

Interactive comment on “On the validity of representing hurricanes as Carnot heat engine” by A. M. Makarieva et al.

A. M. Makarieva et al.

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Prior to publication our paper was evaluated by two anonymous referees. Referee 1 made the following comments:

"The two key points of the paper are: (1) that the hurricane model of Emanuel (1986, 1991, etc) is physically unsound, and (2) an alternative theory for hurricane formation.

I cannot find any basis for the authors criticisms of the Emanuel theory. In particular, the authors' assertions (page2) that the Emanuel theory does not have any cooling at the cold sink and that it implies a zero mechanical efficiency are in direct contradiction equation (E10) and the statement following equation (E11) in Emanuel (1991). Despite their general criticisms, the authors have not pointed to any incorrect statement in the Emanuel papers.

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The authors propose an alternative theory that describes hurricane as an 'anti-explosion'. The proposed theory remains vague and schematic. There is no attempt at building even a quantitative theory, and no statement that could be tested or falsified. As a personal opinion, I find the outline theory at best odd."

To response to the Referee's concern about the overly general character of our claims, under items I-V below we specifically highlight and clarify our key statements.

We start with a brief consideration of a reversible thermodynamic cycle that involves phase transitions of water vapor. For one mol of substance, the first law of thermodynamics then reads as $dQ = pdv + c_v dT + Ld\gamma$, where dQ is heat increment, p is air pressure, v is molar volume, c_v is molar heat capacity of air at constant volume, T is absolute temperature, $\gamma \equiv p_{\text{H}_2\text{O}}/p$ is the relative partial pressure of saturated water vapor, L is molar energy of vaporization in the atmospheric air. The last term in this equation, $Ld\gamma$, describes the energy that is spent on evaporation at $d\gamma > 0$ or released during condensation of water vapor at $d\gamma < 0$ in the atmospheric air. Since under realistic conditions $\gamma \ll 1$, the heat capacity of liquid water is neglected.

The reversible cycle consists of two isotherms ($dT = 0$) at $T = T_s$ and $T = T_0$ and two adiabates ($dQ = 0$). Integrating the above equation for all the four processes, we obtain:

$$1, \text{ isotherm } a-c : \int_a^c dQ \equiv Q_s = A_1 + L\Delta_1\gamma; \quad (1)$$

$$2, \text{ adiabat } c-o : \int_c^o dQ = 0 = A_2 + c_v(T_0 - T_s) - L\Delta_2\gamma; \quad (2)$$

$$3, \text{ isotherm } o-o' : \int_o^{o'} dQ \equiv -Q_0 = -A_3 - L\Delta_3\gamma; \quad (3)$$

$$4, \text{ adiabat } o'-a : \int_{o'}^a dQ = 0 = -A_4 + c_v(T_s - T_0) + L\Delta_4\gamma. \quad (4)$$

Here spatial points a , c , o and o' correspond to those in Fig. 1 of Emanuel (1991, Annu. Rev. Fluid Mech. 23: 179, hereafter (E91)). All terms in Eqs. (1)-(4) are defined to be positive. Along the first isotherm at $T = T_s$ the air receives heat Q_s , expands and *performs work* $A_1 \equiv \int_a^c p dv > 0$, some water vapor evaporates, $L\Delta_1\gamma \equiv \int_a^c L d\gamma > 0$. Then the air expands adiabatically and *performs work* $A_2 \equiv \int_c^o p dv > 0$; as the air cools, water vapor condenses; this is indicated by the minus sign at $L\Delta_2\gamma \equiv \int_o^c L d\gamma > 0$. Air loses heat $Q_0 > 0$ (hence the minus sign at this term) and compresses along the second isotherm at $T = T_0$; here *work is exerted on the air* and water vapor condenses, as indicated by the minus signs at $A_3 \equiv \int_{o'}^o p dv > 0$ and $L\Delta_3\gamma \equiv \int_{o'}^o L d\gamma > 0$. Finally, the air compresses adiabatically and warms; work is again *exerted on the air*, while water evaporates, hence the minus and plus signs at $A_4 \equiv \int_a^{o'} p dv > 0$ and $L\Delta_4\gamma \equiv \int_{o'}^a L d\gamma > 0$, respectively.

For the cumulative work $A \equiv \oint p dv$ performed by the heat engine we have from Eqs. (1)-(4):

$$A \equiv \oint p dv = A_1 + A_2 - A_3 - A_4 = \oint dQ = Q_s - Q_0. \quad (5)$$

Summing Eq. (2) and Eq. (4) we have

$$A_2 - A_4 = L(\Delta_2\gamma - \Delta_4\gamma). \quad (6)$$

Differentiating the ideal gas equation $pv = RT$ (here R is the universal gas constant) for the isotherms $dT = 0$ we have $p dv = -v dp$. This allows one to calculate work A (5) by integrating A_1 and A_3 and using (6):

$$A = RT_s \ln \frac{p_a}{p_c} - RT_0 \ln \frac{p_{o'}}{p_o} + L(\Delta_2\gamma - \Delta_4\gamma). \quad (7)$$

Here low indices at p refer to air pressure in the corresponding points. Note that Eq. (7) allows for a unambiguous calculation of A given p_a , p_c , T_0 and T_s . The values of p_o and $p_{o'}$ are related to p_a and p_c by the moist adiabat equations (2) and (4), respectively.

In the simple case of the "dry" Carnot cycle ($\gamma = 0$) the dry adiabat equation relates pressures p_1 and p_2 as $p_1/p_2 = (T_1/T_2)^{c_v/R}$. Since $T_o/T_c = T_{o'}/T_a = T_0/T_s$, we have $p_o/p_c = p_{o'}/p_a$ and

$$A = R(T_s - T_0) \ln \frac{p_a}{p_c}. \quad (8)$$

We also note that

$$\oint d\gamma = \Delta_1\gamma - \Delta_2\gamma - \Delta_3\gamma + \Delta_4\gamma = 0, \quad (9)$$

i.e., all energy that is released in the reversible moist adiabatic cycle during condensation is spent on evaporation within the cycle; and that

$$Q_0 = (1 - \varepsilon)Q_s = \frac{T_0}{T_s} \left(RT_s \frac{p_a}{p_c} + L\Delta_1\gamma \right), \quad (10)$$

where $\varepsilon = (T_s - T_0)/T_s$ is efficiency of the reversible Carnot cycle; $A = \varepsilon Q_0$.

We now proceed to the list of specific criticisms.

I. Formula (16) in (E91) for work $A = \oint dQ = \oint T ds = \varepsilon T_s \Delta s$,

$$\varepsilon T_s \Delta s = RT_s \ln \frac{p_a}{p_c} + \frac{1}{4} f^2 r_a^2,$$

where the last term is negligible (E91), is incorrect (see Section 3.1, pp. 17427–17428 in the Discussion paper, hereafter DP). The correct formula is that of Eq. (7). A helpful observation is that in the "dry" limit ($\gamma \rightarrow 0$) the correct formula for work of the "moist" engine should tend to Eq. (8) for the "dry" engine. Formula (16) in (E91) does not depend on γ altogether; in the dry limit, it overestimates the real work given by Eq. (8) by three times given the characteristic values of $T_s = 300$ K and $T_0 = 200$ K.

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II. The above inconsistency results from an incorrect integration of Bernoulli's equation in (E91) (see Section 3.1, pp. 17427-17428 in DP). Specifically, formula (15) in (E91)

$$\int_a^c \mathbf{F} dl = - \int_a^c \alpha dp,$$

is incorrect, as it ignores the change of velocity V , see p. 17428 in DP. As shown above, see Eqs. (1)-(10), consideration of the first law of thermodynamics for the four processes of the reversible thermodynamic cycle is sufficient for calculation of work performed by the engine. For this reason, applying Bernoulli's equation for calculation of the engine's work is needless; it is also useless, as it involves an additional unknown variable, namely, velocity V .

If one assumes that all kinetic energy undergoes dissipation in the hurricane area (along the horizontal streamline), then from Bernoulli's equation, Eq. (1) in (E91), we obtain $dV^2/2 + \mathbf{F} dl = 0$ and, hence, $\alpha dp = 0$, i.e. the hurricane does not exist, see. p. 17428 in DP. If, on the other hand, one takes into account that in reality kinetic energy dissipates outside the hurricane area, then formula (4) in (E91) does not contain any information about the hurricane area and is unrelated to Bernoulli's equation.

III. Formula (6) in Emanuel (2003, Annu. Rev. Earth Planet. Sci., 31: 75, hereafter (E03)),

$$P = \frac{T_s - T_0}{T_s} \left(2\pi \int_a^c C_k \rho |V| (k_0^* - k) r dr + D \right),$$

where $D = 2\pi \int_a^c C_D \rho |V|^3 r dr$ is "the net dissipative energy" ("the vertically integrated dissipative heating"), see formula (7) and text on p. 84 in (E03), as well as the resulting expression for the maximum hurricane wind speed V_{\max} ,

$$|V_{\max}|^2 \approx \frac{C_k}{C_D} \frac{T_s - T_0}{T_0} (k_0^* - k),$$

where Carnot efficiency $\varepsilon = \frac{T_s - T_0}{T_s}$ is replaced by $\frac{T_s - T_0}{T_0}$ and which, starting from S7329

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the work of Bister and Emanuel (1998, *Meteorol. Atmos. Phys.*, 65: 233, hereafter (BE98)), is present in Emanuel (1999, *Nature*, 401: 665), (E03), Emanuel (2005, *Divine wind: The history and science of hurricanes*, OUP), Emanuel (2006, *Physics Today*, 59: 74), are incorrect. These formulae conflict with the energy conservation law and the second law of thermodynamics, as explained in Section 2, pp. 17426-17427; Section 3.2, p. 17431 in DP and additionally clarified below.

The Carnot heat engine receives heat $Q_{in} = Q_s$ at $T = T_s$, produces work $A = \varepsilon Q_s$ and gives away heat $Q_0 = (1 - \varepsilon)Q_s$ at $T = T_0 < T_s$, where $\varepsilon = (T_s - T_0)/T_s$. Importantly, in the environment where the Carnot heat engine operates work A accumulates with time t at a rate

$$\frac{dA}{dt} = (Q_s - Q_0)n, \quad (11)$$

where n is the number of engine cycles per unit time. If there is no dissipation (all energy is stored in the environment in the form of work, no heat is produced), entropy s of the environment does not change:

$$\frac{ds}{dt} = \left(\frac{Q_s}{T_s} - \frac{Q_0}{T_0} \right) n = 0, \quad (12)$$

which is the physical essence of the Carnot cycle. If the work produced dissipates to heat, $dQ/dt > 0$, entropy of the environment increases.

In the cycle modified by Bister and Emanuel (BE98) it is assumed that work A (P in the notation of (E03)) dissipates to heat $Q_A = A$ (D in the notation of (E03)) at temperature T_s and is added (recycled) to heat Q_s ($Q_s = 2\pi \int_a^c C_k \rho |V| (k_0^* - k) r dr$ in formula (7) of (E03), see above). So, for this engine heat input Q_{in} becomes $Q_{in} = Q_s + Q_A$. Additionally imposing the stationarity condition and demanding that A does not change with time gives $A = \varepsilon Q_{in} = \varepsilon(Q_s + Q_A) = \varepsilon(Q_s + A)$. This is the physical content of formula (7) in (E03). From this one has $A = [\varepsilon/(1 - \varepsilon)]Q_s$ and the final formula for squared maximum wind speed, see above, follows. Heat outflow occurring in this

engine is then

$$Q_{out} = (1 - \varepsilon)(Q_s + A) = Q_s, \quad (13)$$

i.e. it coincides with the external heat input. Hence, energy content would not change in the environment where such an engine were operating:

$$\frac{dA}{dt} = \frac{dQ}{dt} = 0. \quad (14)$$

This implies that entropy of the environment does not change, since $ds = dQ/T = 0$. However, this stationarity condition cannot be reconciled with another inherent feature of the considered engine, namely with the fact that the net flux of entropy from the environment to the heat sink is now positive, so that the entropy of the environment diminishes (!), cf. (12) and replace Q_0 by Q_s as prescribed by Eq. (13):

$$\frac{ds}{dt} = \left(\frac{Q_s}{T_s} - \frac{Q_s}{T_0} \right) n < 0. \quad (15)$$

If, on the other hand, one demands that $ds = 0$ and $Q_{out} = \varepsilon Q_s = Q_0 \neq Q_s$, as it should be in Carnot cycle, see Eq. (12), then formula (7) in (E03) and the resulting formula for hurricane wind speeds will come in conflict with the energy conservation law, see p. 17427 in DP.

These fundamental contradictions stem from the fact that the modified cycle of (BE98) involves a process prohibited by the second law of thermodynamics. Importantly, at the warmer isotherm $T = T_s$ of the Carnot cycle *all heat* introduced to the engine is converted to work. For the modified cycle of (BE98) it means $A_1 = Q_s + Q_A = Q_s + A$, i.e. work A dissipates to heat and is regenerated back to work at one and the same temperature T_s . Such recycling of dissipated energy is an inherent feature of a perpetual motion machine of the second kind. Indeed, in order to dissipate work A isothermally at $T = T_s$ to heat $Q_A = A$, the heat produced should be removed from the environment during dissipation (otherwise the environment would warm) to a colder

medium with $T_1 < T_s$. The removed heat should be afterwards gained back to the environment, but this would imply heat transport from the cooler to the warmer object, which is physically impossible. (Note that for clarity this consideration is made for the dry Carnot cycle, but equally applies to the "moist" cycle of Eqs. (1)-(4).)

The above criticisms (items I-III) pertain to the theoretical treatment of the Carnot cycle irrespective of its (in)applicability for the description of hurricanes. We will now show that the observed quantitative features of hurricanes unambiguously testify that hurricanes are not a thermodynamic heat engine.

IV. If hurricane is viewed as a thermodynamic cycle, the amount of heat that has to be released to space from the hurricane area is of the order of $Q_0 \sim 4 \times 10^3 \text{ W m}^{-2}$. This is physically prohibited, as it implies $Q_0 \ll Q_s$ and $\varepsilon \approx 1$, see Section 3.1, pp. 17428-17429 and Section 3.4 in DP. Hurricane is not a thermodynamically closed system.

The *reversibility* of the moist thermodynamic cycle, Eqs. (1)-(4), (and the Carnot maximum efficiency formula is only valid for reversible cycles!) implies that moisture that condenses during the moist adiabatic ascent $c - o$ and during the isothermal compression $o - o'$ remains within the air parcel to evaporate back during its adiabatic descent and warming. (Only in this case water vapor will be close to saturation in all phases of the cycle – note that evaporation at relative humidity less than unity is an *irreversible* process.) This could only be the case if the rate of removal of the condensed moisture from the atmosphere by rain would either be absent or negligibly small compared to the flux of latent heat release during the adiabatic ascent, which is approximately given by $L\Delta_2\gamma$ in Eq. (2).

The available estimates of precipitation rates are in the order of $r \sim 5 \text{ mm hour}^{-1} = 1.4 \times 10^{-6} \text{ m s}^{-1}$ (Miller, 1964, Mon. Wea. Rev., 92: 389). This corresponds to the heat release flux of $I = r\rho_l L_v \sim 3.4 \times 10^3 \text{ W m}^{-2}$, where $\rho_l = 5.6 \times 10^4 \text{ mol m}^{-3}$ is molar density of liquid water, $L_v \approx 44 \times 10^3 \text{ J mol}^{-1}$ is the molar heat of vaporization.

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This estimate coincides, in its order of magnitude, with the available estimates of latent heat flux release (see, e.g., Black and Holland (1995, Mon. Wea. Rev., 123: 2007) and p. 17433 in DP. (One should also take into account that a considerable part of condensed moisture can be transported away from the hurricane area.) This means that a significant part of moisture condensed within the hurricane is removed from the atmosphere via precipitation.

This important fact has a two-sided implication for the view of hurricanes as a thermodynamic cycle. First, since the air volumes ascending within the hurricane area reach the upper cold atmosphere practically totally depleted of (condensed and precipitated) moisture, the descent of these air volumes can only occur along a dry adiabat, not along a moist adiabat, as the irreversibility of the cycle, Eqs. (1)-(4), prescribes. In such a case relative humidity of the descending air parcels at the sea surface would have been close to zero. This strongly contradicts the observations (relative humidity at the sea surface is around 80%, i.e., close to unity), which indicates that there is a strong admixture of moist air (and, hence, latent heat) into the descending air parcels. This means that there is a very significant import of latent energy (in the order of 10^3 W m^{-2}) from the external environment to the hurricane and that **hurricane is not therefore a thermodynamically closed system (cycle). It exchanges not only mass, but also latent energy with the external environment. The statement that "there is little thermodynamic contribution" from the descending leg of the hurricane (E91, p. 184), is not supported by the available evidence.**

Second, even if one considered hurricane as, at least to some approximation, a thermodynamically closed cycle with air parcels descending approximately dry adiabatically ($L\Delta_4\gamma \approx 0$, see Eq. (4)), then there arises the problem of disposing of latent heat released during the moist adiabatic ascent. At $L\Delta_4\gamma \approx 0$ we have from Eq. (9):

$$\Delta_1\gamma \approx \Delta_2\gamma + \Delta_3\gamma \approx \Delta_2\gamma. \quad (15)$$

The last approximate equality results from the observation that at low temperature

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$T = T_0$ the absolute value of γ and its change is negligibly small. From Eq. (15) and Eq. (10) for the outflowing heat flux Q_0 we have

$$Q_0 \approx \frac{T_0}{T_s} \left(RT_s \frac{p_a}{p_c} + L \Delta_2 \gamma \right). \quad (16)$$

With $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$, $T_s = 300 \text{ K}$, $T_0 = 200 \text{ K}$, $p_a/p_c \sim 0.1$, $\Delta_2 \gamma \sim \gamma_s \sim 0.06$, $L \sim 5 \times 10^4 \text{ J mol}^{-1}$, we obtain $Q_0 \sim [2/3(250 + 3000)] \sim 2 \times 10^3 \text{ J mol}^{-1}$. It is easy to see that the release of latent energy (the second term in Eq. (16)) makes by far the major contribution into the outflowing heat flux compared to the first one. The obtained order-of-magnitude accurate value estimates the release of energy in one mol of air that has reached the upper atmosphere. To express this on a surface-specific basis, one needs to know the mean velocity of ascent. At vertical velocity $u_z \sim 0.1 \text{ m s}^{-1}$ and air molar density $N = 45 \text{ mol m}^{-3}$ the outgoing flux should have been $Q_0 u_z N \sim 10^4 \text{ W m}^{-2}$. As explained in Section 3.4 in DP, such a flux cannot be radiated to space by an atmosphere with $T < T_s = 300 \text{ K}$, see p. 17433 in DP. Since the real flux is virtually zero compared to the estimated Q_0 in Eq. (16) (see the last equation on p. 17428 in DP), the formalism adopted in the work of (E91) results in the conflict with the energy conservation law, see p. 17429 in DP.

V. New physical approach to hurricanes. We did not consider developing a detailed theory as the focus of the present paper. Such a theory is available in the preprint (Gorshkov, Makarieva, 2008); it is cited in DP and publicly available. Here we have aimed at introducing a transparent physical concept which, stripped of unnecessary sophistications, could be more readily evaluated by the broader scientific community.

According to Dalton's law, pressure p of atmospheric air is the sum of partial pressures p_i of the air mixture constituents:

$$p = \sum p_i = p_{\text{O}_2} + p_{\text{N}_2} + p_{\text{CO}_2} + p_v + \dots \quad (17)$$

Change of any p_i will induce an equal change of air pressure p . Partial pressure p_v of

water vapor increases during evaporation and decreases during condensation. For example, if one splashes a cupful of water onto the metallic surface of a glaring oven, the instantaneous increase of p_v and, hence, p , will produce a mini-explosion: the resulting wave of hot moist air radiating away from the oven can hurt the observer if he/she does not haste to retreat. Conversely, massive removal of water vapor from the atmosphere during condensation/precipitation (the processes that invariably accompany hurricanes and tornadoes) causes local air pressure to drop. In the result, the surrounding air is "sucked" into the area of condensation from the adjacent areas, hence the proposed analogy with the reversed explosion. The maximum drop Δp of air pressure is, apparently, $\Delta p \sim p_v$. Maximum wind velocities that this effect can produce are readily derived from Bernoulli's equation (see p. 17434 in DP) and constitute, depending on temperature, from 50 to 120 m s⁻¹, i.e. precisely the magnitudes observable in hurricanes and tornadoes.

Since the saturated partial pressure of water vapor depends exponentially on temperature, the proposed approach makes a clear quantitative prediction for the exponential dependence of maximum wind speeds in hurricanes and tornadoes on air temperature. It can also be predicted that hurricanes and tornadoes should arise in the regions where water vapor concentration and, hence, the intensity of condensation, is maximum and should further move in the direction of maximal concentration of water vapor.

Interactive comment on Atmos. Chem. Phys. Discuss., 8, 17423, 2008.

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