

## ***Interactive comment on “On the validity of representing hurricanes as Carnot heat engine” by A. M. Makarieva et al.***

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### **REVISED MANUSCRIPT, part II**

#### **4. Water vapor condensation as driver of atmospheric circulation**

##### **4.1 Partial pressure of water vapor as store of potential energy**

We will now outline a perspective of a quantitative unifying physical description of intense circulation events like hurricanes and tornadoes as adiabatic processes involving gas-liquid phase transitions<sup>1</sup> Briefly, during condensation, water vapor disappears from the gas phase; in the result, local air pressure drops; this leads to the appearance of

<sup>1</sup> Gorshkov, V. G. and Makarieva, A. M.: The osmotic condensational force of water vapor in the terrestrial atmosphere, Preprint 2763, Petersburg Nuclear Physics Institute, Gatchina, 43 pp., available at: <http://www.bioticregulation.ru/2763.php>, 2008.

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the wind-inducing pressure gradient force proportional in magnitude to the amount of water vapor in the atmosphere. The volume-specific store of potential energy responsible for hurricane formation can be thus estimated as the value of partial pressure  $p_{\text{H}_2\text{O}}$  of saturated water vapor. (Vertical distribution of saturated partial pressure  $p_{\text{H}_2\text{O}}$  departs significantly from the aerostatic equilibrium; at any height  $p_{\text{H}_2\text{O}}$  is over five times larger than the weight of water vapor column above this height (Makarieva et al., 2006; Makarieva and Gorshkov, 2007). For this reason practically all water vapor ascending in the hurricanes undergoes condensation, so the condensational potential energy coincides with  $p_{\text{H}_2\text{O}}$  to a good approximation.)

According to Bernoulli's equation, potential energy  $p_{\text{H}_2\text{O}}$  ( $\text{J m}^{-3}$ ) is transformed to kinetic energy  $\rho u_{\text{max}}^2/2$  ( $\text{J m}^{-3}$ ) of air masses having density  $\rho$  and moving at velocity  $u_{\text{max}}$  as  $p_{\text{H}_2\text{O}} = \rho u_{\text{max}}^2/2$ . At  $\gamma \equiv p_{\text{H}_2\text{O}}/p = 0.04$  (at 30 °C) and  $\gamma = 0.08$  (at 40 °C on land) (Bolton, 1980), moist air pressure  $p = 10^5$  Pa and  $\rho = 1.2$   $\text{kg m}^{-3}$  we have  $u_{\text{max}} = 80$   $\text{m s}^{-1}$  and  $u_{\text{max}} = 120$   $\text{m s}^{-1}$ , respectively. These upper limit theoretical estimates agree with observations of maximum wind velocities observed in hurricanes and tornadoes (Zrnić and Istok, 1980; Samsury and Zipser, 1995; Wurman et al., 1996; Businger and Businger, 2001). The outlined approach also explains the pronounced dependence of hurricane's intensity on surface temperature and predicts that maximum hurricane intensity should grow exponentially with increasing surface temperature, following the exponential temperature dependence of saturated partial pressure of water vapor. Thus, with temperature increasing from 20 to 30 °C, the maximum amount of potential energy available for conversion into the kinetic energy of air masses increases from  $p_{\text{H}_2\text{O}} \sim 2 \times 10^3$  Pa =  $2 \times 10^3$   $\text{J m}^{-3}$  to  $4 \times 10^3$   $\text{J m}^{-3}$ . Below we provide a more detailed outline of the framework.

#### 4.2 Evaporative-condensational pressure gradient force

Condensation of water vapor lowers air pressure and creates a pressure gradient force. What are the physical determinants of this force? In contrast to dry air, atmospheric water vapor finds itself under the action of two independent physical factors. On the

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one hand, as all other air gases, water vapor tends to aerostatic equilibrium described by the equation  $\partial p_v / \partial z - \rho_v(z)g = 0$ , where  $p_v$  is partial pressure of water vapor,  $\rho_v$  is its mass density. Using the equation of state for ideal gas  $\rho_v / M_v = p_v / (RT)$ ,  $M_v = 18 \text{ g mol}^{-1}$  is water vapor molar mass,  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$  is the universal gas constant,  $T$  is absolute temperature, the condition for aerostatic equilibrium of water vapor can be written as

$$\frac{1}{p_v(z)} \frac{\partial p_v}{\partial z} = -\frac{1}{h_v(z)}, \quad h_v(z) \equiv \frac{RT(z)}{M_v g}, \quad h_v(0) = 13.5 \text{ km}. \quad (1)$$

Eq. (12) means that in aerostatic equilibrium (Makarieva and Gorshkov, 2007) partial pressure  $p_v(z)$  of water vapor diminishes exponentially with height, by  $e$  times per each 13.5 km or twice per each 9 km.

On the other hand, partial pressure of water vapor cannot exceed the saturated partial pressure  $p_{\text{H}_2\text{O}}(z)$ ,  $p_v(z) \leq p_{\text{H}_2\text{O}}(z)$ , the latter depending on temperature  $T(z)$  as dictated by Clausius-Clapeyron equation. This equation can be written in a form similar to that of Eq. (12):

$$\frac{1}{p_{\text{H}_2\text{O}}(z)} \frac{\partial p_{\text{H}_2\text{O}}(z)}{\partial z} = -\frac{1}{h_{\text{H}_2\text{O}}(z)}, \quad h_{\text{H}_2\text{O}}(z) \equiv h_v(z) \frac{\Gamma_{\text{H}_2\text{O}}(z)}{\Gamma}, \quad h_{\text{H}_2\text{O}}(0) = 2.4 \text{ km}. \quad (2)$$

$$\Gamma \equiv -\frac{dT(z)}{dz}, \quad \Gamma_{\text{H}_2\text{O}}(z) \equiv \frac{T(z)M_v g}{L}, \quad \Gamma_{\text{H}_2\text{O}}(0) \equiv \Gamma_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1}. \quad (3)$$

Here  $L$  is molar vaporization heat of water vapor (latent heat). Note that for the circulation patterns to be considered, the above use of Clausius-Clapeyron equation is the single connection with thermodynamics.

It follows immediately from Eqs. (12) and (13) that saturated water vapor can only be in aerostatic equilibrium when  $h_v(z) = h_{\text{H}_2\text{O}}(z)$ , i.e.  $\Gamma = \Gamma_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1}$  (Makarieva and Gorshkov, 2007). At  $\Gamma > \Gamma_{\text{H}_2\text{O}}$  saturated water vapor cannot be in equilibrium. The

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difference between the right-hand and the left-hand parts of Eq. (12) is then not equal to zero and represents an upward force  $f_E$  acting on a unit mass of moist air:

$$f_E(z) = \frac{\partial p_{\text{H}_2\text{O}}(z)}{\partial z} - g\rho_{\text{H}_2\text{O}}(z) = \frac{p_{\text{H}_2\text{O}}(z)}{h_{\text{H}_2\text{O}}(z)} \left( 1 - \frac{\Gamma_{\text{H}_2\text{O}}(z)}{\Gamma} \right). \quad (4)$$

The observed tropospheric mean temperature lapse rate is  $\Gamma = 6.5 \text{ K km}^{-1}$ , which is six times a larger value than  $\Gamma_{\text{H}_2\text{O}}$ , Eq. (14). Therefore the last term,  $\Gamma_{\text{H}_2\text{O}}/\Gamma$ , in brackets of Eq. (15) is only 1/6.

Work  $A_E = \int_0^\infty f_E(z)dz$  performed by force  $f_E$  when raising a unit volume of moist air along the entire atmospheric column is, to the accuracy of a few per cent neglecting the change of absolute temperature in the lower part of the atmosphere up to  $h_{\text{H}_2\text{O}}$ ,  $[(T(z) - T(0))/T(0) \leq \Gamma h_{\text{H}_2\text{O}}/T(0) = 0.05]$ , equal to

$$A_E = f_E(0)h_{\text{H}_2\text{O}}(0) = p_{\text{H}_2\text{O}}(0) \left( 1 - \frac{\Gamma_{\text{H}_2\text{O}}}{\Gamma} \right) = \Delta p \equiv \rho \frac{u_E^2}{2}. \quad (5)$$

Velocity  $u_E$  has the meaning of vertical velocity of air that has been accelerated by force  $f_E$  along the entire atmospheric column. Force  $f_E$  and work  $A_E$  are present everywhere in the atmosphere where there is saturated water vapor. Since force  $f_E$  arises due to condensation of water vapor in the atmosphere, which can only be sustained if there is a compensating process of evaporation from the hydrosphere, it is logical to term this force as the evaporative-condensational force or E-force, for brevity. Note that since  $\Gamma_{\text{H}_2\text{O}}/\Gamma = 1/6$ , work  $A_E$  does not depend on latent heat to the accuracy of 17%, Eq. (16).

Note that in the process of evaporation latent heat  $L$  is spent on the dissociation of bonds between molecules of liquid water,  $L_v$ , and on latent work that is performed to place the evaporated molecules within the moist air volume. Latent work per unit mol of saturated water vapor is  $p_{\text{H}_2\text{O}}v_{\text{H}_2\text{O}} = RT$ , so that  $L = L_v + RT$ . When water vapor undergoes condensation, the release of heat corresponding to bond energy  $L_v$

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and dissipation of latent work  $RT$  occurs on different time scales: on a microscopic time scale for the release of  $L_v$  in the process of liquid water formation and on a much longer macroscopic time scale corresponding to dissipation of kinetic energy that arises from latent work  $RT$  (Makarieva et al., 2008).

The moist atmosphere of Earth, which exists in contact with liquid hydrosphere in the presence of a supracritical vertical lapse rate of air temperature  $\Gamma > \Gamma_{\text{H}_2\text{O}}$ , is not in equilibrium. (As such, it cannot be characterized by the notions of stable, unstable or neutral equilibrium.) Force  $f_E$  that arises due to the absence of equilibrium causes the atmosphere to circulate continuously along a variety of linear scales. The associated horizontal mixing of atmospheric layers with different water vapor content results in the fact that the global mean relative humidity  $R_H$  at the surface becomes less than unity,  $R_H \sim 0.8$  (Held and Soden, 2000). This means that water vapor becomes saturated starting from some height  $z_H \sim 500$  m. This does not change the value of  $A_E$  in any considerable way.

The physics of the evaporative-condensational force can be interpreted as a very peculiar case of the well-known phenomenon of osmosis. The nature of osmosis consists in the fact that partial pressures of particular constituents of gas mixtures (or liquid solutions) tend to spatial homogeneity independently of each other (Dalton's law). Consider two mixtures with different concentrations of various constituents that are separated by a semipermeable membrane. If this membrane impedes spatial propagation of one of the constituents and prevents it from reaching the equilibrium distribution, then the resulting equilibrium distribution of partial pressures of other constituents will be associated with a pressure gradient across the membrane. If the membrane is removed, the dynamic fluxes of liquid or gas will follow governed by the osmotic pressure gradient until the mixture pressures and concentrations of all constituents in the two areas equate. In the atmosphere, the role of semipermeable membrane of a unique nature is played by the vertical temperature gradient – it selectively removes, via condensation, one of the gases from the mixture (water vapor). At the same time, lacking material essence, this unusual "membrane", unlike the conventional osmotic membrane, is penetrable to

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the dynamic flow of mixture as a whole, sustaining continuous air circulation.

Depending on the rate at which condensation of water vapor occurs, the resulting atmospheric circulation phenomena can be either stationary or episodic. In hurricanes and tornadoes the rate of water vapor condensation is determined by the maximum velocity  $u_E$  of air masses accelerated by force  $f_E$ ; condensation rate exceeds the evaporation rate dictated by solar power by many orders of magnitude. The stationary pattern is therefore composed of long periods of relative calmness (when water vapor slowly accumulates in the atmosphere) intermitted by intense hurricane or tornado events accompanied by rapid condensation of the previously accumulated water vapor.

However, under conditions when the turbulent surface friction is sufficient to impede acceleration of air masses, a stationary state is possible when the rate of condensation is equal to the rate of evaporation at any moment of time. Such a situation arises when there are two large adjacent areas of comparable linear size of the order of several thousand kilometers and when the evaporation rate in one of the two areas (the acceptor area) is, for some physical reason, consistently higher than in the other (the donor area). In this case there forms a large-scale atmospheric circulation pattern with surface air flowing from the donor to acceptor area. This can be called the evaporative-condensational pump. This horizontal air flow compensates the ascending and descending motion of air masses in the acceptor and donor areas, respectively. Essentially, in this case pressure difference  $\Delta p$ , Eq. (16), is distributed along the total length of the streamline, which horizontal part is three orders of magnitude larger than the vertical one (atmospheric height) (see Appendix A and below). For this reason, the non-equilibrium pressure difference in the vertical dimension of the streamline becomes thousand times smaller than  $\Delta p$ , Eq. (16), which prohibits the origin of hurricanes and tornadoes (as well as of other circulation irregularities associated with floods and droughts (Makarieva and Gorshkov, 2007)). Such large regions with inherently different evaporation rates can be exemplified by large river basins covered by natural forests (acceptor) + the adjacent ocean (donor); ocean (acceptor) and adjacent desert (donor), as well as by the part of Hadley circulation cell in the Intertropical Convergence

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Zone. This explains, for example, why there are no hurricanes either in the Amazon basin or in the oceanic region immediately adjacent to it (Nobre, 2008).

Hurricanes and tornadoes arise when the linear size of the donor and acceptor areas diminishes to the values when the power of turbulent surface friction becomes sufficiently smaller than the power of the evaporative-condensational force. Consider the following analogy (be warned that it has its limitations). A diesel locomotive pulls a train consisting of 50 carriages at a constant low velocity. In this stationary case the total pulling power of the locomotive is equal to 50 times the power of friction between each carriage and the railway. This is comparable to a stationary large-scale atmospheric circulation. Now if one detached 49 carriages from the train and left the locomotive to haul only one carriage, the locomotive power would appear much greater than the diminished friction power. In the result, the train would move with acceleration until all the fuel were used up. The resulting velocity would be much higher than in the first case of a long train. This situation could be compared to hurricane or tornado. The locomotive pulling force, which is equivalent to the evaporative-condensational force in the atmosphere, is the same in both cases, yet it produces fundamentally different patterns of motion. Below we consider these patterns in quantitative terms.

#### 4.3 Tornado, hurricane and large-scale stationary circulation

The simplest case, when work  $A_E$  is released and the locally accumulated potential energy per unit volume  $p_{\text{H}_2\text{O}} \approx \Delta p$  is converted to kinetic energy  $\rho u_E^2/2$ , is the case when intense condensation of water vapor occurs in a local area surrounded by dry areas where water vapor concentration is very low, like when tornadoes develop in the semi-deserts of North America. In this case the ascending air flow in the condensation area sucks in dry air masses from the neighboring areas. The converging air streams approach the condensation area with some non-zero angular momenta, which inevitably results in a spiral-like rotating structure of the tornado. This structure is therefore an immediate consequence of the vertical force  $f_E$  acting in the three-dimensional space. Tornado further moves in the direction of maximum water vapor concentration. When all local water vapor is used up, tornado is extinguished, since the neighboring areas are dry and cannot supply more water vapor. The next tornado occurrence will be due to a moment when, after a prolonged period of water vapor accumulation in the

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atmosphere, moist air masses again become spatially concentrated amidst the otherwise dry area.

We now consider two extensive (oceanic and/or land) regions where water vapor is present in the atmosphere in approximately equal quantities, but condensation in one region, termed here the acceptor region, is more intense than in the other (donor) region. The ascending air flow caused by condensation in the acceptor region leads to the inflow of moist air masses from the donor region; the imported water vapor serves to sustain the condensation process in the acceptor region. We denote the approximately equal lengths of acceptor and donor regions for  $L_E$ , their width (equal to the length of the border between them) for  $D$ , vertical velocity of air masses ascending within the donor region for  $w$ , horizontal air velocity for  $u$ . The mass-conserving equality between the horizontal air flux entering the donor region via vertical atmospheric cross-section  $Dh_{\text{H}_2\text{O}}$  with velocity  $u$  ( $h_{\text{H}_2\text{O}}$  is the scale height of the vertical distribution of atmospheric water vapor, see Eq. (13), and, hence, describes the part of the atmospheric column where the evaporative-condensational force  $f_E$  is in action) and the vertical air flux leaving the donor region via horizontal atmospheric area  $DL_E$  (the integral continuity equation) reads as

$$L_E w = h_{\text{H}_2\text{O}} u. \quad (6)$$

In order to understand the structure of atmospheric circulation in these regions it is necessary to take into account the turbulent friction forces that impede acceleration of air masses (that would otherwise occur under the action of force  $f_E$  and work  $A_E$ ) and lead to the formation of a stationary circulation pattern.

Total turbulent friction force  $f_T$  consists of two parts: the one independent of wind velocity – this part is determined by weight of atmospheric column proportional to  $\rho g$ , and the one dependent on vertical velocity of air masses ascending within the atmospheric column. From the dimensional considerations  $f_T$  acting on unit air volume can be written as

$$f_T = \frac{\rho g z_T}{2h_{\text{H}_2\text{O}}} + \frac{\rho w^2}{2h_{\text{H}_2\text{O}}}. \quad (7)$$

Here  $z_T$  is the characteristic height up to which the horizontal air flow is influenced by surface roughness. The value of  $\rho g z_T / 2$  represents turbulent friction force per unit surface area (turbulent friction pressure, multiplier  $1/2$  is introduced for convenience). The corresponding force per unit air volume averaged over the atmospheric column is therefore  $\rho g z_T / 2h_{\text{H}_2\text{O}}$ , Eq. (18).

The second term describes power loss of the streamflow due to formation of turbulent eddies. In similarity to Eq. (16), the second term represents force which work is equal to kinetic energy  $\rho w^2/2$  of an eddy budding from the main flow and developing along the atmospheric height  $h_{\text{H}_2\text{O}}$  (Appendix B).

The main horizontal streamflow that enters the acceptor region via a vertical cross-section of area  $\sim Dh_{\text{H}_2\text{O}}$  with velocity  $u$  and propagates in the surface atmospheric layer below  $z \sim h_{\text{H}_2\text{O}}$ . Governed by the continuity equation, Eq. (11), within the acceptor area this flow changes its direction and is transformed into vertical flow moving with velocity  $w$  via horizontal area  $L_E D$ . Then, at heights greater than  $z \sim h_{\text{H}_2\text{O}}$ , it turns again into a reverse horizontal flow moving at velocity  $\sim u$  via vertical area  $\sim Dh_{\text{H}_2\text{O}}$ , to ultimately undergo descent somewhere in the donor region. All the different-sized eddies formed by turbulent friction continue to travel together with the main streamflow (first horizontally along the surface towards the acceptor area, then upward in the acceptor region, then horizontally back towards the donor area and, ultimately, downward in the donor area) until their complete dissipation to heat.

The circulation is stationary in strict terms when the rate of condensation is equal to the rate of evaporation sustained by solar power for any period of time and when the power of evaporative-condensational force  $f_E$  is equal to the power of turbulent friction force  $f_T$ . These conditions can be written for work  $A_E$  or power  $W_E$ , see Eq. (17):

$$W_E = f_E \bar{w} = f_T \bar{u}, \quad A_E = f_E h_{\text{H}_2\text{O}} = f_T L_E. \quad (8)$$

Here vertical velocity  $w = \bar{w}$  is the average vertical velocity of upwelling moist air masses that corresponds to the mean evaporation rate from the surface,  $\bar{w} \sim 10^{-3} \text{ m s}^{-1}$  (Makarieva and Gorshkov, 2007). Due to the very small magnitude of  $\bar{w}$  the last term in Eq. (18) can be neglected. Along the horizontal part of the streamline turbulent friction force  $f_T$  remains therefore constant and independent of velocity (Appendix B), so its work  $f_T L_E$  grows linearly proportionally to length  $L_E$  of the acceptor region. Using Eq. (19), for average horizontal velocity  $\bar{u}$  and linear dimension of the acceptor region  $L_E$  we have

$$\bar{u} = K \bar{w}, \quad L_E = K h_{\text{H}_2\text{O}}, \quad K \equiv \frac{f_E}{f_T} = \frac{u_E^2}{gz_T}. \quad (9)$$

In the expression for the dimensionless coefficient  $K$  all magnitudes except for  $z_T$  are quantified theoretically, see Eqs. (13) and (16). Height  $z_T$  can be determined phenomenologically by

comparing Eq. (20) with the available empirical data. Global mean wind velocity estimates as  $\bar{u} \sim 7 \text{ m s}^{-1}$  (Gustavson, 1979) and  $\bar{w}$  as  $\bar{w} \approx 1.3 \text{ mm s}^{-1}$  (Makarieva and Gorshkov, 2007), which gives  $K \approx 5400$  in Eq. (20). (For  $u_E = 60 \text{ m s}^{-1}$  we have  $z_T \sim 0.1 \text{ m}$ .) Finally, using Eqs. (20) and (13), we have  $L_E \sim 10^4 \text{ km}$ . This approximate estimate shows that, in order to enjoy a stable stationary circulation continuously sustained by evaporation and solar radiation, the donor and acceptor regions must be very large.

Note an essential detail. According to Eqs. (19) and (16) the turbulent friction force is equal to  $f_T = \Delta p / L_E$ . In the stationary case, when the horizontal acceleration is absent, pressure gradient force should be equal to turbulent friction force. This means that pressure difference  $\Delta p$  is uniformly distributed along the horizontal part of the streamline with a constant pressure gradient equal to  $\Delta p / L_E = f_T = \rho g z_T / 2 h_{\text{H}_2\text{O}}$ . As the horizontal part of the streamline is thousand of times longer than the vertical part,  $L_E \sim 10^3 h_{\text{H}_2\text{O}}$ , the non-equilibrium pressure difference along the vertical part of the streamline is thousand of times smaller than the total pressure difference  $\Delta p$  (see also Appendix A). In the considered case the deviation from hydrostatic equilibrium for air as a whole is negligibly small.

With decreasing linear size  $L_E$  of the donor and acceptor regions work  $f_T L_E$  of the turbulent friction force diminishes and becomes smaller than work  $f_E h_{\text{H}_2\text{O}}$  of the evaporative-condensational force. Work of the turbulent friction force becomes negligibly small when  $L_E$  diminishes by one order of magnitude as compared to the stationary value and becomes less than  $10^3 \text{ km}$ . This is the spatial domain for hurricane development:

$$h_{\text{H}_2\text{O}} \sim 2 \text{ km} \ll L_E < 10^3 \text{ km}. \quad (10)$$

In the view of Eq. (18), vertical velocity  $w$  becomes tens and hundreds of times larger than the stationary value  $\bar{w}$ . The rate of condensation is, unlike in the stationary case, no longer related to the evaporation rate. However, since  $L_E \gg h_{\text{H}_2\text{O}}$  and, hence,  $w \ll u$ , the major part of pressure difference  $\Delta p$  that appears due to water vapor condensation still falls on the horizontal part of the streamline. Note that the velocity-dependent turbulent friction force described by the second term in Eq. (18) remains negligibly small within the hurricane due to  $w \ll u$ . (This term becomes the major one in tornado at  $w \sim u$ , where it may limit the stationary value of velocity.) Hurricane intensity is limited not by the turbulent friction force, but by the inertial forces of air masses that are sucked in towards the hurricane center from the neighbouring areas. Radial

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velocity  $u$  developed near the windwall is determined by Bernoulli's equation (Appendix A):

$$\rho \frac{u^2}{2} = \Delta p = p_{\text{H}_2\text{O}} \left( 1 - \frac{\Gamma_{\text{H}_2\text{O}}}{\Gamma} \right). \quad (11)$$

As the converging air masses approach hurricane center they curve as dictated by their initial angular momenta and Coriolis acceleration. This leads to a non-uniform distribution of pressure gradient along the streamline and results in the formation of the hurricane eye where air pressure drops towards the center compensating the centrifugal forces (e.g., Holland, 1980). However, the primary determinant of the hurricane wind structure is the pressure difference  $\Delta p$ , Eq. (22), related to condensation of water vapor. Hurricane arises in the region of maximum intensity of water vapor condensation. Hurricane intensity is therefore largely determined by the intensity of condensation of water vapor accumulated within the very area of hurricane development rather than by the intensity of water vapor import from the neighboring areas.

Finally, when horizontal size  $L_E$  diminishes to the values of the order of atmospheric height  $h_{\text{H}_2\text{O}}$ , there appears a possibility of tornado formation, as discussed above.

The outlined framework also allows one to determine velocity  $U$  of hurricane and tornado movement. Let us denote linear dimensions of the donor and acceptor regions as  $L_d$  and  $L_a$ , respectively. Since the source of kinetic energy of hurricanes and tornadoes is the potential energy accumulated in the form of partial pressure of water vapor, the condition for calculating  $U$  is that as soon as the local store of water vapor in the donor region is depleted, the wind structure must either dissipate or move to another area. Total store of water vapor in the donor region is  $S_d = N_{\text{H}_2\text{O}}(L_d^2 h_{\text{H}_2\text{O}})$ , where  $N_{\text{H}_2\text{O}} = p_{\text{H}_2\text{O}}/(RT)$  is molar density of atmospheric water vapor ( $\text{mol m}^{-3}$ ). Rate of water vapor "production" (via evaporation) per unit surface area is  $N_{\text{H}_2\text{O}}\bar{w}$ , where  $\bar{w} \sim 10^{-3} \text{ m s}^{-1}$  is the upward velocity corresponding to global mean evaporation rate (Makarieva and Gorshkov, 2007). The rate at which water vapor is "spent" (undergoes condensation) within the acceptor region is given by  $W_a = N_{\text{H}_2\text{O}}(w - \bar{w})L_a^2$ , where  $w$  is vertical wind velocity in the acceptor region occupied by considered the wind structure (hurricane, tornado). Time  $\tau$  during which all water vapor accumulated in the donor region is "spent" in the acceptor region is given by  $\tau = S_d/W_a = (L_d^2 h_{\text{H}_2\text{O}})/[(w - \bar{w})L_a^2]$ . During this time the wind structure should either cease to exist or move to the neighboring donor region (if such exists). The resulting velocity of its movement will be  $U = L_d/\tau = (w - \bar{w})L_a^2/(L_d h_{\text{H}_2\text{O}})$ . In the stationary case of a large-scale circulation, when  $L_d \approx L_a \equiv L_E$ , the circulation remains immobile, as

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$w = \bar{w}$  and  $U = 0$ . For a hurricane with  $w \sim 0.1 \text{ m s}^{-1} \gg \bar{w}$  we have  $U = wL_a^2/(L_d h_{\text{H}_2\text{O}})$ . Taking  $L_d \sim 10^3 \text{ km}$ ,  $L_a \sim 4 \times 10^2 \text{ km}$ ,  $h_{\text{H}_2\text{O}} \sim 2 \text{ km}$ , we obtain  $U = 2 \text{ m s}^{-1}$ . Conversely, knowing vertical velocity  $w$  of air masses in the hurricane, velocity of hurricane movement  $U$  and hurricane linear scale  $L_a$ , it is possible to estimate the linear dimension of the donor region  $L_d = (w/U)L_a^2/h_{\text{H}_2\text{O}}$ . For tornado with  $w \sim u \sim 10^2 \text{ m s}^{-1}$  and  $L_a \sim L_d \sim h_{\text{H}_2\text{O}}$  we have  $U \sim u$ , where  $u$  is the horizontal wind speed.

## 5. Conclusions

Hurricanes and tornadoes can be compared to an explosion reversed and prolonged in time. In the ordinary explosion potential energy concentrated in the explosion center is released in a burst, making local air pressure rise sharply and causing dynamic air movement in the direction away from the explosion center. Conversely, condensation of saturated water vapor within the column of ascending air in hurricanes and tornadoes leads to a sharp drop of local air pressure. This further enhances the ascending motion of yet accelerating air masses, as well as the compensating radial fluxes of moist air incoming to the area where the process of condensation is most intensive. Water vapor contained in the incoming air undergoes condensation in the same area; this sustains the pressure difference between the hurricane center and its environment. Hurricane could also be compared to a black hole, which sucks the surrounding air into the center, where it partially "annihilates" due to condensation of water vapor and its disappearance from the gas phase. Thus, hurricane is an "anti-explosion". While in explosion the gas phase appears from either liquid or solid phase, in hurricanes and tornadoes, conversely, the gas phase of water vapor partially disappears from air due to condensation.

Unlike in explosion, the velocity of air masses in hurricanes and tornadoes is significantly lower than the velocity of thermal molecular motion. In consequence, all air volumes are in thermodynamic equilibrium, so that air pressure, temperature and density within the hurricane conform to equilibrium thermodynamics. The driving force of all hurricane processes is a *rapid* release, as in compressed spring, of potential energy previously accumulated in the form of saturated water vapor in the atmospheric column during a *prolonged* period of water vapor evaporation under the action of the absorbed solar radiation. Since the power of the practically instantaneous energy release in the hurricane greatly exceeds the power of energy exchange with the environment, all hurricane processes can be described as adiabatic. The outlined approach predicts that winds can develop anywhere in the atmosphere (over land as well as over the ocean), where absolute humidity is high and the process of condensation is spa-

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tially non-homogeneous. It thus provides a unifying theoretical framework for understanding both hurricanes and tornadoes, as well as the large-scale stationary atmospheric circulation on Earth, including biotic pump of atmospheric moisture in the forested river basins (Makarieva and Gorshkov, 2007).

#### Appendix A Spatial distribution of pressure difference $\Delta p$

Euler equation for the horizontal part of the streamline  $d\mathbf{l}$ , which is parallel to horizontal velocity  $\mathbf{u}$ , has the following form:

$$\frac{1}{2}\rho\frac{\partial\mathbf{u}^2}{\partial\mathbf{l}} + \frac{\partial p}{\partial\mathbf{l}} + f_T\frac{\mathbf{u}}{u} = 0. \quad (\text{A1})$$

For the vertical part of the streamline Euler equation becomes

$$\frac{1}{2}\rho\frac{\partial\mathbf{w}^2}{\partial z} + \frac{\partial p}{\partial z} + \rho g = 0. \quad (\text{A2})$$

In the stationary case, when air masses move with constant horizontal velocity  $u$  along distance  $L_E$  from the donor region with lower evaporation rate to the acceptor region with higher evaporation rate characterized by vertical air velocity  $\bar{w}$ , we have, see Eq. (18):

$$\frac{\partial\mathbf{u}^2}{\partial\mathbf{l}} = 0, \quad -\frac{\partial p}{\partial\mathbf{l}} = f_T = \text{const}, \quad w \sim \bar{w} = \frac{h_{\text{H}_2\text{O}}}{L_E}u \ll u. \quad (\text{A3})$$

From Eq. (A3) we have for the integral along the whole streamline

$$\Delta p = f_T L_E, \quad \int_0^{h_{\text{H}_2\text{O}}} \left( \frac{\partial p}{\partial z} + \rho g \right) dz = \frac{1}{2}\rho\bar{w}^2 \ll \Delta p = \frac{1}{2}\rho u_E^2. \quad (\text{A4})$$

That is, total pressure difference  $\Delta p$ , Eq. (16), that forms due to water vapor condensation, is distributed along the horizontal part of the streamline, see Eq. (19), while air along the vertical part, due to its small linear size, practically remains in hydrostatic equilibrium.

In the hurricane due to the smaller value of  $L_E$  the turbulent friction force is much smaller than the second term in Eq. (A1) and can be neglected. Euler equation for the horizontal part of the streamline takes the form

$$\frac{1}{2}\rho\frac{\partial\mathbf{u}^2}{\partial\mathbf{l}} + \frac{\partial p}{\partial\mathbf{l}} = 0. \quad (\text{A5})$$

Integral along the vertical part of the streamline still satisfies Eq. (A4) due to  $w = uh_{\text{H}_2\text{O}}/L_E \ll u$ . Therefore, integrating Eq. (A5) and taking into account the equation of state  $p = \rho gh$ ,  $h \equiv RT/(Mg)$ ,  $M = 29 \text{ g mol}^{-1}$  is air molar mass, we have ( $\Delta p \ll p$ ):

$$\frac{1}{2}\mathbf{u}^2 = -gh \ln \frac{p - \Delta p}{p} = -\frac{p}{\rho} \ln \left( 1 - \frac{\Delta p}{p} \right) = \frac{\Delta p}{\rho}, \quad (\text{A6})$$

which coincides with Bernoulli's equation for the incompressible liquid.

Finally, for tornado with  $L_E \sim h_{H_2O}$ , we have  $w \sim u$ , so that a major part of pressure difference  $\Delta p$  (8) falls on the vertical part of the streamline, Eq. (A2). This results in the maximum observed vertical velocities  $w \sim u_E$ , see Eqs. (15), (16).

### Appendix B Turbulent friction force

Here we give a more detailed derivation of Eq. (19) that includes turbulent friction force  $f_T$  (18). Power of the upward evaporative-condensational force is equal to  $\rho(u_E^2/2)wDL_E$  for the entire atmospheric column and  $\rho(u_E^2/2)w$  per unit area of the Earth's surface. Under the action of turbulent friction air eddies bud off from the main streamflow as it propagates along the surface. These eddies continue to move within the main streamflow filling the atmospheric column. Rotational velocity  $u_T$  and kinetic energy density  $\rho u_T^2/2$  of these small eddies does not depend on streamflow velocity  $u$ , but is determined by weight of atmospheric column and linear size  $z_T$  related to the characteristic roughness height of the surface,  $\rho u_T^2 = \rho g z_T$ . As the main stream passes along the surface, a new small eddy is formed every  $z_T/u$  seconds. This confines the total power of the turbulent friction force at the surface as  $\rho(u_T^2/2)(u/z_T)DLz_T$ , where  $DLz_T$  is the volume where the surface turbulent friction force is acting and the small eddies are formed. Power of turbulent friction force per unit surface area is then  $\rho(u_T^2/2)u$ .

Further above the surface the main horizontal streamflow is decelerated by the ascending flows having vertical velocity  $w$  that form eddies with linear size of the order of atmospheric scale height  $h_{H_2O}$ . New large eddies bud off from the main streamflow every  $h_{H_2O}/u$  seconds. Total power of turbulent friction force far from the planetary surface is  $\rho(w^2/2)(u/h_{H_2O})DLh_{H_2O}$ , where  $DLh_{H_2O}$  is the atmospheric volume where this force acts and the large eddies are formed; per unit surface this power is  $\rho(w^2/2)u$ . Equating the powers of the evaporative force and the total turbulent friction force one obtains

$$\frac{1}{2}\rho u_E^2 w = \frac{1}{2}\rho u_T^2 u + \frac{1}{2}\rho w^2 u \approx \frac{1}{2}\rho u_T^2 u, \quad u_T^2 = g z_T, \quad (B1)$$

which, given Eq. (18), coincides with Eq. (19).

Note that in the stationary case when the evaporative-condensational pump is in action, so that air flows without acceleration, total power  $W_E$  of the evaporative-condensational force  $f_E$  is spent on formation of turbulent eddies filling the atmospheric column as the streamflow propagates along the surface. In hurricanes and tornadoes power  $W_E$  significantly exceeds the power of turbulent eddies. It is converted to the power of the main streamflow, which results in high wind velocities.

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