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Atmospheric Chemistry and Physics Discussions

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Interactive Comment

Interactive comment on "On the validity of representing hurricanes as Carnot heat engine" *by* A. M. Makarieva et al.

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In this commentary we provide additional quantitative details on the energetics of evaporation that are essential for evaluation of the proposed physical mechanism for hurricane and tornadoes.

When moisture evaporates from the liquid hydrosphere, the appearing water vapor has to get uniformly distributed in the moist air. That is, the newly coming water vapor molecules have to perform some work to secure themselves some space in the air. It is natural to term this work "latent work", by analogy with the existing notion of "latent heat". Namely this work is released, together with latent heat, during water vapor condensation. This work feeds with energy all dynamic wind processes of atmospheric circulation.

1 Introducing latent work

Consider one mol of moist air occupying volume v in contact with the hydrosphere. When ambient temperature T increases by dT, water evaporates and partial pressure p_v of saturated water vapor increases by dp_v. Clausius-Clapeyron equation relates dT and dp_v as

$$dp_v = p_v \xi \frac{dT}{T}, \quad \xi \equiv \frac{L_v}{RT} \equiv \frac{T_{\mathrm{H_2O}}}{T}, \quad T_{\mathrm{H_2O}} \equiv \frac{L_v}{R}, \tag{1}$$

where L_v is molar heat of vaporization for water vapor (latent heat), $R = 8.3$ J K^{-1} mol^{-1} is the universal gas constant. The added water vapor molecules should take their place in moist air, so the number of moist air mols per unit volume increases, while the molar volume v (the volume occupied by one mol of moist air) decreases by dv_v. In order to secure space for added water vapor it is necessary to perform latent work $dA_L \equiv p dv_v$, where p is the pressure of moist air. Let us find the magnitude of latent work.

From two equations of state for ideal gas (prior to and after evaporation, respectively):

$$pv = RT, \tag{2}$$

$$(p + dp)(v - dv_v) = R(T + dT); \tag{3}$$

and equation for the increment of moist air pressure dp, which comes as the sum of increment dp_v of the partial pressure of water vapor and increment $dp_T = p dT/T$ of moist air pressure due to temperature increase:

$$dp = dp_v + p\frac{dT}{T} = dp_v + \frac{RdT}{v}; \tag{4}$$

we have

$$dA_L \equiv p dv_v = dp_v v. \tag{5}$$

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Evaporation changes the relative partial pressure $\gamma \equiv p_v/p$ of water vapor by $d\gamma$. Using Eqs. (4), (5) and (1) we have

$$d\gamma \equiv d\frac{p_v}{p} = \frac{dp_v}{p} - \gamma\frac{dp}{p} = \frac{dp_v}{p}(1-\gamma) - \gamma\frac{dT}{T} = \frac{dp_v}{p}(1-\gamma-\frac{1}{\xi}) \tag{6}$$

In the result, for latent work we have

$$dA_L = dp_v v = pvd\gamma\frac{1}{1-\left(\gamma+\frac{1}{\xi}\right)} = RTd\gamma\frac{1}{1-\left(\gamma+\frac{1}{\xi}\right)} \approx RTd\gamma. \tag{7}$$

The last approximation diminishes dA_L by less than 10% at $\gamma < 0.1$ characteristic for the Earth's atmosphere.

2 Latent work and the first law of thermodynamics

The first law of thermodynamics for atmospheric air in the presence of evaporation and condensation should, in the view of the introduced latent work, read as

$$dQ = c_p dT - vdp + Ld\gamma, \quad L \equiv L_v + RT. \tag{8}$$

Note that $d\gamma \neq 0$ if and only if there are phase transitions (condensation or evaporation) of water vapor.

Using the ideal gas equation of state (2) and Clausius-Clapeyron law (1) the first law of thermodynamics (8) can be written as

$$\frac{dQ}{T} = c_p\left\{\frac{dT}{T}[1+\gamma\mu\xi(\xi+1)] - \mu\frac{dp}{p}[1+\gamma(\xi+1)]\right\}, \tag{9}$$

$$\mu \equiv \frac{R}{c_p} = \frac{2}{7} = 0.29, \quad \xi \equiv \frac{L_v}{RT} = 18, \quad \gamma \equiv \frac{p_v}{p}.$$

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In adiabatic processes $dQ = 0$ and we have

$$\frac{dT}{T} = \frac{dp}{p}\mu\varphi(\xi,\gamma), \quad \varphi(\xi,\gamma) \equiv \frac{1+\gamma(\xi+1)}{1+\gamma\mu\xi(\xi+1)}. \tag{10}$$

Under the assumption of hydrostatic equilibrium, saturation-adiabatic (moist-adiabatic) lapse rate Γ_m in the view of (9) and (10) becomes

$$\Gamma_m \equiv \frac{\partial T}{\partial z} = \frac{Mg}{c_p}\varphi(\xi,\gamma), \quad -\frac{1}{p}\frac{\partial p}{\partial z} = \frac{Mg}{RT}, \tag{11}$$

where $M = 29$ g mol^{-1} is molar mass of atmospheric air, $g = 9.8$ m s^{-2} is the acceleration of gravity. (This formula differs from the one listed in meteorological textbooks and glossaries, where $(\xi+1)$ in (10) is replaced by ξ. This indicates that latent work has not so far been taken into account in meteorology. Note that in the majority of meteorological glossaries (e.g., Glossary of Meteorology, amsglossary.allenpress.com) function $\varphi(\xi,\gamma)$ is written in mass rather than molar variables, so it has a somewhat more complicated form.) At $\gamma \sim 0.05$ the account of latent work changes the values of $\varphi(\xi,\gamma)$ and Γ_m by 9% due to the large value of $\xi \sim 20$.

3 Physical meaning of latent work

While latent heat L_v mainly describes energy needed to overcome the forces of intermolecular attraction of the liquid water, latent work A_L describes energy needed to "squeeze" air molecules into a smaller molar volume to secure space for water vapor molecules that are added to the moist air during evaporation.

Complete condensation of water vapor corresponds to the release of volume-specific work $a_L \equiv A_L/v \approx RT\gamma/v = p_v$ (dimension J m^{-3}), where p_v is the saturated partial pressure of water vapor. This consideration emphasizes once again that, in the

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presence of a sufficiently large, supracritical vertical lapse rate of air temperature (Makarieva, Gorshkov, 2007, HESS 11: 1013), partial pressure of water vapor represents a store of potential energy in the atmosphere. It exponentially depends on temperature and increases approximately twofold per each ten degrees of temperature rise. As shown in the discussion paper, in hurricanes and tornadoes this potential energy is converted to kinetic energy of moving air masses with the resulting maximum velocities u calculable from Bernoulli's equation as $p_v = \rho u^2/2$.

We would also like to correct four misprints in our first comment (Makarieva et al. 2008 ACPD 8: S7325):
p. S7328, line 2 from top: $p_1/p_2 = (T_1/T_2)^{c_v/R}$ should read $p_1/p_2 = (T_1/T_2)^{c_p/R}$;
p. S7328, line 10 from bottom: $A = \varepsilon Q_0$ should read $A = \varepsilon Q_s$;
p. S7331, line 12 from top: $Q_{out} = \varepsilon Q_s$ should read $Q_{out} = (1-\varepsilon)Q_s$;
p. S7333, line 11 from top: "irreversibility" should read "reversibility".

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Discussion Paper