^2/2)w$  per unit area of the Earth's surface. Under the action of turbulent friction air eddies bud off from the main streamflow as it propagates along the surface. These eddies continue to move within the main streamflow filling the atmospheric column. Rotational velocity  $u_T$  and kinetic energy density  $\rho u_T^2/2$  of these small eddies does not depend on streamflow velocity u, but is determined by weight of atmospheric column and linear size  $z_T$  related to the characteristic roughness height of the surface,  $\rho u_T^2 = \rho g z_T$ . As the main stream passes along the surface, a new small eddy is formed every  $z_T/u$  seconds. This confines the total power of the turbulent friction force at the surface as  $\rho(u_T^2/2)(u/z_T)DLz_T$ , where  $DLz_T$  is the volume where the surface turbulent friction force is acting and the small eddies are formed. Power of turbulent friction force per unit surface area is then  $\rho(u_T^2/2)u$ .

Further above the surface the main horizontal streamflow is decelerated by the as-

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cending flows having vertical velocity w that form eddies with linear size of the order of atmospheric scale height  $h_{H_2O}$ . New large eddies bud off from the main streamflow every  $h_{H_2O}/u$  seconds. Total power of turbulent friction force far from the planetary surface is  $\rho(w^2/2)(u/h_{H_2O})DLh_{H_2O}$ , where  $DLh_{H_2O}$  is the atmospheric volume where this force acts and the large eddies are formed; per unit surface this power is  $\rho(w^2/2)u$ . Equating the powers of the evaporative force and the total turbulent friction force one obtains 1 - 2 - 1 - 2 - 1 - 2 - 1 - 2 - 2

$$\frac{1}{2}\rho u_E^2 w = \frac{1}{2}\rho u_T^2 u + \frac{1}{2}\rho w^2 u \approx \frac{1}{2}\rho u_T^2 u, \quad u_T^2 = gz_T,$$
(12)

which, given Eq. (7), coincides with Eq. (8).

Note that in the stationary case when the evaporative-condensational pump is in action, so that air flows without acceleration, total power  $W_E$  of the evaporative-condensational force  $f_E$  is spent on formation of turbulent eddies filling the atmospheric column as the streamflow propagates along the surface. In hurricanes and tornadoes power  $W_E$  significantly exceeds the power of turbulent eddies. It is converted to the power of the main streamflow, which results in high wind velocities.

#### Appendix 2. Spatial distribution of pressure difference $\Delta p$

Euler equation for the horizontal part of the streamline dl, which is parallel to horizontal velocity u, has the following form:

$$\frac{1}{2}\rho\frac{\partial \boldsymbol{u}^2}{\partial \boldsymbol{l}} + \frac{\partial p}{\partial \boldsymbol{l}} + f_T\frac{\boldsymbol{u}}{\boldsymbol{u}} = 0.$$
(13)

For the vertical part of the streamline Euler equation becomes

$$\frac{1}{2}\rho\frac{\partial \boldsymbol{w}^2}{\partial z} + \frac{\partial p}{\partial z} + \rho g = 0.$$
(14)

In the stationary case, when air masses move with constant horizontal velocity u along distance  $L_E$  from the donor region with lower evaporation rate to the acceptor region

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with higher evaporation rate characterized by vertical air velocity  $\overline{w}$ , we have, see Eq. (7):

$$\frac{\partial \boldsymbol{u}^2}{\partial \boldsymbol{l}} = 0, \quad -\frac{\partial p}{\partial l} = f_T = \text{const}, \quad \boldsymbol{w} \sim \overline{\boldsymbol{w}} = \frac{h_{\text{H}_2\text{O}}}{L_E} \boldsymbol{u} \ll \boldsymbol{u}. \tag{15}$$

From Eq. (15) we have for the integral along the whole streamline

$$\Delta p = f_T L_E, \quad \int_0^{M_2 0} \left(\frac{\partial p}{\partial z} + \rho g\right) dz = \frac{1}{2} \rho \overline{w}^2 \ll \Delta p = \frac{1}{2} \rho u_E^2. \tag{16}$$

That is, total pressure difference  $\Delta p$ , Eq. (5), that forms due to water vapor condensation, is distributed along the horizontal part of the streamline, see Eq. (8), while air along the vertical part, due to its small linear size, practically remains in hydrostatic equilibrium.

In the hurricane due to the smaller value of  $L_E$  the turbulent friction force is much smaller than the second term in Eq. (13) and can be neglected. Euler equation for the horizontal part of the streamline takes the form

$$\frac{1}{2}\rho\frac{\partial u^2}{\partial l} + \frac{\partial p}{\partial l} = 0.$$
(17)

Integral along the vertical part of the streamline still satisfies Eq. (16) due to  $w = uh_{H_2O}/L_E \ll u$ . Therefore, integrating Eq. (17) and taking into account the equation of state  $p = \rho gh$ ,  $h \equiv RT/(Mg)$ , M = 29 g mol<sup>-1</sup> is air molar mass, we have

$$\boldsymbol{u}^{2} = -gh\ln\frac{p-\Delta p}{p} = -\frac{p}{\rho}\ln\left(1-\frac{\Delta p}{p}\right) = \frac{\Delta p}{\rho}, \quad (\Delta p \ll p), \quad (18)$$

which coincides with Bernoulli's equation for the incompressible liquid.

Finally, for tornado with  $L_E \sim h_{H_2O}$ , we have  $w \sim u$ , so that a major part of pressure difference  $\Delta p$  (8) falls on the vertical part of the streamline, Eq. (14). This results in the maximum observed vertical velocities  $w \sim u_E$ , see Eqs. (4), (5).

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