Atmos. Chem. Phys. Discuss., 8, S11826–S11832, 2009
www.atmos-chem-phys-discuss.net/8/S11826/2009/

Atmospheric Chemistry and Physics Discussions

ACPD

8, S11826–S11832, 2009

Interactive Comment

Interactive comment on "On the validity of representing hurricanes as Carnot heat engine" *by* A. M. Makarieva et al.

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Received and published: 22 March 2009

Here we present several additional considerations of the role of turbulent friction in the formation of atmospheric circulation patterns driven by the process of water vapor condensation. A stationary constant value of velocity along some part of the streamline indicates absence of air acceleration, which corresponds to the equality between the pressure gradient force F_A that accelerates air and the force of turbulent friction F_T that opposes the air motion (forces are taken per unit area of the Earth's surface and have the dimension of $\mathrm{N\,m^{-2} = J\,m^{-3} = Pa}$):

$$F_A = \Delta p h_a / L = F_T. \quad (1)$$

Here Δp is the horizontal pressure difference observed over distance L, L is horizontal dimension of the considered circulation pattern, $\Delta p/L$ is the accelerating pressure

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Interactive Discussion
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gradient force per unit air volume, h_a is characteristic height of the part of atmospheric column where the horizontal wind retains its direction. We will now explore the nature of the turbulent friction force or surface friction stress F_T.

When considering motion of terrestrial animals and transport one takes into account two types of resistance forces. These are the surface friction force, which is independent of velocity and proportional to the weight of the body, and air resistance, which is proportional to the squared movement velocity u. Out of dimensional considerations these two friction terms per unit surface area can be written in the following form (Gorshkov, 1983):

$$F_T = \mu\rho g l + C_D \rho u^2. \tag{2}$$

Here ρ is mass density of the body, $g = 9.8$ m s^{-2} is acceleration of gravity, h is the mean height of the body, $\mu \sim 1$ is the dimensionless coefficient of friction of rest, C_D is the dimensionless drag coefficient reflecting the geometry of the moving body (Landau & Lifshitz, 1987, § 45). Drag coefficient C_D is of the order of unity for spherical bodies; in the general case it is proportional to the ratio of the vertical cross-section area of the body to the area of its projection on the Earth's surface, i.e. to ratio h/L, where h is height, L is length of the body (Gorshkov, 1983).

In meteorology turbulent friction is formally represented by the second term of Eq. (2) with drag coefficient C_D being of the order of 10^{-3} (Garratt, 1977). If we apply an analogy to the motion of animals, C_D can be estimated, in its order of magnitude, as

$$C_D \sim h_a/L, \tag{3}$$

i.e., to the ratio of the vertical to horizontal dimensions of the circulation pattern. Taking a characteristic horizontal pressure gradient on Earth to be in the order of $\Delta p/L \sim 1$ mbar $(100\,\text{km})^{-1} = 1$ Pa km^{-1}, height $h_a \sim h_{H_2O} \approx 2$ km (e.g., Miller, 1964, Fig. 5), air density $\rho = 1.3$ kg m^{-3}, global mean velocity $\overline{u} \sim 7$ m s^{-2} (Gustavson, 1979) and comparing f_A and f_T in Eq. (1) we have:

$$f_A \sim 2\ \text{N m}^{-2} \gg C_D \rho u^2 \sim 0.06\ \text{N m}^{-2}. \tag{4}$$

That is, the accelerating pressure gradient force exceeds the aerodynamic drag force by more than thirty times. This comparison shows that the account of aerodynamic turbulent friction $C_D \rho u^2$ is essentially insufficient for explaining the observed nearly constant horizontal velocities of the order of a few meters per second that can be maintained on the major part of the streamline over horizontal distances of the order several thousand kilometers. For example, air circulation over the Amazon river basin is characterized by wind velocities $u \sim 5$ m s^{-1} maintained practically year round over distances of $L \sim 2 \times 10^3$ km (Zhou & Lau, 1998). This implies that the air masses travel for several days inland at nearly constant velocity before they ascend and reverse the direction of their motion in the upper atmosphere. In the meantime, the force imbalance implied by Eq. (4) would correspond to an unrealistic acceleration $a \sim \Delta p/(L\rho) \sim 1.5 \times 10^{-4}$ m s^{-2} ≈ 13 m s^{-1} day^{-1} even if a conservative estimate of horizontal pressure gradient $\Delta p/L \sim 0.2$ Pa km^{-1} (10 hPa over 5 thousand kilometers) is applied.

The problem can be solved by the proposition (Makarieva et al. 2009 ACPD 8: S8904, http://www.cosis.net/copernicus/EGU/acpd/8/S8904/acpd-8-S8904.pdf; Makarieva & Gorshkov 2009 HESSD 6: S59, http://www.cosis.net/copernicus/EGU/hessd/6/S59/hessd-6-S59.pdf) that there is another term in the cumulative friction force F_T that is analogous to the first term in Eq. (2) and independent of horizontal velocity u. Total friction force can then be written as

$$F_T = F_{T0} + F_{Ta}, \quad F_{T0} = \rho g z_T, \quad F_{Ta} = C_D \rho u^2. \tag{5}$$

Here F_{Ta} characterizes the aerodynamic turbulent friction in the lower part atmospheric column $z \leq h_a$, while F_{T0} represents surface turbulent friction and z_T is the vertical scale of surface roughness. Let us discuss this term in greater detail.

Air pressure p at the Earth's surface is approximately equal to the weight of atmospheric column. According to the equation of state it can be written as $p = \rho g h = 10^5$ J m^{-3} (1 J m^{-3} = 1 N m^{-2}), $h = RT/Mg = 8.4$ km is the atmospheric exponential scale height, $R = 8.3$ J mol^{-1} K^{-1} is the universal gas constant, $M = 29$ g mol^{-1} is molar

ACPD
8, S11826–S11832, 2009
Interactive Comment

mass of air. Due to surface roughness, horizontal air movement leads to formation of turbulent eddies at the surface. The energy of these eddies is substracted from the kinetic energy of the mean horizontal air flow, resulting in its slow down. Interaction of turbulent eddies with the surface and with each other leads to their dissipation into yet smaller eddies and ultimately to dissipation to heat. Energy of the turbulent eddies is proportional to atmospheric pressure and to linear size z_T of surface roughness. By analogy to the first term in Eq. (2), F_{T0} can then be written as

$$F_{T0} = \rho g z_T = \mu \rho g h, \;\; \mu \equiv z_T/h. \tag{6}$$

Since on the global average $F_T = F_A \gg C_D \rho u^2$, Eq. (4), this means that $F_A \sim \rho g z_T$. Eq. (4) in this case gives $z_T \sim 0.2$ m and $\mu \sim 2 \times 10^{-5}$. In other words, compared to the case of the motion of solid bodies along a rough surface that is characterized by $\mu \sim 1$, Eq. (2), surface turbulent friction for air moving along a rough surface is, as is well-known, vanishingly small (i.e., if the atmospheric column of the same weight were a solid body, its movement along the Earth's surface would be characterized by $\mu \sim 1$ instead of $\mu \sim 2 \times 10^{-5}$). However, the absolute value of surface turbulent friction F_{T0} appears large enough for it to be the major source of resistance in large-scale circulation patterns where horizontal wind velocities of the order of several meters per second are maintained along the major part of the horizontal streamline of the order of 10^3 km:

$$F_A = F_T \approx f_{T0} \gg F_{Ta}. \tag{7}$$

Note that z_T is not related to the vertical profile of wind velocity (logarithmic or any other) near the surface, but, instead, represents an independent linear scale corresponding to the actual linear size of surface inhomogeneities. Therefore, if one chooses an area with z_T much smaller than the mean value for which Eq. (7) holds, then over such a surface term F_{T0} may become negligible. Then the only term that can be empirically measured on such an area will be F_{Ta} (see, e.g., Sheppard, 1947 on C_D measurements on a smooth concrete field with $z_T \sim 10^{-3}$ m $\ll 0.2$ m).

ACPD
8, S11826–S11832, 2009

Interactive Comment

Let us now explore the nature of the aerodynamic turbulent friction in the atmospheric column, $F_{Ta} = C_D \rho u^2$. It can be written in the standard form involving turbulent kinematic viscosity ν_T as $F_{Ta} = \nu_T du/dz$, where z is height above the surface. Assuming that $\nu_T = h_{\mathrm{H_2O}} w$ (Makarieva & Gorshkov, 2007), where w is vertical wind velocity and $h_{\mathrm{H_2O}} \approx 2$ km is the exponential scale height of the water vapor distribution, and taking into account that $du/dz \sim u/h_a \sim u/h_{\mathrm{H_2O}}$, we have $F_{Ta} = \rho w u$. Integral continuity equation for circulation of horizontal length L and height h_a reads as

$$uh_a = wL. \tag{8}$$

From this equation and considering that, on the other hand, $F_{Ta} = C_D \rho u^2$, we derive an order of magnitude estimate for drag coefficient

$$C_D = ch_a/L, \quad c \sim 1. \tag{9}$$

This theoretical estimate agrees, in the order of magnitude, with the available observational evidence. At global mean velocity $\overline{u} = 7$ m s^{-1} the observations give a mean $C_D \approx 1.2 \times 10^{-3}$ (Garratt, 1977). Taking mean cyclone radius $L \approx 600$ km (Simmonds, 2000), we obtain from Eq. (8) $c \approx 0.5$ and

$$C_D = h_a/2L. \tag{10}$$

Aerodynamic turbulent friction $f_{Ta} = C_D \rho u^2 = \rho u w/2$ can thus be interpreted as kinetic energy of turbulent eddies that are characterized by mean velocity $v_e = \sqrt{uw}$.

Change of horizontal velocity u along the streamline l is determined by Euler equation

$$\frac{1}{2}\rho\frac{\partial u^2}{\partial l} + \frac{\partial p}{\partial l} + \frac{F_T}{h_a} = 0. \tag{11}$$

Here F_T/h_a is the mean force of turbulent friction acting on unit air volume in the part of atmospheric column $z \le h_a$, F_T is defined in Eq. (6).

ACPD
8, S11826–S11832, 2009
Interactive Comment

Taking into account that along the entire streamline $\Delta p \ll p \approx \rho g h$, one can neglect the change of air density ρ along the streamline putting $\partial p/\partial l = \Delta p/L$, where L is length of the horizontal part of the streamline. We can now determine horizontal dimension $L = \overline{L}$ of such a circulation pattern where acceleration is absent $(\partial u^2/\partial l = 0)$ and velocity $u = \overline{u}$ is constant over the major part of the streamline. From Eqs. (1), (5), (8) and (11) we have

$$\overline{L} = h_a \frac{\Delta p}{\rho g z_T}, \quad \text{or} \quad \overline{L} = h_a \frac{\overline{u}}{\overline{w}}, \quad \overline{u} = \overline{w} \frac{\Delta p}{\rho g z_T}, \tag{12}$$

where $\overline{w}$ is the mean vertical velocity of ascending air that can be determined from the rate of precipitation.

In circulation patterns where length L is smaller than $\overline{L}$ (12) equations (1) and (12) do not hold. From Eq. (11) we then obtain the following relationship for the increase of radial velocity u:

$$\frac{1}{2}\rho \frac{\partial u^2}{\partial l} = -\frac{\Delta p}{L} \gg F_T, \quad \text{or} \quad u^2 = \frac{\Delta p}{L}\frac{2l}{\rho}, \quad l \leq L_0 < L. \tag{13}$$

Radial velocity ceases to grow ($\partial u^2/\partial l = 0$) at a distance $l = L_0$, where the aerodynamic resistance F_{Ta} (5), (10), grows up to F_A (1). In this case from Eq. (11) we have

$$\frac{\Delta p}{L} = \frac{F_{Ta}}{h_a} = \frac{1}{2}\rho \frac{u^2}{L_0}, \quad \text{or} \quad u^2 = \frac{\Delta p}{L}\frac{2L_0}{\rho}, \quad L_0 \leq l < L. \tag{14}$$

This coincides with the second equality in Eq. (13) at $l = L_0$. Equation (15) means that the aerodynamic turbulent friction (5), (10) coincides with the accelerating pressure gradient force and sets the limit to the increase of wind velocity. Since $L - L_0 \ll L$, in a good approximation we have $1/2\rho u^2 = \Delta p$, see Eq. (A6) in the revised manuscript (ACPD 2009 8: S11275-S11289).

Within the biotic pump theory at $L \gg h_a$ the horizontal pressure difference Δp in the circulation pattern driven by the evaporative force is related to the vertical non-equilibrium

ACPD

8, S11826–S11832, 2009

Interactive Comment

pressure difference Δp_v of water vapor as $\Delta p \approx \Delta p_v = f_E h_{H_2O} = p_v \times 0.82$, where f_E is given by Eq. (16) of Makarieva & Gorshkov (2007), p_v is partial pressure of water vapor at the surface.

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Interactive comment on Atmos. Chem. Phys. Discuss., 8, 17423, 2008.