

## ***Interactive comment on “The von Kármán constant retrieved from CASE-97 dataset using a variational method” by Y. Zhang et al.***

**Y. Zhang et al.**

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Following the three reviewers' suggestions and comments, we have made major revisions to the manuscript. We have made every effort to address the reviewers' questions and comments on the manuscript. Our detailed, point to point responses to three reviewers' comments have been submitted and also attached below.

Response to RC S5218, Prof. Larry Mahrt

In response to the reviewer's comments, we have made relevant revisions on the manuscript. Listed below are answers and changes made to the manuscript according to the questions and suggestions given by the reviewer. The original comments questions from the reviewer are listed first followed by our responses.

1. Is the difference between  $\kappa = 0.39$  and  $0.40$  significant, considering the mea-

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surement errors?

Response: The measurement errors were considered in the variational computation by introducing the dimensionless weights for wind, air temperature and humidity profiles in the cost function Eq. (1), defined to be inversely proportional to their respective observation error variances. This is one of advantages of the variational technique. After relaxing the constraint  $0.35 \leq k \leq 0.45$  imposed to the variational calculation, we obtain a  $k$  value at 0.384. Thus the variational computed  $k$  value with and without the constraint ranges from 0.384 to 0.390, just within the error range of Andreas et al's (2006) value at  $0.387 \pm 0.003$ . This point has been added to the revised paper. We further estimated the statistical difference between variational computed  $k$  values and the  $k$  value at 0.4, which is determined using a t-test. Based on the calculation, the statistic  $t$  under the null hypothesis  $H_0$  is equal to 6.27 ( $> t_{0.005/2} = 2.6$ ) with the statistically significant level of 99.5%. This suggests that the statistical difference between  $k$  values at 0.39 and 0.4 is significant. These statements have been added to the revised manuscript.

2. Because the profiles are not linear, one might expect sensitivity to the choice of observation levels?

Response: It is not clear if the variational calculated von Kármán constant would be sensitive to wind, air temperature and humidity profiles at different observation levels. The CASES-97 dataset provided only wind, air temperature and humidity at two vertical levels. Further study on this aspect is needed by using multiple levels observations. The reviewer's question has been addressed in the revised manuscript.

3. The imposed condition  $0.35 < k < 0.45$  probably strongly influences the results. Unless the distribution of  $k$  within this allowed range is strongly asymmetric, the mean value will be necessarily close to 0.40. Is it possible to put conditions on stability and/or nonstationarity instead of conditions on the von Kármán constant? I think some discussion would be helpful.

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Response: The imposed condition  $0.35 \leq k \leq 0.45$  was used only in the output of model result, rather than used in the variational calculation. Nevertheless, following the reviewer's suggestions and comments, additional computations were conducted to test the sensitivity of the von Kármán constant to atmospheric stability by relaxing the imposed constraint  $0.35 \leq k \leq 0.45$ . Instead, we simply impose a condition which requiring  $k \leq 0.6$ . This yields  $k$  values at 0.428 for stable condition and 0.340 for unstable condition. The mean  $k$  value with total 3563 samples is 0.384 under all atmospheric stability conditions. We further relax all conditions that imposed to  $k$  values and introduce constraints on stable atmospheric conditions by setting the Obukhov length  $L > 10, 20, \dots, 10000$ . Results show that the von Kármán constant tends asymptotically to 0.4 from very stable to neutral condition. A new paragraph and a new figure (Fig. 3 in the revised manuscript) describing and showing these results have been added to the revised manuscript.

4. If I understand correctly, the majority of the stable cases are rejected by the restrictions on kappa. I think this is a very important finding. I agree with the authors that it is probably due to failure of Monin-Obukhov similarity theory, at least at the available observational levels. Since Monin-Obukhov similarity theory is generally applied in models to all conditions, further investigation of the frequent noncompliance cases would be valuable. Presumably the situation becomes rapidly more complex due to the influence of additional length scales. In addition, the relative insensitivity to the choice of the coefficients in the stability functions, found in the present analysis, probably breaks down. I realize this is a major task.

Response: As the reviewer noticed, the determination of reasonable  $k$  values was failed under the majority of stable conditions. As our response to the reviewer's question 3 and shown by new figure 3, if we use the restriction  $0.35 \leq k \leq 0.45$ ,  $k$  values in the stable cases with  $L$  (Obukhov length)  $< 60$  would be rejected. If we impose the condition  $k \leq 0.7$  in the variational calculation, we obtain  $k = 0.453$  for stable conditions. The number of samples satisfying the condition for the stable atmosphere ( $k$

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$\leq 0.7$ ) increases from 778 (for  $k \leq 0.6$ ) to 859. This suggests greater uncertainties in determination of the von Kármán constant in the stable boundary-layer compared with unstable conditions. It is important to indicate that, because the majority of the stable cases are rejected by the restrictions on  $k$  in our calculations, the mean  $k$  value of 0.384 is, in reality, weighted to unstable conditions. These statements have been added to the revised manuscript. This is certainly a challenge to the application of Monin-Obukhov similarity theory. Analogous to our response to the reviewer's question 3, we have relaxed the constraint  $0.35 \leq k \leq 0.45$  imposed to the variational computed results but simply set a condition which only rejects calculated  $k$  values that are greater than 0.8, we found that under the unstable conditions  $k = 0.340$  using the first group of profile constants and 0.351 using the second group of constants. For the stable cases,  $k = 0.493$  using the first group of profile constants and 0.490 using the second group of constants. Nevertheless, we agree with the reviewer that further investigation of the frequent noncompliance cases need to be carried out.

#### Response to RC S5998

In response to the reviewer's comments, we have made relevant revisions on the manuscript. Listed below are answers and changes made to the manuscript according to the questions and suggestions given by the reviewer. The original comments and questions from the reviewer are listed firstly followed by our responses.

Comment: Could it be possible that after all  $k$  is in fact constant, and the variation stems from the measurement inaccuracies or the incomplete theory/model, where the variation of some other variables or some unknown, generally minor phenomena, are not taken into account?; I am not sceptical that  $k$  would be a changing variable but I would like to see the authors' more strong statement on this; the possible invalidity of M-O theory under highly stable and unstable conditions is discussed but could it be possible that there is still even something else or if we had a corrected M-O theory the  $k$  would be constant? - I. 159: LE was computed from the energy budget method; how accurate is this concerning that there exists generally the

known mismatch between net radiation and turbulent fluxes in a way that often the sum of fluxes are only 80% - 90% of the net radiation, over long averaging periods when storages should be negligible; may this affect somehow on the analysis in the paper? - How sensitive are the results for the mathematical form of Eqs. 5a &#8211; 6b; these forms are quite standard ones but sometimes a bit different formulas are also used - I would omit Table 1 and give the information only in the main text: different min and max limits of used weights of constraints and report the variation/max-min limits of the resulting k values - I would omit Fig. 3

Responses: The von Kármán constant was firstly introduced and defined in scaling wind profiles in the neutral boundary-layer as a constant. Our study did confirm the constant value of  $k = 0.4$  under the neutral condition. While the flux-gradient relationship is extended to non-neutral stratifications, k value would vary in M-O theory. Changes in k value with stability has been also detected in Andreas et al.'s evaluations of the von Kármán constant using the measured data over Arctic sea ice (see ref.). Though these data were collected under much more neutral conditions, their results are quite similar to our figure 2, showing the decreasing of k values from stable to unstable conditions. As a response to the reviser's question, following Andreas et al (2006), a stratification correction is made to the variational calculated k values. This yields a mean value of the von Kármán constant at 0.401. This value is the same as the k value in the neutral stability. This point has been added in the revised paper.

The measurement errors were considered in the variational computation by introducing the dimensionless weights for wind, air temperature and humidity profiles in the cost function Eq. (1), defined to be inversely proportional to their respective observation error variances. This is one of advantages of the variational technique. After relaxing the constraint  $0.35 \leq k \leq 0.45$  imposed to the variational calculation, we obtain a k value at 0.384. Thus the variational computed k value with and without the constraint ranges from 0.384 to 0.390, just within the error range of Andreas et al's (2006) value at  $0.387 \pm 0.003$ . This point has been added to the revised paper

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There are likely other factors that affect the determination of the von Kármán constant, e.g., the mismatch between net radiation and turbulent fluxes, as the reviewer indicated. In the present study, the physical constraints in the cost function (Eq. (1)) are expressed by the Monin-Obukhov similarity relationships from which the von Kármán constant is retrieved. Therefore, uncertainties in the MOST in estimation of profiles of wind, air temperature and humidity as well as momentum and heat fluxes, especially under very stable and strong convection conditions, would inevitably yield errors in the evaluation of the von Kármán constant. On the other hand, given that the advantage of the variational method is that it is able to fully take into account information of the existing MOST and measured meteorological conditions over an underlying surface, the variational method has led to substantial improvements over the conventional MOST-based flux-gradient method (Cao and Ma, 2005, Cao et al., 2006, see Ref). We would expect that the variational estimated  $k$  values would be more accurate than that derived from the conventional method. These statements have been added in the revised manuscript. We further estimated the statistical difference between variational computed  $k$  values and the  $k$  value at 0.4, which is determined using a  $t$ -test. Based on the calculation, the statistic  $t$  under the null hypothesis  $H_0$  is equal to 6.27 ( $> t_{0.005/2} = 2.6$ ) with the statistically significant level of 99.5%. This suggests that the statistical difference between  $k$  values at 0.39 and 0.4 is significant. These statements have been added to the revised manuscript.

We still keep Table 1 in the revised manuscript. The von Kármán constant was introduced in the early time as a scaling factor to scale the logarithmic law of mean wind profile, and subsequently extended to scale the mean temperature and humidity profile. The results listed in table 1 confirm that the scaling factor  $k$  can be applied in all logarithmic laws for  $u$ ,  $T$  and  $q$ . Fig. 3 in the original version of the paper in ACPD is omitted following the reviewer's suggestion. A new figure 3 has been added as a response to other reviewers.

Minor/Technical - I. 152: I think 2.95 m is too accurate, it should be 3.0 m; also 1 and

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2 m should 1.0 and 2.0 m

Done!

- I. 158: 1-D sonic anemometer?; I guess it should be 3-D

It is CSI 1-D sonic anemometer as described by Dr. Russell J. Qualls at [http://data.eol.ucar.edu/datafile/nph-get/20.05/qualls\\_readme](http://data.eol.ucar.edu/datafile/nph-get/20.05/qualls_readme)

- I. 347: how were sensible heat fluxes computed?

The way to calculate sensible heat flux has been defined and described in the revised paper.

Table 1 and Fig. 1: why Fig. 1a gives 0.42 for  $k$  while Table 1 gives 0.41?

$K$  values shown in Fig. 1 used the constraint of  $0.35 < k < 0.45$ , whereas Table 1 used a constraint of  $0.35 < k < 0.42$ . To be consistent with Fig. 1, revision was made. The revised table 1 used the same constraint as that in Fig. 1.

- Fig. 4: what is the fundamental reason that two quarters (upper right and lower left) are empty? Can it be seen easily for example from theory?

The results presented in Fig. 4b are, in fact, consistent with Fig. 2 and the finding described in the manuscript. Namely,  $k$  is greater than 0.4 under stable conditions and smaller than 0.4 under unstable conditions. This point has been added to the revised manuscript.

Response to SC S6655, Prof. Thomas Foken

In response to the reviewer (Prof. Foken)'s comments, we have made relevant revisions on the manuscript. Listed below are answers and changes made to the manuscript according to the questions and suggestions given by the reviewer. The original comments and questions from the reviewer are listed on the first follow by our responses.

Comment: The flux-gradient similarity according to the Monin-Obukhov similarity the-

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ory depends on parameters which must be determined experimentally. These are the von-Kármán constant and the coefficients of the universal function and, in the case of the sensible and latent heat flux, the turbulent Prandtl and Schmidt numbers, respectively. The normal way to determine these parameters is firstly to use near neutral cases of the momentum flux to determine the von-Kármán-constant, and secondly to use near neutral cases of the sensible and latent heat flux to determine the turbulent Prandtl and Schmidt numbers. The third step is to use data of all stratifications to determine the coefficients of the universal function. The authors have done this the opposite way. They assumed correct coefficients of the universal function and determined errors in the von-Kármán-constant. The turbulent Prandtl and Schmidt numbers are ignored, while Businger et al. (1971) determined a turbulent Prandtl number of 0.74 (Foken, 2006). This can be done when the numbers are included in the universal function (Högström, 1988), but this was not done.

Response: As the reviewer correctly indicated, we determined the von-Kármán constant in an opposite way by assuming that coefficients of the universal function were correct. In fact, though the focus of this study was on the estimate of the von-Kármán constant, we also intend to demonstrate the capability of variational method in retrieving parameters in Monin-Obukhov similarity relationships. It is deducible that, if the von-Kármán-constant (= 0.4) was assumed to be correct, we could also determine the coefficients of the universal function and Prandtl number in a similar manner as we did in this study. We agree with the reviewer that the turbulent Prandtl and Schmidt numbers could be included in the universal function. In the revise paper we have indicated this.

Comment: It is not understandable why the authors used, from the large amount of universal functions (Foken, 2008b; Högström, 1988) available, the universal function by Businger et al. (1971) with a von-Kármán-constant of 0.35, which was determined under non-ideal measuring conditions (Wieringa, 1980; Wyngaard et al., 1982) and was corrected by Högström (1988). The authors incorrectly tested their method with an

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independent universal function by Wieringa (1980), because this is the same function but re-determined with another von-Kármán-constant of 0.41.

Response: The use of Businger et al. (1971)'s universal function was that this set of constants were widely used in micrometeorology. Nevertheless, following the reviewer's comments, in the revised paper we have used Högström's universal function) = (19.3, 11.6, 6, 7.8) to recalculate the von Kármán constant in the variational method. The results showed almost no difference with the universal function by Businger et al (1971). In the revised manuscript we have deleted Fig. 3 of the previous version of the paper in ACPD and added a new figure 3 that presents the change in the von Kármán constant with the Obukhov length under stable surface boundary-layer using Högström's universal function. Foken and Högström's works in this aspect have been also cited in the references of the revised paper.

Comment: It is not possible to determine the latent heat flux from an energy balance calculation, because of the "unclosed" energy balance at the surface (Foken, 2008a). Furthermore the radiation sensors used are probably not of a high accuracy (Kohsiek et al., 2007). The input data for the latent heat flux have an error of at least 20 %.

Response: To address the reviewer's point, in the revised paper we have added a new paragraph (the second paragraph from the bottom paragraph). We indicated that, though there are some system errors, because the variational method minimizes the differences between the computed and the observed meteorological variables, it can adjust the computed flux toward the measured one. Through this process, the observed meteorological and surface conditions are sufficiently taken into account in the variational computation. In addition, as shown in Table 1 (the table has been recalculated), the elimination of  $W_h$  (the weight for humidity profile) had very little effect on calculation of the von Kármán constant.

Comment: The CASES-97 (Poulos et al., 2002) data set is, of course, one of the best of the last decade, but to use only two levels is not adequate. The ratio of the measuring

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heights (2 and 1 m) is much too low to determine gradients in the surface layer with a high accuracy (Foken, 2008b). Furthermore the lowest level can always be influenced by the roughness sublayer. This can only be checked if one has a profile with at least 4-5 levels. Furthermore any information about the canopy, the zero-plane displacement and the roughness height are missing. Therefore systematic errors can be assumed.

Response: We acknowledge that the use of the CASES-97 data is likely a weakness of this work. It is not clear if the variational calculated von Kármán constant would be sensitive to wind, air temperature and humidity profiles at different observation levels. The CASES-97 dataset provided only wind, air temperature and humidity at two vertical levels. One of consideration of using the CASES-97 data was that this dataset covered all stratifications and the diurnal variation of wind, air temperature and humidity profile was significant. We also intend to use the SHEBA data with multiple level measurements of wind, temperature and humidity. Further study on this aspect is planned. In the revised paper, the reviewer's concern has been addressed in a new paragraph. Because in the present study we used the information of wind, air temperature and humidity at the two measurement levels (1 and 2 m), the roughness length for momentum, heat and humidity as well as the zero-displacement height were not taken into account. These texts have been added in the revised paper.

Comment: Measurements under stable stratification need a very carefully conducted data analysis because of, for example, intermittencies or decoupling. Often a local Obukhov length must be used, and not the Obukhov length. For details about the determination of universal functions under these conditions see Handorf et al. (1999), Andreas et al. (2006; 2005) and others. Generally nothing is said about a data selection according to the fulfilment of turbulent conditions (Foken and Wichura, 1996).

Response: More calculations have been done for stable stratification. If we impose the condition  $k \leq 0.7$  in the variational calculation, we obtain  $k = 0.453$  for stable conditions. The number of samples satisfying the condition for the stable atmosphere ( $k \leq 0.7$ ) increases from 778 (for  $k \leq 0.6$ ) to 859. This suggests greater uncertainties

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in determination of the von Kármán constant in the stable boundary-layer compared with unstable conditions. In the revised paper, to address the reviewer's comment, we have added a new Fig. 3 illustrating the change in the von Kármán constant with the Obukhov length under stable stratification. As our above response to the reviewer, further study is planned using the SHEBA data (Andreas et al, 2006) which were collected under more stable conditions.

Comment: Högström (1996) found, after a very careful analysis of universal functions, that their accuracy for a given von-Kármán-constant of about 0.40 is, in a range of not very strong stable and unstable stratification, about 10-20%. The authors found, for the opposite method of calculations for more neutral conditions, the same error. Therefore the results are absolutely not new, are based only on the Kansas experiment (Izumi,1971) and ignore many other experiments, and the method of calculation has many weaknesses. It may be interesting to use the variational method for different examples, which you have already done, but the determination of the von Kármán constant is probably not the best example.

Response: We agree with the reviewer's comments. We did notice and cite Högström (1996)'s work. As we indicated, one of objectives in this study was to demonstrate the capability of variational method in retrieving parameters in Monin-Obukhov similarity relationships. As the reviewer indicated, this method indeed repeats and confirms Högström (1996) and Andreas et al (2006)'s finding, and therefore is useful. We have also inserted a new sentence in the end of the first paragraph of section "Concluding Remarks" to address this point. In fact, we have applied this method to retrieve the roughness lengths and heat fluxes (e.g., Ma and Daggupaty, 1999; Cao and Ma, 2005). More works using this method will be reported.

Minor remarks: What the authors call a universal function is the integrated form. This integration of the Dyer-Businger-type of universal functions was firstly done by Paulson (1970).

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We agree!

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Interactive comment on Atmos. Chem. Phys. Discuss., 8, 13667, 2008.

**ACPD**

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